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Trajectory Planning and Tracking of Bilinear Systems Using Orthogonal Functions

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Abstract: This paper proposes a trajectory planning and tracking approach for bilinear systems that approximate weakly nonlinear systems, based on orthogonal functions and especially the use of operational integration and product matrices. These operational tools allow the conversion of a bilinear differential state equation into an algebraic one depending on initial and final conditions. Arranging and solving the obtained algebraic equation lead to an open loop control law that allows the planning of a system trajectory. The parameters setting of the tracking state feedback closed loop control is yielded by considering a reference model characterizing the desired performances.

Keywords: *bilinear systems; trajectory planning; orthogonal functions; trajectory tracking.*

Mathematics Subject Classification (2000): 42C05, 05E35, 93B52, 93C10.

1 Introduction

Trajectory planning and tracking are linked subjects. Indeed, trajectory planning is finding an open loop control that permits to reach a final fixed state from a known initial state, and tracking is designing a closed loop control that ensures stability of system round its planned trajectory. These subjects have been considered by different approaches for stationary linear systems and particular classes of nonlinear systems [1]–[4]. Orthogonal functions were used as a powerful tool for systems study, identification [5, 6] and control

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[7, 8]. For this purpose, different orthogonal functions were used as Walsh [9] and Block-pulse [10] functions as well as Laguerre [11], Chebychev [12], Hermite [13] and Legendre polynomials [14]. The projection of the system differential equation on an orthogonal basis leads to an algebraic system representation that turns out to be more convenient for equation resolution especially for bilinear systems. In this work, we start by pointing out that a weakly nonlinear systems can be approximated by a bilinear system [15], and then we propose to use orthogonal functions properties with the aim to turn away the integration difficulty caused by trajectory planning and tracking for bilinear systems. We will point out that the algebraic form of system obtained by the orthogonal basis approximation and the use of tools offered by orthogonal functions such as operational matrix of integration and of product makes possible the characterization of a planned system trajectory and the synthesis of tracking state feedback control.

2 Bilinear Approximation of Weakly Nonlinear Systems

Consider a nonlinear system described by the following state equation

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $f \in \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function with initial condition $x(0) = x_0$,$u(t) \in \mathbb{R}$, $y(t) \in \mathbb{R}$ and $B, C \in \mathbb{R}^n$ are constant vectors.

The system (1) can be linearized around an operating point (u_{op}, x_{op}, y_{op}) as

$$\begin{aligned} \dot{\tilde{x}}(t) &= A\tilde{x}(t) + B\tilde{u}(t), \\ \tilde{y}(t) &= C\tilde{x}(t), \end{aligned} \tag{2}$$

where $\tilde{x} = x - x_{op}$, $\tilde{u} = u - u_{op}$, $\tilde{y} = y - y_{op}$ and $A = \frac{\partial f}{\partial u}|_{x=\tilde{x}}$. The matrix A can be also approached by means of an identification method [16].

The main inconvenience of the obtained linear model that describes the original nonlinear plant is its availability in a limited domain around the operating point. In order to simplify the nonlinear model in a large region, one may look for a bilinear model. In fact, the bilinear structure of dynamical system constitutes a medium structure between the complex nonlinear model and the simple linear one. It represents a good compromise between the simplicity and complexity of dynamical models. It is complex enough to preserve the nonlinear properties of the original system and it is simple enough to recall the linear representation. The bilinear model can be written in the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Nx(t)u(t) + Bu(t), \\ y(t) &= Cx(t). \end{aligned} \tag{3}$$

The bilinearization of a nonlinear plant can be led by different techniques as the determination of A, B, N and C matrices by identification method [16]. Another known technique is the Carlemen bilinearization [15]. This technique is based on the development of the analytic function $f(.)$ in a polynomial form:

$$f(x) = A_1x^{[1]} + A_2x^{[2]} + A_3x^{[3]} + \cdots + A_rx^{[r]}, \tag{4}$$

where $x^{[i]}$ is the i-th Kronecker power of the vector x. Then the nonlinear system (1) with the polynomial approximation (4) can be bilinearized as

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{N}\hat{x}(t)u(t) + \hat{B}u(t), \\ y(t) &= \hat{C}\hat{x}(t), \end{aligned} \tag{5}$$

where $\hat{x}(t) = \begin{bmatrix} x^{(1)T} & x^{(2)T} & \cdots & x^{(r)T} \end{bmatrix}^T$ and $\hat{A}$, $\hat{B}$, $\hat{N}$, $\hat{C}$ are constant matrices, which can be expressed by A_n, B and C. $\hat{A}$ and $\hat{N}$ are square matrices of dimension $n + n^2 + \cdots + n^k$. $\hat{x}$, $\hat{B}$ and $\hat{C}$ are vectors with $n + n^2 + \cdots + n^k$ components.

As example, in particular case where $r = 3$ and $n = 1$ one has

$$\hat{A} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} \\ 0 & \hat{A}_{22} & \hat{A}_{23} \\ 0 & 0 & \hat{A}_{33} \end{bmatrix}, \tag{6}$$

where $\hat{A}_{11} = A_1$, $\hat{A}_{12} = A_2$, $\hat{A}_{13} = A_3$, $\hat{A}_{22} = A_1 \otimes I_n + I_n \otimes A_1$, $\hat{A}_{23} = A_2 \otimes I_n + I_n \otimes A_2$, $\hat{A}_{33} = A_1 \otimes I_{n^2} + I_n \otimes A_1 \otimes I_n + I_{n^2} \otimes A_1$,

$$\hat{B} = \begin{bmatrix} \hat{B}_1 \\ 0 \\ 0 \end{bmatrix} \text{ with } \hat{B}_1 = B, \tag{7}$$

$$\hat{N} = \begin{bmatrix} 0 & 0 & 0 \\ \hat{N}_{21} & 0 & 0 \\ 0 & \hat{N}_{32} & 0 \end{bmatrix}, \tag{8}$$

where $\hat{N}_{21} = B \otimes I_n + I_n \otimes B$, $\hat{N}_{22} = B \otimes I_{n^2} + I_n \otimes B \otimes I_n + I_{n^2} \otimes B$.

In the next section we will consider the class of bilinear system having the same representation as (5) for trajectory planning.

3 Proposed Approach for Trajectory Planning

3.1 Orthogonal functions

Consider a set of orthogonal functions $\Phi = \{\varphi_i(t), i \in \mathbb{N}\}$ defined on $[a, b] \subset \mathbb{R}$. The key idea is that all analytical function $f(t)$ absolutely integrable can be developed as follows

$$f(t) = \sum_{i=0}^{\infty} f_i \varphi_i(t), \ \forall t \in [a, b], \tag{9}$$

where the coefficients f_i are constant and given by

$$f_i = \frac{1}{r_i} \int_a^b w(x)\varphi_i(x)f(x)dx, \ \forall i \in \mathbb{N}. \tag{10}$$

To obtain practice function approximation, the projection (9) is shorten to an order N, such that:

$$f(t) \cong \sum_{i=0}^{N-1} f_i \varphi_i(t) = F_N^T \Phi_N(t), \tag{11}$$

where $F_N = \begin{bmatrix} f_0 & f_1 & \cdots & f_{n-1}^T \end{bmatrix}$ is a constant coefficient vector and $\Phi_N(t) = \begin{bmatrix} \varphi_0(t) & \varphi_1(t) & \cdots & \varphi_{n-1}(t)^T \end{bmatrix}$ is the vector composed by N orthogonal functions. The projection of a matrix $A(t) = [a_{ij}(t)]$ on the basis of the orthogonal functions is given by

$$A(t) \cong \sum_{i=0}^{N-1} A_{Ni}\varphi_i(t), \tag{12}$$

where $A_{Ni} \in \mathbb{R}^{n\times m}$ are constant matrices. More than approximation (12), orthogonal functions offers different operational tools like the operational matrix of integration and the operational matrix of product which are used for solving differential equations. The operational matrix of integration is the constant matrix $P_N \in \mathbb{R}^{N\times N}$ verifying:

$$\int_0^t \Phi_N(t)dt \cong P_N\Phi_N(t), \tag{13}$$

and the operational matrix of product M_{iN} is defined such that one has

$$\varphi_i(t)\Phi_N(t) \cong M_{iN}(V)\Phi_N(t) \tag{14}$$

with

$$M_{iN} = \begin{bmatrix} K_{0i} & K_{1i} & \cdots & f_{n-1,i} \end{bmatrix}, \tag{15}$$

where $\forall i, j \in \{0, 1, \cdots, N-1\}$, $\varphi_i(t)\varphi_j(t) \cong K_{ij}^T\Phi_N(t)$ Thus the following operational relation holds for any constant vector $V \in \mathbb{R}^n$ [16]:

$$\Phi_N(t)\Phi_N^T(t) \cong M_N(V)\Phi_N(t), \tag{16}$$

where $M_N(V) = \begin{bmatrix} M_{0N}V & \vdots & M_{1N}V & \vdots & \cdots & \vdots & M_{(N-1)N}V \end{bmatrix}$.

3.2 Proposed trajectory planning approach

Consider a bilinear system having the following state representation

$$\begin{aligned} \dot{x} = Ax + Bu + \sum_{i=0}^{m} A_i x u_i, \\ y = Cx, \end{aligned} \tag{17}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{1\times n}$. We intend to determine, using orthogonal functions as approximation tool, an open loop control that permits system (17) going to fixed final state $x(T)$ starting from known initial state $x(0)$. The projection of state variables of system (17) on a set of orthogonal functions $\{\varphi_i(t), i = 0, \cdots, N-1\}$ with a truncation of order N allows to write:

$$x(t) = x_N\Phi_N(t), \tag{18}$$

$$u_i(t) = u_{iN}\Phi_N(t), \tag{19}$$

$$u(t) = u_N\Phi_N(t), \tag{20}$$

and the state representation (17) can be put in the following approximated form

$$\dot{x} = Ax_N\Phi_N(t) + Bu_N\Phi_N(t) + \sum_{i=0}^{m} A_i x_N u_{iN}\Phi_N(t). \tag{21}$$

On the other hand, substituting the initial state $x(0)$ by its projection on the orthogonal basis $\Phi_N(t)$:

$$x(0) = x_{N,0}\Phi_N(t), \tag{22}$$

where $x_{N,0} = \left[x(0) \;\vdots\; 0 \;\vdots\; \dots \;\vdots\; 0\right]$ and integrating equation (21) between an initial time ($t_0 = 0$) and a time t and making use of the operational integration and product proprieties(13) and (14) one obtains

$$(x_N - x_{N,0}) = Ax_N P_N + Bu_N P_N + \sum_{i=0}^{m} A_i x_N M_N(u_{iN}) P_N. \tag{23}$$

By using the Vec operator and its main following property [17]

$$Vec(ABC) = (C^T \otimes A)Vec(B), \tag{24}$$

the equation (23) yields the following relation

$$\begin{aligned} Vec(x_N) = \left[I_{nN} - (P_N^T \otimes A) - \sum_{i=0}^{m} P_N^T M_N^T(u_{iN}) \otimes A_i\right]^{-1} \\ \left[(P_N^T \otimes B)Vec(u_N) + Vec(x_{N,0})\right]. \end{aligned} \tag{25}$$

Integrating again relation (21) between instant t and final time T and replacing $x(T)$ by its projection on the orthogonal basis:

$$x(T) = x_{N,T}\Phi_N(t) \tag{26}$$

with $x_{N,T} = \left[x(T) \;\vdots\; 0 \;\vdots\; \dots \;\vdots\; 0\right]$ and using the fact that the orthogonal basis vector at final instant T verifies: $\Phi_N(t) = K_N\Phi_N(t)$ one obtains

$$(x_{N,T} - x_N) = Ax_N P_N(K_N - I_N) + Bu_N P_N(K_N - I_N) + \sum_{i=0}^{m} A_i x_N M_N(u_{iN}) P_N(K_N - I_N), \tag{27}$$

putting $\Pi_N = P_N(K_N - I_N)$ and applying Vec operator yield:

$$\begin{aligned} Vec(x_N) = \left[I_{nN} + (\Pi_N^T \otimes A) + \textstyle\sum_{i=0}^{m} \Pi_N^T M_N^T(u_{iN}) \otimes A_i\right]^{-1} \\ \left[Vec(x_{N,T}) - (\Pi_N^T \otimes B)Vec(u_N)\right]. \end{aligned} \tag{28}$$

By equalizing (25) and (28) one obtains the following relation

$$H_N^{-1}\left[(P_N^T \otimes B)Vec(u_N) + Vec(x_{N,0}\right] = G_N^{-1}\left[Vec(x_{N,T} - (\Pi_N^T \otimes B)\right], \tag{29}$$

where

$$H_N = H_N(u_N) = I_{nN} - R_u, \tag{30}$$

$$R_u = (P_N^T \otimes A) + \sum_{i=0}^{m} P_N^T M_N^T(u_{iN}) \otimes A_i, \tag{31}$$

$$G_N = I_{nN} + (\Pi_N^T \otimes A) + \sum_{i=0}^{m} \Pi_N^T M_N^T(u_{iN}) \otimes A_i, \tag{32}$$

By substitution of Π_N by its expression $\Pi_N = P_N(K_N - I_N)$, one has

$$G_N = H_N + (K_N^T \otimes I_N)R_u, \tag{33}$$

and the relation (29) becomes:

$$\begin{aligned}&\left[I_{nN} - (K_N^T \otimes I_N)\right]\left[(P_N^T \otimes B)Vec(u_N) + Vec(x_{N,0})\right] + \\ &(K_N^T \otimes I_N)H_N^{-1}\left[(P_N^T \otimes B)Vec(u_N) + Vec(x_{N,0}\right] \\ &= (I_{nN} - K_N^T \otimes I_N)\left[(P_N^T \otimes B)Vec(u_N)\right] + Vec(x_{N,T})\end{aligned} \tag{34}$$

Let's put

$$Z(u_N) = H_N^{-1}(u_N)\left[(P_N^T \otimes B)Vec(u_N) + Vec(x_{N,0})\right], \tag{35}$$

$$\Gamma(x_{N,0}, x_{N,T}) = (K_N^T \otimes I_{nN})Vec(x_{N,0}) + Vec(x_{N,T}), \tag{36}$$

the relation (34) yields

$$(K_N^T \otimes I_N)Z(u_N) = \Gamma(x_{N,0}, x_{N,T}). \tag{37}$$

The planning open loop control is then derived by minimizing with respect to u_N the norm of the difference between the two parts of equality (37):

$$\zeta = \|(K_N^T \otimes I_N)Z(u_N) - \Gamma(x_{N,0}, x_{N,T})\|. \tag{38}$$

This minimization can be led using the Matlab optimization toolbox.

4 Trajectory Tracking: Closed Loop Control

Consider the difference variables

$$\delta x = x(t) - x_p(t), \quad \delta u = u(t) - u_p(t) \tag{39}$$

between the trajectory of system (17) and a planned trajectory $(x_p(t), u_p(t))$ verifying the system equation

$$\dot{x}_p = Ax_p + Bu_p + \sum_{i=0}^{m} A_i x_p u_{ip}, \tag{40}$$

the state equation of difference system can be written as

$$\delta\dot{x} = (A + \sum_{i=0}^{m} A_i u_{ip})\delta x(t) + (B + \sum_{i=0}^{m} A_i x_p)\delta u(t) + \sum_{i=0}^{m} A_i \delta x \delta u_i, \tag{41}$$

by neglecting the product term $\delta x \delta u_i$ compared with δx and δu_i, the state equation (41) can be simplified into a linear time variant state equation

$$\delta\dot{x} = \mathcal{A}(t)\delta x(t) + \mathcal{B}(t)\delta u(t), \tag{42}$$

where

$$\mathcal{A}(t) = A + \sum_{i=0}^{m} A_i u_{ip}(t), \quad \mathcal{B}(t) = B + \sum_{i=0}^{m} A_i x_p(t). \tag{43}$$

Our purpose is then to characterize a state feedback control law $\delta u(t) = -K\delta x(t)$ which confers to a controlled LTV (Linear Time Variant) system

$$\delta\dot{x} = (\mathcal{A}(t) - \mathcal{B}(t)K)\delta x(t), \tag{44}$$

desired performances. Such performances can be defined in a convenient linear reference model [18]

$$\delta\dot{x} = E\delta x(t). \tag{45}$$

The expansion of the time variant matrices $\mathcal{A}(t)$, $\mathcal{B}(t)$ and the state vector $\delta x(t)$ into a basis of orthogonal functions as follows

$$\mathcal{A}(t) = \sum_{i=0}^{N-1} \mathcal{A}_{Ni}\varphi_i(t), \tag{46}$$

$$\mathcal{B}(t) = \sum_{i=0}^{N-1} \mathcal{B}_{Ni}\varphi_i(t), \tag{47}$$

$$\delta x(t) = \delta x_N \Phi_N(t), \tag{48}$$

yields the following differential relation

$$\delta\dot{x} = \left[\sum_{i=0}^{N-1} \mathcal{A}_{Ni}\varphi(t) - K\sum_{i=0}^{N-1} \mathcal{B}_{Ni}\varphi(t)\right]\delta x_N \Phi_N(t). \tag{49}$$

Integrating equation (49) and making use of operational matrices of integration and product and the Vec operator one obtains:

$$Vec(\delta x_N) - Vec(\delta x_{Np}) = (\sum_{i=0}^{N-1}(M_{iN}P_N)^T \otimes \mathcal{A}_{Ni} - k(\sum_{i=0}^{N-1}(M_{iN}P_N)^T \otimes \mathcal{B}_{Ni})Vec(\delta x_N). \tag{50}$$

A similar development for the reference model (45) yields:

$$Vec(\delta x_{N,r}) - Vec(\delta x_{N,0}) = (P_N^T \otimes E)Vec(\delta x_N). \tag{51}$$

The equalization of $Vec(\delta x_N)$ coming from (49) and $Vec(\delta x_{N,r})$ derived from (50) allows to have the following linear algebraic equation where unknown is the feedback control gain K:

$$\phi K = \psi \tag{52}$$

with

$$\phi = \sum_{i=0}^{N-1}(M_{iN}P_N)^T \otimes \mathcal{B}_{Ni}, \quad \psi = \sum_{i=0}^{N-1}(M_{iN}P_N)^T \otimes \mathcal{A}_{Ni} - (P_N^T \otimes E). \tag{53}$$

Solving equation (52) by using least squares method leads to a closed loop control feedback law $\delta u(t) = -K\delta x(t)$ that ensures trajectory tracking for bilinear system (17). Note that the development (44) until (51) can be easily extended to look for a time variant feedback control law $\delta u(t) = -K(t)\delta x(t)$ where the time variant gain $K(t)$ can be determined as an expansion of orthogonal functions: $K(t) = \sum_{i=0}^{N-1} K_{Ni}\varphi_i(t)$.

5 Illustrating Example

In this section we present the implementation of the proposed approach for trajectory planning and tracking of the bilinear system described by the following equations

$$\dot{x}_1 = x_1 - x_2 u + u, \quad \dot{x}_2 = -x_2 - x_1 u - u, \\ y = x_2. \tag{54}$$

A state representation of this system is the following

$$\dot{x} = Ax + Nxu + Bu, \\ y = Cx, \tag{55}$$

with $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$, $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $N = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

The application of the proposed planning approach based on modified Legendre orthogonal functions with a truncation order $N = 16$, for system (55) starting from an initial state $x_0 = \begin{bmatrix} 1 & 2 \end{bmatrix}^T$ at initial time $t_0 = 0s$ to the final state $x_T = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ at final time $T = 10s$, yields the planned trajectories $x_{1p}(t)$ and $x_{2p}(t)$ and planning input $u_p(t)$ presented in Figure 1.

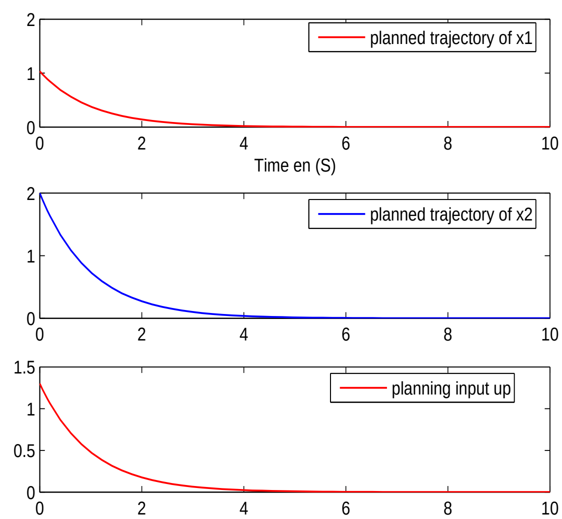

Figure 1: Trajectory planning and planning input.

These simulations show that the use of orthogonal approximation method yields an open loop control that allows to the bilinear system (55) to reach the fixed final state x_T starting from a chosen initial state x_0. Note that initial and final conditions can be modified and one obtains then an open loop control that yields another trajectory planning. The tracking of the obtained trajectory $(x_p(t), u_p(t))$ is given by the application of the orthogonal approximation to the following LTV system

$$\delta\dot{x} = \mathcal{A}(t)\delta x(t) + \mathcal{B}(t)\delta u(t) \tag{56}$$

with $\mathcal{A}(t) = A + Nu_p(t)$ and $\mathcal{B}(t) = B + Nx_p(t)$ derived from the linearization of the bilinear system round the planned trajectory.

The synthesis of the closed loop control $\delta u(t) = -K\delta x(t)$ that ensures the tracking of the planned trajectory is based on the following linear reference system that confers to the state feedback controlled LTV system the desired performances corresponding to the linear reference model:

$$\delta \dot{x} = E\delta x(t), \tag{57}$$

where $E = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$. Figure 2 shows the effect of the obtained control law on the tracking of the planned trajectory affected by two instantenous disturbances at $t_1 = 3s$ and $t_2 = 15s$.

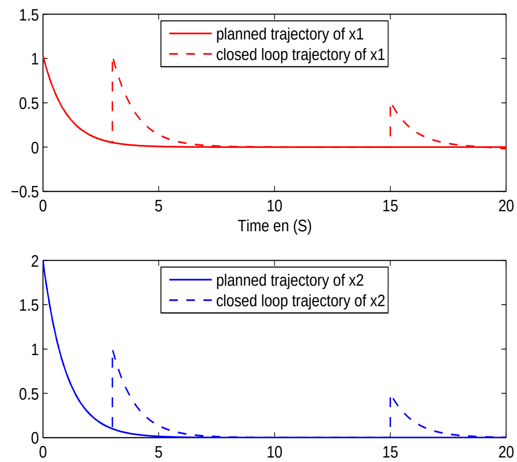

Figure 2: Trajectory tracking.

It appears that the designed control law ensures stability of the system around its planned trajectory. Note that the performances of the closed loop controlled system can be modified by choosing another linear reference model.

6 Conclusion

In this paper, a new approach has been introduced for trajectory planning and tracking of bilinear systems, which approximate weakly nonlinear systems, by using orthogonal functions as a tool of approximation. The presented method was applied to a class of bilinear invariant systems. The use of operational matrices of integration and product in planning problem has allowed the transformation of the system differential equation into an algebraic one depending on the control variable and the initial and final states. For trajectory tracking, this technique has allowed the synthesis of a closed feedback control which ensures for the considered system the performances of a prespecified reference model. Note that the proposed approach can be extended to other classes of systems such as time variant bilinear systems and affine control nonlinear systems.

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