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## Weak Solutions for Boundary-Value Problems with Nonlinear Fractional Differential Inclusions

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**Abstract:** This paper deals with the existence of solutions, under the Pettis integrability assumption, for a class of boundary value problems for fractional differential inclusions involving nonlinear integral conditions. Our results are based on the technique of measures of weak noncompactness and a fixed point theorem of Mönch type.

**Keywords:** boundary value problem; differential inclusion; Caputo fractional derivative; measure of weak noncompactness; Pettis integrals; weak solution.

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## 1 Introduction

This note is concerned with the existence of solutions of the boundary value problem with fractional order differential inclusions and nonlinear integral conditions of the form

$${}^{c}D^{\alpha}x(t) \in F(t, x(t)), \text{ for a.e. } t \in J = [0, T], \ 1 < \alpha \le 2,$$
 (1)

$$x(0) - x'(0) = \int_0^T g(s, x(s)) ds,$$
(2)

$$x(T) + x'(T) = \int_0^T h(s, x(s))ds,$$
(3)

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