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Quasilinearization Method Via Lower and Upper Solutions for Riemann–Liouville Fractional Differential Equations

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Abstract: Existence and comparison results of the linear and nonlinear Riemann–Liouville fractional differential equations of order q, $0 < q < 1$, are recalled and modified where necessary. Generalized quasilinearization method is developed for nonlinear fractional differential equations of order q, using upper and lower solutions. Quadratic convergence to the unique solution is proved via weighted sequences.

Keywords: *fractional differential equations; lower and upper solutions; quasilinearization method.*

Mathematics Subject Classification (2000): 34A34, 34A45.

1 Introduction

Fractional differential equations have various applications in widespread fields of science, such as in engineering [9], chemistry [10, 17, 18], physics [3, 4, 11], and others [12, 13]. In the majority of the literature existence results for Riemann–Liouville fractional differential equations are proven by a fixed point method. Initially we will recall existence by lower and upper solution method, which is more comparable to our main results. Despite there being a number of existence theorems for nonlinear fractional differential equations, much as in the integer order case, this does not necessarily imply that calculating a solution explicitly will be routine, or even possible. Therefore, it may be necessary to employ an iterative technique to numerically approximate a solution to a needed solution. In this paper we construct such a method.

The iterative technique we manufacture is the method of quasilinearization for nonlinear Riemann–Liouville fractional differential equations of order q, $0 < q < 1$. This

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