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Exponentially Long Orbits in Boolean Networks with Exclusively Positive Interactions

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Abstract: The absence of negative feedback in Boolean networks tends to result in systems with relatively short orbits. We present a construction of N-dimensional Boolean networks that use only AND, OR, COPY gates and nevertheless have an exponentially large orbit (of size c^N for arbitrary $c < 2$). The construction is based on pseudorandom number generation algorithms. A previously obtained nontrivial upper bound on the orbit length under certain limitations on the number of outputs per node is shown to be optimal.

Keywords: *Boolean networks; monotone systems; gene networks; systems biology.*

Mathematics Subject Classification (2000): 06E99, 34C12, 39A33, 92B99, 94C10.

1 Introduction

The concept of a *Boolean network* was originally proposed in the late 1960's by Stuart Kauffman to model gene regulatory behavior at the cell level [13]. This type of modeling can sometimes capture the general dynamics of continuous systems in a simplified framework without the choice of specific nonlinearities or parameter values; see for instance [1]. Boolean networks are used in several other disciplines such as electrical engineering, computer science, and control theory, and analogous definitions are known under various names such as sequential dynamical systems [16] or Boolean difference equations [6].

An N-dimensional *Boolean dynamical system* or *Boolean network* (Π, g) consists of N variables $s_1, \ldots, s_N$, each of which can have value 0 or 1 at any given time step t. The variables are updated according to $s_i(t+1) = g_i(s_1(t), \ldots, s_N(t))$.

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