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Positive Solutions to an Nth Order Multi-point Boundary Value Problem on Time Scales

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Abstract: In this paper, we consider an nth order multi-point boundary value problem on time scales. We establish criteria for the existence of at least one or two positive solutions. We shall also obtain criteria which lead to nonexistence of positive solutions. Examples applying our results are also given.

Keywords: *positive solutions; fixed-point theorems; time scales; dynamic equations; cone.*

Mathematics Subject Classification (2000): 39A10.

1 Introduction

We are concerned with the following boundary value problem (BVP) on time scales $\mathbb{T}$:

$$\begin{cases} y^{\Delta^n}(t) + \lambda f(y^{\sigma}(t))) = 0, \quad t \in [a,b] \subset \mathbb{T}, \\ y^{\Delta^i}(a) = 0, \quad 0 \le i \le n-2, \\ \sum_{i=1}^{m} \alpha_i y^{\Delta^{n-2}}(\xi_i) = y^{\Delta^{n-2}}(\sigma(b)) \end{cases} \tag{1.1}$$

where $\lambda > 0$ is a parameter, $f \in \mathcal{C}([0,\infty),[0,\infty))$, $n \ge 3$, $m \ge 1$ are integers, $a < \xi_1 < \xi_2 < ... < \xi_m < b$, $\alpha_i \in (0,+\infty)$ for $1 \le i \le m$ and $\sum_{i=1}^{m} \alpha_i < 1$.

We assume that $D = \sigma(b) - a - \sum_{i=1}^{m} \alpha_i(\xi_i - a) > 0$ and $\sigma(b)$ is right dense so that $\sigma^j(b) = \sigma(b)$ for $j \ge 1$.

The study of dynamic equations on time scales goes back to its founder Stefan Hilger [10]. Some preliminary definitions and theorems on time scales can be found in the books [2, 3] which are excellent references for the calculus of time scales.

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