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Homoclinic Orbits for a Class of Second Order Hamiltonian Systems

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Abstract: A new result for existence of homoclinic orbits is obtained for the second order Hamiltonian systems $\ddot{x}(t)+V'(t,x(t)) = f(t)$, where $t \in \mathbb{R},\ x \in \mathbb{R}^N$, $V \in C^1(\mathbb{R}\times\mathbb{R}^N,\mathbb{R}),\ V(t,x) = -K(t,x) + W(t,x)$ is $T-$periodic in t, $T > 0$ and $f : \mathbb{R} \longrightarrow \mathbb{R}^N$ is a continuous bounded function, under an assumption weaker than the so-called Ambrosetti–Rabinowitz-type condition.

Keywords: *homoclinic orbits; Hamiltonian systems; critical point; diagonal method.*

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1 Introduction

In this paper we are concerned with the study of the existence of homoclinic solutions for second order time-dependent Hamiltonian systems of the type

$$\ddot{x}(t) + V'(t,x(t)) = f(t), \qquad (HS)$$

where $x = (x_1,...,x_N),\ V \in C^1(\mathbb{R}\times\mathbb{R}^N,\mathbb{R}),\ V'(t,x) = \frac{\partial V}{\partial x}(t,x)$ and $f : \mathbb{R} \longrightarrow \mathbb{R}^N$ is a continuous function. Here, as usual, we say that a solution x of (HS) is homoclinic $(to\ 0)$ if $x(t) \to 0$ as $t \to \pm\infty$. In addition x is called nontrivial if $x \not\equiv 0$.

The existence of homoclinic solutions for (HS) has been extensively investigated in many papers via the critical point theory, see [8, 11]. These results were obtained under the fact that the potential V is of the type

$$V(t,x) = -\frac{1}{2}L(t)x.x + W(t,x),$$

where $L \in C(\mathbb{R},\mathbb{R}^{N^2})$ is a symmetric matrix-valued function and $W \in C^1(\mathbb{R}\times\mathbb{R}^N,\mathbb{R})$.

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