Nonlinear Dynamics and Systems Theory, 12 (2) (2012) 171-178



Positive Solutions for a Fourth Order Three Point Focal Boundary Value Problem

J. R. Graef^{1*}, L. Kong¹, and B. Yang²

 ¹ Department of Mathematics, University of Tennessee at Chattanooga, Chattanooga, TN 37403, USA
 ² Department of Mathematics and Statistics, Kennesaw State University, Kennesaw, GA 30144, USA

Received: January 04, 2011; Revised: March 16, 2012

Abstract: The authors consider a fourth order three point boundary value problem. Some a priori estimates to positive solutions for the boundary value problem are obtained. Sufficient conditions for the existence and nonexistence of positive solutions for the problem are established.

Keywords: fixed point theorem; cone; nonlinear boundary value problem; positive solution.

Mathematics Subject Classification (2010): 34B18.

1 Introduction

In this paper, we consider the fourth order differential equation

$$u''''(t) + g(t)f(u(t)) = 0, \quad 0 \le t \le 1,$$
(1)

together with the boundary conditions

$$u(0) = u'(p) = u''(1) = u'''(1) = 0.$$
(2)

Throughout this paper, we assume that

(H1) p is a real constant such that $1 - \sqrt{3}/3 \le p \le 1$, $f: [0, \infty) \to [0, \infty)$ and $g: [0, 1] \to [0, \infty)$ are continuous functions, and $g(t) \ne 0$ on [0, 1].

^{*} Corresponding author: mailto:John-Graef@utc.edu

^{© 2012} InforMath Publishing Group/1562-8353 (print)/1813-7385 (online)/http://e-ndst.kiev.ua 171