Existence, Uniqueness and Asymptotic Stability of Solutions to Non-Autonomous Semi-Linear Differential Equations with Deviated Arguments

Rajib Haloi [1*], Dwijendra N. Pandey [2] and D. Bahuguna [3]

[1,3] *Department of Mathematics, Indian Institute of Technology, Kanpur – 208 016, India.*
[2] *Department of Mathematics, Indian Institute of Technology, Roorkee – 247667, India.*

Abstract: We consider a non-autonomous semi-linear differential equation of parabolic type with a deviated argument in an arbitrary Banach space. Using the Sobolevskiĭ-Tanabe theory of parabolic equations, we prove the existence and uniqueness of a solution. We also discuss the asymptotic stability of a solution. As an application, we give an example to illustrate the main results.

Keywords: *analytic semigroup, parabolic equation, differential equation with a deviated argument, Banach fixed point theorem.*

Mathematics Subject Classification (2010): 34G10, 34G20, 34K30, 35K90, 47N20.

1 Introduction

The purpose of this article is to study the following differential equation in a Banach space $(X, \| \cdot \|)$:

$$\left. \begin{aligned} \frac{du}{dt} + A(t)u(t) &= f(t, u(t), u(h(u(t), t))), \ t > 0; \\ u(0) &= u_0, \ u_0 \in X. \end{aligned} \right\} \tag{1}$$

We assume that for each $t \geq 0$, $-A(t)$ generates an analytic semigroup of bounded linear operators on X, $f \colon [0, \infty) \times X \times X \to X$ and $h \colon X \times [0, \infty) \to [0, \infty)$. The nonlinear continuous functions f and h satisfy suitable growth conditions in their arguments stated in Section 2.

---

* Corresponding author: `mailto:rajib.haloi@gmail.com`