



# Travelling Wave Solutions of Nonlocal Models for Media with Oscillating Inclusions

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**Abstract:** Continual model of a complex medium with oscillating inclusions is considered. Travelling wave (TW) solutions to the source system are shown to satisfy a four-dimensional dynamical system. Qualitative study of the factorized system enables to show the existence of homoclinic and heteroclinic contours in vicinities of fixed points. Existence of the homoclinic loops results in the complex global behavior of phase trajectories, including the bifurcations of tori, that are investigated numerically.

**Keywords:** *travelling wave solutions; homoclinic curve; invariant tori; nonlinear normal modes.*

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## 1 Introduction

Experimental investigations of deformations of geomedia in the wide range of loading velocities, carried out in the last decades, testify that geomedia possess two basic features, namely, a discrete structure and oscillating motion of the discrete elements [1, 2].

Oscillating modes can be incorporated into the continual model by means of adding extra volumetric forces, causing the movements of the elements of the structure. In the papers [3, 4] a linear mathematical model for structured media taking into account the oscillations of structural elements has been suggested. In the simplest form the equations of motion can be written as follows:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} - m\rho \frac{\partial^2 w}{\partial t^2}, \quad \frac{\partial^2 w}{\partial t^2} + \omega^2 (w - u) = 0, \quad (1)$$

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