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A Compactness Condition for Solutions of Nonlocal Boundary Value Problems of Orders $n = 3, 4$ & 5 [†]

J. Henderson [*]

Department of Mathematics, Baylor University, Waco, TX 76798-7328, USA

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Abstract: For the ordinary differential equation, $y^{(n)} = f(x, y, y', \dots, y^{(n-1)})$, of order $n = 3, 4$, or 5, it is shown that the existence of unique solutions of certain 4-point nonlocal boundary value problems implies a compactness condition on uniformly bounded sequences of solutions.

Keywords: *boundary value problem; nonlocal; continuous dependence; compactness condition.*

Mathematics Subject Classification (2010): 34B10, 34B15.

1 Introduction

In a recent paper, for $n \geq 3$ and $1 \leq k \leq n-1$, Henderson [6] studied solutions of the ordinary differential equation,

$$y^{(n)} = f(x, y, y', \dots, y^{(n-1)}), \quad a < x < b, \tag{1}$$

satisfying the $(k+2)$-point nonlocal boundary conditions,

$$\begin{aligned} &y^{(i-1)}(x_j) = y_{ij},\ 1 \leq i \leq m_j,\ 1 \leq j \leq k, \\ &y(x_{k+1}) - y(x_{k+2}) = y_n, \end{aligned} \tag{2}$$

for positive integers $m_1, \dots, m_k$ such that $m_1 + \cdots + m_k = n-1$, points $a < x_1 < x_2 < \cdots < x_k < x_{k+1} < x_{k+2} < b$, real values $y_{ij}, 1 \leq i \leq m_j, 1 \leq j \leq k$, and $y_n \in \mathbb{R}$. In particular, sufficient conditions were given under which the existence of solutions for 4-point nonlocal boundary value problems for (1), (2), (that is, when $k = 2$), led to the existence of unique solutions of $(k+2)$-point nonlocal boundary value problems for (1), (2), for all $1 \leq k \leq n-1$.

Fundamental to that paper's main result was the following list of assumptions on solutions of (1).

[†] In memory of Professor Keith W. Schrader, April 22, 1938 – December 27, 2010.

[*] Corresponding author: `mailto:Johnny_Henderson@baylor.edu`