



# Mathematical Modeling of the Hydro-Mechanical Fluid Flow System on the Basis of the Human Circulatory System

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**Abstract:** The primary objective of this study is to examine a human/mammal circulatory system. Considering structures and operating rules of a natural, biological circulatory system one can easily state that it is possible to create an analogous hydromechanical dynamic system. Noting the similarities and taking into account blood and vessels features there is a mathematical model given that includes differential equations of the fluid mechanics. Additionally a stand/analog consisted of hydraulic and electronics elements is presented. A prototype of the circulatory system is proposed with a construction of the heart as a bicapsular pumping unit powered by external pneumatic system. Solving the equations describing biological system, gives opportunities to examine some external and internal risk factors, model input signals and activity under different conditions.

**Keywords:** *mathematical modeling; fluid flow; circulatory system; biomechanics.*

**Mathematics Subject Classification (2010):** 93A10, 93A30.

## 1 Introduction

Considering anatomy and operating principles of biological human/mammal circulatory system (cardiovascular system) one can state that structurally it should be regarded as a hydro-mechanical closed-loop system. Because of the fact that anatomically, considered system is well-examined structure with application of a wide range of fields of science devoted to it (see [4-6]), we are able to exploit available knowledge combining with

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mechanics to create both simulation and mathematical system and laboratory structure, which yields possibility to make a comparison of numerical and experimental results.

Analyzing the state of knowledge regarding the mentioned problem and owing to the results presented in reference [7], the functional analysis of the biological circulatory system has been carried out to develop hydraulic system scheme. The proposed hydraulic system dynamics is governed by ordinary differential equations. An important feature of presented model is an oscillating forcing function that refers to the periodic signal of the heartbeat with the highlighted following phases of the heart muscle contractions. According to the biological data, selected parameters have been empirically estimated that characterize the mechanical properties (see [3]) of the system (the vascular elasticity, the flow resistance, the viscosity of a hydraulic fluid, the stroke volume) while checking the reliability of the results against the statistical data for the interesting physical quantities of the circulatory system. It seems to be obvious that the complexity of the biological structure of the vascular system carries implications in terms of structural simplification of the laboratory. An enormous number of small body capillary vessels or bronchioles capillary net and their branches, where gas exchange occurs, additionally the strict connection with opened lymphatic system is structurally an issue not to wade. To get the highly comparative to natural flow conditions there were implemented both the flow direction and resistance control hydraulic components. Moreover, because of the fact that one of the most significant natural blood vessels properties elasticity, has to be considered, the rigid hydraulic hoses were complemented by adjustable, vulnerable capsules so that the ripple of the blood vessels can be reflected during the operation of the system (including different venous and arteria compliance). The artificial heart construction (pump) was also proposed as a pneumatically controlled bicapsular, diaphragm pumping unit whereby the greatest structural and functional similarity to anatomical system was obtained. The hydraulic structure was expanded by a measurement system using absolute pressure sensors and measurement data acquisition module. Thanks to this construction, we are able to keep up with changes of the measured relevant values.

The creation of a complete simulation model and developing laboratory rig by another mechanical components gives possibility to examine the system behaviour under various conditions meaning input pattern signal disorders and changes (heart rythm). Moreover, it is also possible to generate artificial pathological conditions such as for example ventricular tachycardia or cardiac arrhythmias.

## 2 Simulation Model

### 2.1 The hydro-mechanical model basis

The following assumptions are used while carrying out the analysis of the working fluid flow in the considered closed system:

- the working fluid in the system is incompressible;
- differences in elevation as it flows in a horizontal arrangement do not exist;
- mass forces (inertial forces) do not affect the movement of the fluid;
- the flow is laminar.

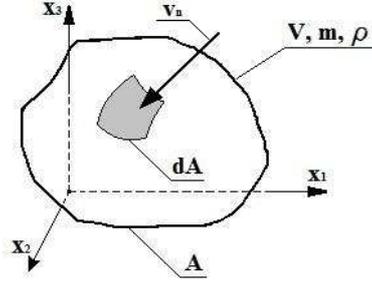
It can be concluded that in the proposed model, the only physical factor causing the movement of the fluid is external, pressure forcing signal (eg, the pressure generated by

the movement of the piston pump). It is convenient, therefore, to describe the flow in the system, using the mass conservation principle (indestructibility of matter).

As shown in Figure 1, we can write the law of conservation of mass in the analytical form for the volume  $V$ , and the surrounding control surface  $S$ :

$$\frac{dm}{dt} = \int \int_S \rho v_N dA, \quad (1)$$

where  $m$  is the fluid mass contained in the volume  $V$ ,  $\rho$  is the fluid density,  $v_N$  is the velocity vector, normal to the element of the control surface  $dS$ .



**Figure 1:** Model of fluid volume area.

Because in the real system, important for the analysis, flows through hydraulic elements (throttle valves, leakage, fluid supply elements, the hydraulic casing and fluid compliance, the volume changes of the cylinder-source of the fluid motion) are modeled as discrete object/elements, we are able to use the discrete instead of continuous model. As a result of the discretization of the model (see Figure 1), the equation (1) takes the following form:

$$\frac{dm}{dt} = \sum_{i=1}^n \rho v_{N_i} \Delta A_i, \quad (2)$$

This is the datum form of the equation, and after some trivial transformations (see [1]) the mass elementary flows balance equation is obtained:

$$\sum_{i=1}^n Q_i = \pm A \dot{x} + (C_0 + C_0 \frac{p}{E} + \frac{V_0}{E} \pm \frac{A}{E} x) \dot{p}, \quad (3)$$

where: dotted  $x$  and  $p$  in the notation given are respectively  $\frac{dx}{dt}$  and  $\frac{dp}{dt}$ ,  $A$  is the cylinder working surface,  $x$  is the movement coordinate of the piston,  $C_0$  is coefficient of compliance of the deformable elements in the system,  $p$  is a pressure,  $V_0$  is the initial volume of the fluid,  $E$  is the Young's modulus of the hydraulic fluid.

Of course, the applied notation of either '+' or '-' in a component of the equation is based on the direction of the forcing flow (in the present case the hydraulic cylinder piston movement direction). Other terms of the equation express the elementary flows associated with the deformation of both the elements and medium.

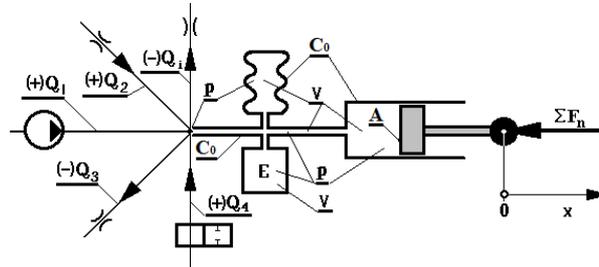


Figure 2: The physical model of the hydro-mechanical system [1].

## 2.2 The human cardiovascular system versus hydro-mechanical model

The diagram of the physical model of the hydro-mechanical system (shown in Figure 2) can be easily applied to a biological system of blood circulation in the human body. It should be emphasized that the circulatory system is a closed-loop system so the overall balance of mass elementary flows must be zero.

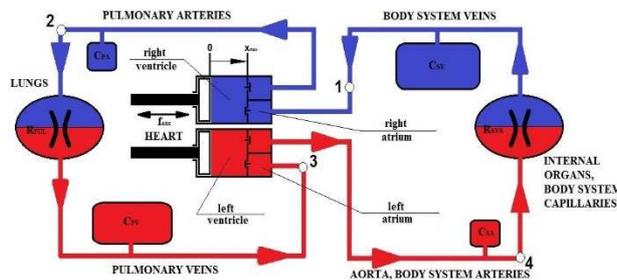


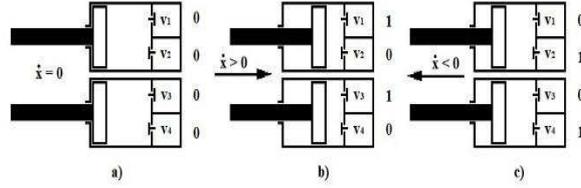
Figure 3: Hydro-mechanical model of the human circulatory system.

In order to simulate the respective mechanical properties of components of the human circulatory system, the model includes some important quantities that describe selected parameters (according to [3]). Referring to Figure 3, the chosen values follow:

- $C_{SV}$  – reduced, average compliance of body system veins;
- $C_{PV}$  – reduced, average compliance of pulmonary veins;
- $C_{SA}$  – reduced, average compliance of body system arteries;
- $C_{PA}$  – reduced, average compliance of pulmonary arteries;
- $R_{SYS}$  – reduced flow resistance through internal organs and body system capillaries;
- $R_{PUL}$  – reduced flow resistance through pulmonary vessels.

In the developed model (Figure 3), there is the clear separation of the two subcircuits (analogous to a biological system - the small and large circulatory system). Because both subcircuits are powered by a dual-chamber pumping unit (by assumption) that simulate

human heart, there are necessary elements which separate both circuits and perform biological function of the atria and vessels valves that are simply return one-way valves operating on the logical scheme based on the forcing cylinder movement direction. Since each of the four valves (two for each chamber) can have two states: 'open' or 'closed', they act as hydraulic return valves and from the mathematical point of view we can easily describe their operation by using the indicator function of the piston velocity



**Figure 4:** Valves states depending on the cylinder movement direction.

Of course, there are three cases that must be taken into account during the operation of the system (Figure 4). If the pressure cylinder is fixed, the system has no flow, and the valves  $v_1$ ,  $v_2$ ,  $v_3$  and  $v_4$  are closed (Figure 4a). In two other cases (Figure 4b,c), one of the pairs of valves in each chamber will remain open to allow flow in a closed system.

An important issue is the forcing (cadency/timing) function with the course corresponding to the various stages (phases) of contraction of the heart muscle. The cylinder motion, and thus the cyclic velocity changes which determine the flow control, is enforced by a periodic function with three phases distinguished: pause, diastole - atrial contraction, systole - ventricular contraction. Distribution of the following phases in the one time period of the function has a significant impact on the speed of the cylinder, and consequently on the waveform of the pressure pulsations in the system.

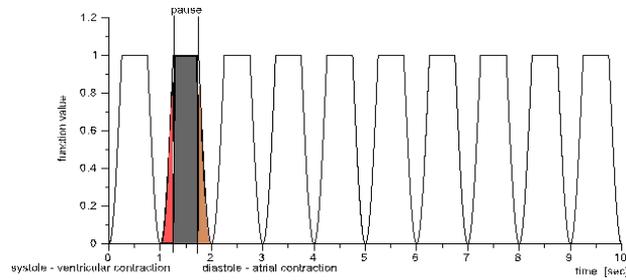
Obviously the nature of the excitation shown (Figure 5) can be disrupted by changing the period or duration of the different phases, thereby simulating changes in heart rate or some pathological cardiac states. Excitation function can be expressed as some conversion of a sinusoidal function, and it may have the form as given:

$$f(t) = -\sin(t)[\text{sgn}(\sin(t)) + 1] + 1. \quad (4)$$

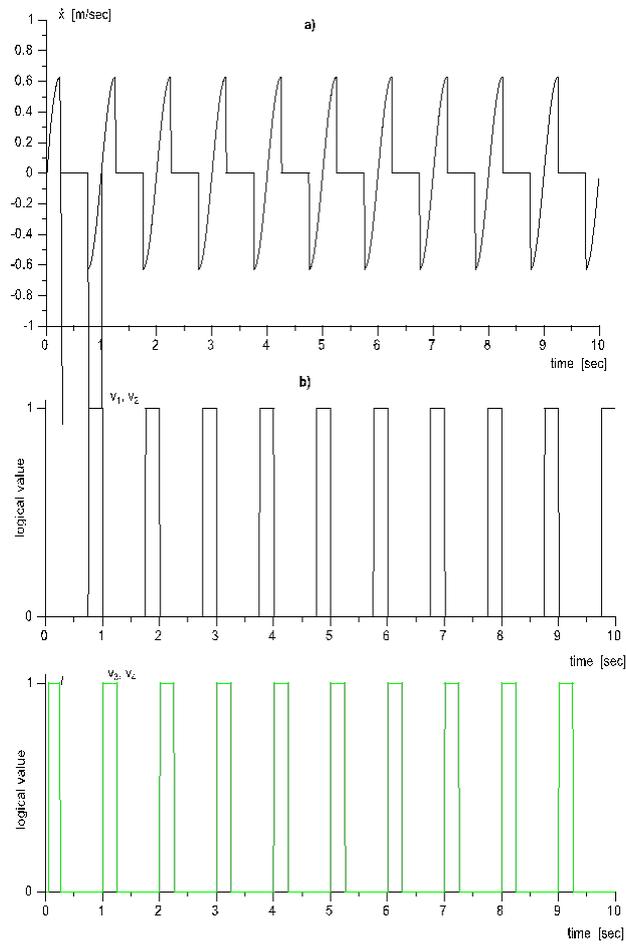
Closely related to the function of the excitation is the velocity of the piston and thus the status of the valves in the chambers (Figure 4). The dynamics of a hydraulic closed structure (Figure 3) is governed by a system of four equations of elementary mass flow balance in the selected points shown in Figure 3. The number 1 indicates the point at which the flow in body system veins is being balanced. As the 2-nd we established the system of the pulmonary arteries, as 3-rd pulmonary venous system and 4-th the body system arteries. The equations follow:

$$(p_4 - p_1) \cdot R_{SYS} = A_{CYL} \cdot \dot{x} \cdot \begin{cases} -1 & \text{for } \dot{x} < 0 \\ 0 & \text{for } \dot{x} > 0 \\ 0 & \text{for } \dot{x} = 0 \end{cases} + C_{SV} \cdot \dot{p}_1, \quad (5)$$

$$-(p_2 - p_3) \cdot R_{PUL} = A_{CYL} \cdot \dot{x} \cdot \begin{cases} 1 & \text{for } \dot{x} > 0 \\ 0 & \text{for } \dot{x} < 0 \\ 0 & \text{for } \dot{x} = 0 \end{cases} + C_{PA} \cdot \dot{p}_2, \quad (6)$$



**Figure 5:** The forcing cadency function waveform.



**Figure 6:** An example piston velocity waveform (a) and corresponding states of the valves (b).

$$(p_2 - p_3) \cdot R_{PUL} = A_{CYL} \cdot \dot{x} \cdot \begin{cases} -1 & \text{for } \dot{x} < 0 \\ 0 & \text{for } \dot{x} > 0 \\ 0 & \text{for } \dot{x} = 0 \end{cases} + C_{PV} \cdot \dot{p}_3, \quad (7)$$

$$-(p_4 - p_1) \cdot R_{SYS} = A_{CYL} \cdot \dot{x} \cdot \begin{cases} 1 & \text{for } \dot{x} > 0 \\ 0 & \text{for } \dot{x} < 0 + C_{SA} \cdot \dot{p}_4. \\ 0 & \text{for } \dot{x} = 0 \end{cases} \quad (8)$$

The left side of each equation contains the flow associated with the pressure difference on both sides of the point at which there is an increased flow resistance. The right side is reduced and simplified sum written for equation (3) of the elementary mass flow balance, wherein the first component of the total flow is associated with the movement of the piston with a working area and stroke defined. The second component is associated with compliance of hydraulic lines (blood vessels) system. Contained in brace bracket element is the mentioned pump piston velocity indicator, which describes the valves operations mathematically depending on the sign of the velocity. All equations (5) – (8) are coupled in pairs due to the construction of the entire system consisting of two sealed subsystems with a common pumping unit, with the work of which a common element is linked for all equations. It can be said that the simplified model of hydro-mechanical circulatory system has a common element in the form of heart unit that binds the entire differential equations system.

### 2.3 The numerical simulation results

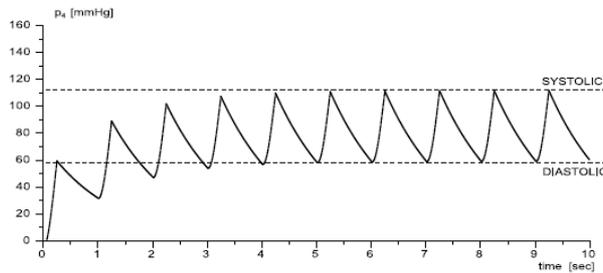
The system dynamics equations (5) – (8) form the basis of the simulation model. They have been implemented as a block diagram in the SciLab Xcos system. Each of the described features, including timed transformed sine function and the pump piston velocity indicator is mapped to the system as a model block components associated with each other by signal connections.

Because of the fact that the essential element for the assessment of cardiovascular efficiency is the pressure in the blood vessels, the analyzed output signals of the system are arterial and venous pressures calculated for the given points of the system (Figure 3). Moreover, it is convenient to calculate fluid flows for each point mentioned that constitute a particular addend in equations given. By selecting appropriate coefficients of the equations of dynamics, based on biological data for compliance and flow resistance, it was possible to obtain pressure waveforms similar to real data (Table 1). It is obvious that selected human physiological conditions are considered, moreover the obtained results of the simulation are closely linked with the body system physiological state assumed in advance (heart rate, symmetry or asymmetry of pumping chambers operation, the length of the following phase in the cycle).

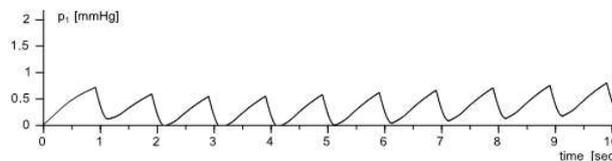
**Table 1:** The biological data of significant circulatory system parameters.

Resting heart rate		1.1 [Hz] (66 beats per minute)
Resting pressures	Body system veins pressure SYSTOLIC/DIASTOLIC $p_1$	2/0 [mmHg]
	Body system arteries pressure SYSTOLIC/DIASTOLIC $p_4$	120/70 [mmHg]
	Pulmonary veins pressure SYSTOLIC/DIASTOLIC $p_3$	5/0 [mmHg]
	Pulmonary arteries pressure SYSTOLIC/DIASTOLIC $p_2$	15-25/8 [mmHg]
Blood vessels compliance (veins to arteries)		$\frac{C_V}{C_A} \approx 20$
Blood flow resistance (system to pulmonary)		$\frac{R_{SYS}}{R_{PUL}} \approx 10$
Stroke volume (average)		$V_{STR} \approx 80[ml] = 80[cm^3]$

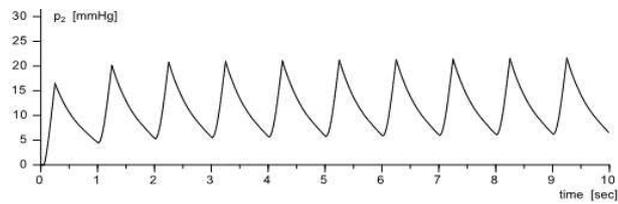
The numerical simulation results are presented, as the most significant pressure (Figure 7-10) and flow (Figures 11–15) waveforms and at selected points of the hydro – mechanical structure (Figure 3). For comparison purposes, the absolute pressure values are given in units of [mmHg], which are typical for the medical description. Because of the fact that flows in particular points are relatively small, they are presented in units of [cm<sup>3</sup>/sec].



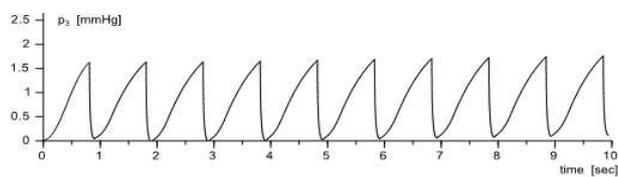
**Figure 7:** An example resting pressure waveform for a body system arterial structure.



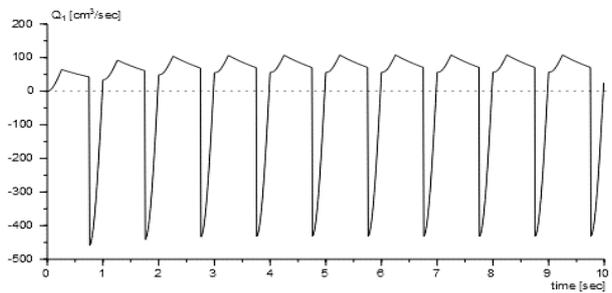
**Figure 8:** An example resting pressure waveform for a body system venous structure.



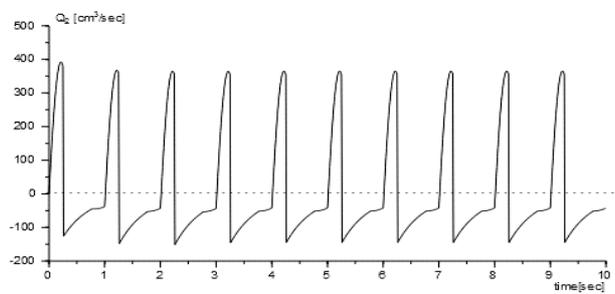
**Figure 9:** An example resting pressure waveform for a pulmonary arterial structure.



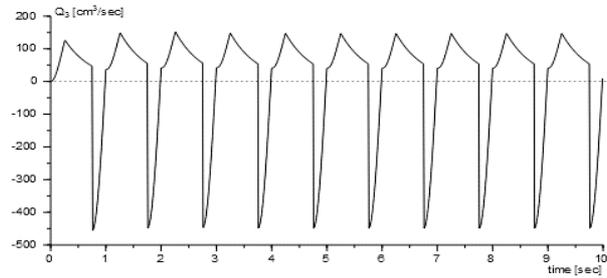
**Figure 10:** An example resting pressure waveform for a pulmonary venous structure.



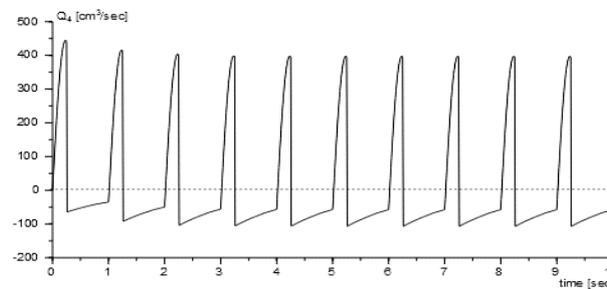
**Figure 11:** An example fluid volume flow waveform for a body system venous structure.



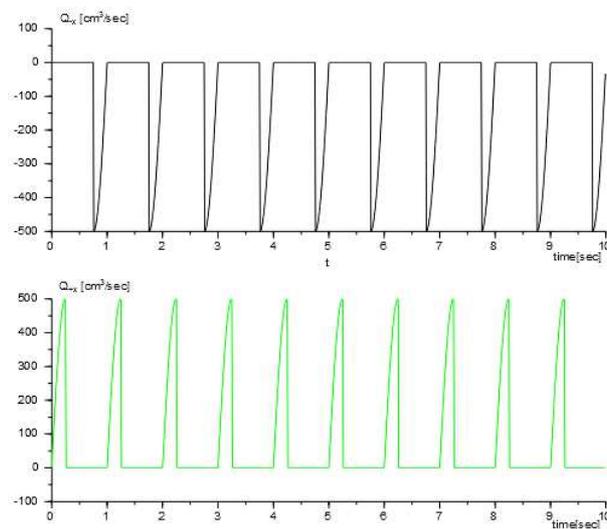
**Figure 12:** An example fluid volume flow waveform for a pulmonary arterial structure.



**Figure 13:** An example fluid volume flow waveform for a pulmonary venous structure.

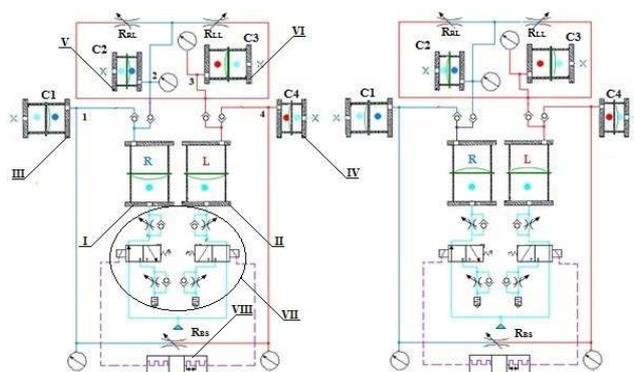


**Figure 14:** An example fluid volume flow waveform for a body system arterial structure.



**Figure 15:** An example fluid volume flow waveforms for the flow occurrence caused by the piston movement: a) return movement direction '-'; b) right movement direction '+'.  
 '-'; b) right movement direction '+'.

Analysis of the exemplary pressure waveforms, shown in Figures 7-10, allows to detect similarities for the extreme pressure values in each waveform in comparison with biological data. Additionally, a common noticeable thing for both pressure and volume flow charts is a pulsation of each variables that is strictly connected with a pumping element operating phases. It has to be emphasize that fluid volume flow "-" sign (below zero point line) means that the fluid outflows from the particular point (on the other hand "+" sign represents fluid inflow). It should not be considered as a fluid movement direction changes, because it is stable by assumption. Observe that the described model is approximately idealized, which means that mechanical parameters used in the model were averaged and reduced to the selected points. The correctness of the simulation results consists in appropriate assumption of the hydro-mechanical structure operation, at this stage the numerical value is of the secondary importance.



**Figure 16:** The scheme diagram of the pneumo-hydraulic structure: I – the right ventricle of the pumping unit, II – the left ventricle of the pumping unit, III – the compliance tank of the body venous system, IV – the compliance tank of the body arterial system, V – the compliance tank of the pulmonary arterial system, VI – the compliance tank of the pulmonary venous system, VII – pneumatic control system of the pumping unit operation, VIII – the electronic timing system (pattern generator).

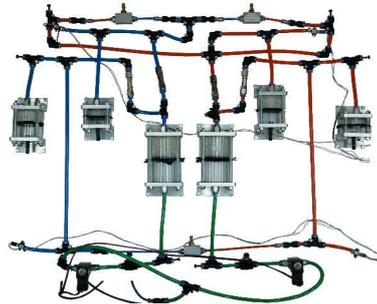
### 3 The Experimental System Description

On the basis of the hydro-mechanical model shown in Figure 3, the laboratory rig was designed to achieve the flow effect for the proposed system. In contrast to the model system, the experimental rig is based on pneumatic and hydraulic structures combination, which optimally reflects the kind of work of the cardiac muscle and blood vessels. Figure 16 presents the schematic construction of the so far described measurement system.

Compared to the simulation model, structurally the dual mechanical pumping cylinder was replaced by diaphragm pneumo-hydraulic chambers. Deformable membrane separates the hydraulic and pneumatic parts. The increase in air pressure of the pneumatic system – setting the pneumatic valves to the open state by the electronic timing unit causes the deformation of the latex membrane, and thus the fluid flow in the system. The fluent and frictionless membranes movement reflects cardiac muscle operation in a better way. Furthermore, marked as III, IV, V and VI (see Figure 16) regulated compli-

ance tanks imitate respective blood vessels parameters. The system was also expanded by throttle valves corresponding to the flow resistance for the following:  $R_{RL}$  – blood flow resistance of the right lung,  $R_{LL}$  – blood flow resistance of the left lung,  $R_{BS}$  – blood flow resistance of the body vascular system. Moreover, in order to follow the pressure changes in the interesting points (indicated in Figure 16 as 1, 2, 3, 4) corresponding to the simulation model points four pressure sensors connected to the data acquisition unit were provided. Figure 17 shows the prototype laboratory appearance which is the first trial version being developed successively.

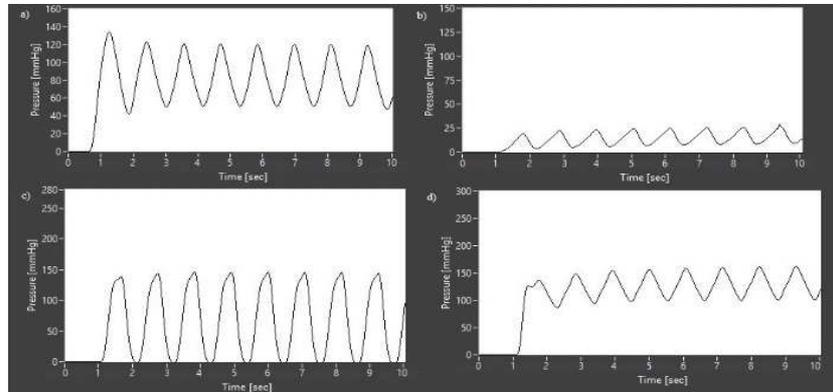
In order to check the quality and trend of measured pressure signals, there were some example, trial data registered in LabView environment which are shown below in Figure 18. Because of the fact that none of appropriate system parameters was selected/calculated at this stage, the resulting pressure waveforms should be regarded as trial and their accuracy should be evaluated only in terms of trend/course quality. It should be noted that the pressure waveforms in the recorded time period correspond to the subsequent pumping cycles (heart rate). Highly clear pressure signal pulsation may correspond to a systolic and diastolic pressure state. The experimental rig does not give an opportunity for tracking a volume flow values, so the comparison of this variable signal with simulation results was neglected at this stage. By calculating appropriate parameters for each hydraulic and pneumo-hydraulic laboratory components, we are able to obtain the correct pressure waveforms including also numerical values.



**Figure 17:** The operating part of the laboratory unit.

#### 4 Summary

Starting from the basic laws of mechanics and taking into account certain simplifications and assumptions connected with the properties and phenomena occurring in the system, we are able to provide a mathematical description of almost any system. The really big challenge is trying to move a perfect biological system to the level of its corresponding mechanical analogue. Although a huge degree of complexity of the human cardiovascular system requires numerous simplifications, however, as it was shown it is possible to create a closed-loop hydraulic circuit and the simulation model that correspond to anatomical indeed. Interactive simulation model developed in any computing environment is only a general description of the purpose object. The main focus of the analysis on the further level is the appropriate parameters selection in order to obtain specific simulation and measurement results [2]. Further exploration of the simulation model and improving the



**Figure 18:** Trial, pressure waveforms recorded for selected points (Figure 11): a) in the point 4, b) in the point 1, c) in the point 3, d) in the point 2.

experimental laboratory will better define the operation of the system, and thus will give the possibility of a more reliable comparison of operating a closed hydraulic system in terms of biological circulatory system.

### Acknowledgment

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