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Transcritical-like Bifurcation in a Model of a Bioreactor

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Abstract: A non-standard bifurcation, similar to a transcritical one, in a model of a bioreactor has been detected. This happens in a periodically-forced system with restrictions on the state space. The bioreactor is periodically fed with substrate. In the mathematical model, a periodic orbit approaches (without hitting) the restriction surface as a bifurcation parameter is varied. The way the orbit approaches the switching surface in the three-dimensional state space is such that it becomes parallel to the restriction surface. This phenomenon is somehow analogous to a transcritical bifurcation since another periodic orbit exists inside the restriction surface, but they do not collide. Full model and bifurcation description are shown.

Keywords: *bifurcation; bioreactor; periodically-forced; nonlinearity.*

Mathematics Subject Classification (2010): 34C23, 34C25, 34D20, 37C27.

1 Introduction

The biological wastewater treatment uses different techniques to create optimum environmental conditions that promote the removal of organic matter by using microorganisms. One of the most common is the activated sludge system, which uses aeration for bacteria [1, 2].

A least-used system, although it is a current research topic, is the Anaerobic Digestion, which operates in the absence of oxygen. The UASB (Upflow Anaerobic Sludge Blanket, or Upflow Anaerobic Reactor) is a type of tubular bioreactor operating in continuous mode and in upflow. These systems have an additional advantage because they can treat effluents with high organic load wastewater from agriculture and food industry tasks. The

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efficiency of this system is estimated to be a reduction of the COD (Chemical Oxygen Demand) of approximately 80%. COD is the parameter used to characterize the organic water pollution. This organic matter, in natural conditions, may be slowly biodegradable to CO2 and H2O. It can take from several days to a few million years, depending on the type of organic matter and biodegradation conditions. Another advantage is the energy production (methane). This also implies a low sludge production compared to aerobic digestion [5, 8].

1.1 Operation

In high-load reactors, UASB is one of the most used in the world. Its main advantage is due to the fact that it retains the biomass without the need for support. It does so through the formation of grains, which makes the reactor more economical and gives technical advantages over other advanced reactors. In general, the success of this reactor relies in that the grains forming the bioparticles are very active and thick. This gives the characteristics of a compact reactor without plugging problems, and without the high costs of traditional packaging.

Anaerobic degradation process is carried out in four stages: hydrolysis, acidogenesis, acetogenesis and methanogenesis.

- Hydrolysis : In this stage the extracellular enzymes of fermentative bacteria are responsible for converting the insoluble organic matter into soluble molecules. This complex molecules breaking is carried out in order for the bacteria to digest organic matter. This stage is very important for effluents treatment with a high content of organic matter.

- Acidogenesis : The compounds formed in the hydrolytic stage are absorbed through the cell wall of acidogenic bacteria. This performs an internal degradation process through microorganisms metabolism which produces carbon dioxide, hydrogen and volatile fatty acids (AGV).

- Acetogenesis : AGVs are converted into acetic acid through the effect of acetogenic bacteria, which also produce hydrogen and carbon dioxide.

- Metanogenesis : At this stage, methane is produced by the activity of a group of bacteria, which have two routes for gas generation. On the one hand, the acetoclastic path, where acetic acid molecules are converted into methane and carbon dioxide. On the other hand, we have the hydrogenophilic path, where methane is produced by a reduction reaction of carbon dioxide with hydrogen.

A complete and comprehensive model for anaerobic digestion called "The IWA (International Water Association) Anaerobic Digestion Model No 1 (AMD1)" features 26 state variables and 19 biological reaction schemes [6]. This model allows for standardization of existing systems and is a starting point for the development of specific models. Without losing sight of its limitations, a mathematical model can be an important tool to understand the kinetics of the processes involved, and to develop and implement good systems for the design and control of wastewater treatment plants.

Our model in this paper takes into account the material balance equations of three state variables: biomass (expressed as volatile suspended solids concentration), substrate (expressed as chemical oxygen demand COD) and volatile fatty acids (VFA). It is based on a kinetic model unstructured and non-segregated. It considers a total reaction which

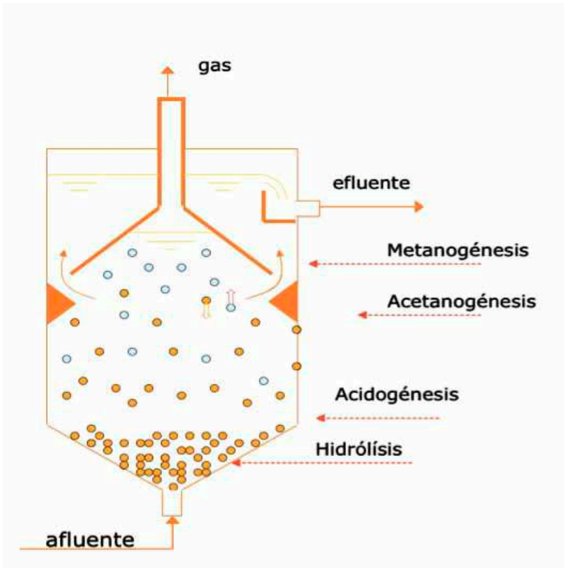

Figure 1: Scheme of an UASB.

groups various microbial colonies within the volatile soluble solids (VSS), and substrates involved in chemical oxygen demand (COD) [4].

2 Modeling

Round 1950, Monod suggested that the bacteria growth rate depends not only on the microorganisms concentration, but also on the substrate concentration [3]. It is currently accepted that conversion of soluble substrates during anaerobic stage is governed by Monod equation, which describes this relationship in a way similar to the one proposed for the Michaelis-Menten enzyme-substrate interaction.

$$\mu = \mu_{max}\frac{S}{K+S} \tag{1}$$

and we have

$$\dot{S} = D(S^{in} - S) - Y(\mu_{max}\frac{S}{(K+S)})X, \tag{2}$$

$$\dot{X} = D(X^{in} - \alpha X) + (\mu_{max}\frac{S}{(K+S)})X. \tag{3}$$

In this paper we consider the concentrations of substrate and biomass in the input current as periodic functions like

$$S^{in}(t) = S^{in}(1 + \beta\cos(\omega t)), \tag{4}$$

$$X^{in}(t) = X^{in}(1 + \delta\cos(\omega t)). \tag{5}$$

Thus our final model is

$$\dot{S} = D(S^{in}(1+\beta\cos(\omega t)) - S) - Y((\mu_{max}\frac{S}{(K+S)})X, \tag{6}$$

$$\dot{X} = D(X^{in}(1+\delta\cos(\omega t)) - \alpha X) + ((\mu_{max}\frac{S}{(K+S)})X, \tag{7}$$

where S is the substrate concentration in the reactor (mg / l COD), D is the dilution factor (d^{-1}), S^{in} is the substrate concentration in the inlet stream (mg / l COD), $\alpha = 1-\eta$ where η is the efficiency of the separator (0.93), X is the biomass concentration in the reactor (mg / l VSS), X^{in} is the biomass concentration in the inlet stream, μ_{max} is the maximum growth rate of microorganisms (d^{-1}), K is the Monod constant, also called half-saturation constant (kg COD/m3), β is the amplitude of the forcing function for the substrate, δ is the amplitude of the forcing function for the biomass and finally, ω is the frequency of the forcing functions.

Since both S and X must fulfill $S \geq 0$ and $X \geq 0$ for meaningful operation, we consider $X = 0$ and $S = 0$ as restriction surfaces.

3 Analysis

We study four cases:

3.1 First case

We first consider that we have no forcing. This is, the amplitude for the forcing is zero. Also we consider that the inflow wastewater contains biomass.

Thus we take

$$\beta = 0, \quad \delta = 0, \quad X^{in} = 240,$$

(certain values for parameters are taken from experiments in [7]).

Then, for the equilibrium points we have

$$D(S^{in} - S) - Y((\mu_{max}\frac{S}{(K+S)})X = 0, \tag{8}$$

$$D(X^{in} - \alpha X) + ((\mu_{max}\frac{S}{(K+S)})X = 0. \tag{9}$$

After some algebraic operations we get

$$X = (S^{in} + YX^{in} - S)/(\alpha Y)$$

and thus

$$(\mu_{max} - D\alpha)S^2 + (DS^{in}\alpha - DK\alpha - \mu_{max}S^{in} - Y\mu_{max}X^{in})S + DK\alpha S^{in} = 0.$$

The values for the parameters were taken from the experimental work in Munoz [7]. $D = 3$, $S^{in} = 3000$, $X^{in} = 240$, $\mu_{max} = 1.32$, $\alpha = 0.07$, $Y = 3.35$ and $K = 5522$.

Two equilibriums are obtained, but one of them is not physically possible.

3.2 Second case

In general, the input current does not contain biomass composition. Thus we consider now that we have no forcing and no biomass in the inflow

$$\beta = 0, \quad \delta = 0, \quad X^{in} = 0.$$

We have then

$$D(S^{in} - S) - Y((\mu_{max}\frac{S}{(K+S)})X = 0, \tag{10}$$

$$D(-\alpha X) + ((\mu_{max}\frac{S}{(K+S)})X = 0. \tag{11}$$

By inspection, we can deduce that both equations are fulfilled when $X = 0$, which corresponds to bioreactor washout condition. This is bad operation for the reactor.

Also, an equilibrium point is found for

$$S = DK\alpha/(\mu_{max} - D\alpha)$$

and

$$X = (S^{in}\mu_{max} - DS^{in}\alpha - DK\alpha)/(Y\alpha),$$

which is physically possible.

3.3 Third case

Now we consider that $\omega = 0$, thus we do not have periodic forcing. But we still have the effect of parameters β and δ. We also consider that wastewater has biomass. Thus,

$$\omega = 0, \quad X^{in} = 240,$$

and the equations for the equilibrium points are

$$D(S^{in}(1+\beta) - S) - Y((\mu_{max}\frac{S}{(K+S)})X = 0, \tag{12}$$

$$D(X^{in}(1+\delta) - \alpha X) + ((\mu_{max}\frac{S}{(K+S)})X = 0. \tag{13}$$

After some algebra we get

$$X = (S^{in}(1+\beta) + YX^{in}(1+\delta) - S)/(\alpha Y)$$

and then

$$(\mu_{max} - D\alpha)S^2 + (DS^{in}(1+\beta)\alpha - DK\alpha - \mu_{max}S^{in}(1+\beta)\mu_{max}X^{in}(1+\delta))S + DK\alpha S^{in} = 0.$$

For positive β and δ we obtain two equilibriums, but one of them is always unfeasible.

3.4 Forth case

We consider again that we have no periodic forcing but we have influence through β and δ. Moreover, we assume now that we have no biomass in the input flow. Thus,

$$\omega = 0 \quad X^{in} = 0.$$

Then, the equations for the equilibrium points are

$$D(S^{in}(1+\beta) - S) - Y((\mu_{max}\frac{S}{(K+S)})X = 0, \tag{14}$$

$$D(-\alpha X) + ((\mu_{max}\frac{S}{(K+S)})X = 0. \tag{15}$$

Similarly to the second case, we have an equilibrium point which corresponds to washout condition ($X = 0$), and a second equilibrium at

$$S = DK\alpha/(\mu_{max} - D\alpha)$$

and

$$X = (S^{in}(1+\beta)\mu_{max} - DS^{in}(1+\beta)\alpha - DK\alpha)/(Y\alpha),$$

which is physically feasible.

In summary, balance characteristics depend on the value of X^{in}. When the input current is present, biomass generates two equilibrium points, but only one is physically meaningful. On the other hand, when we have no input biomass, one of the equilibrium points corresponds to washout.

4 Numerical Results

We consider now real periodic forcing where $\omega \neq 0$. No closed-form solutions are available now and thus we rely on numerical simulations.

The following figures (Figs. 2–4) show waveforms, orbits in the phase space and bifurcation diagrams when we vary the most significant parameter, namely the dilution factor D. The almost non-smooth point in the bifurcation diagrams is due to washout condition. At this point, both periodic orbits (one corresponding to washout, inside the restriction surface; and another one without touching the restriction surface) get infinitely closer. We called this phenomenon a transcritical-like bifurcation since although there is not really a qualitative change in the state space (and thus it is not a bifurcation), it resembles very much a transcritical bifurcation. Moreover, from a practical view, both orbits are indistinguishable and can be considered as if the washout orbit is the only one which exists.

4.1 Transcritical-like bifurcation

In this subsection we describe an exotic bifurcation which is found in the system.

Namely, since in this case we have a periodic forced system, the natural phase space is $\mathbb{S} \times \mathbb{R}^2$, where $\mathbb{S}$ corresponds to the unit circle.

Then, as parameter D is increased, two periodic orbits, one lying in the washout condition, with $X = 0$ and another one with $X \neq 0$ approach. The non-washout periodic orbits aligns with the washout one as parameter D increases, and gets infinitely closer to it. This behaviour is not generic and thus we consider it as a non-standard bifurcation (see Figs. 5–7).

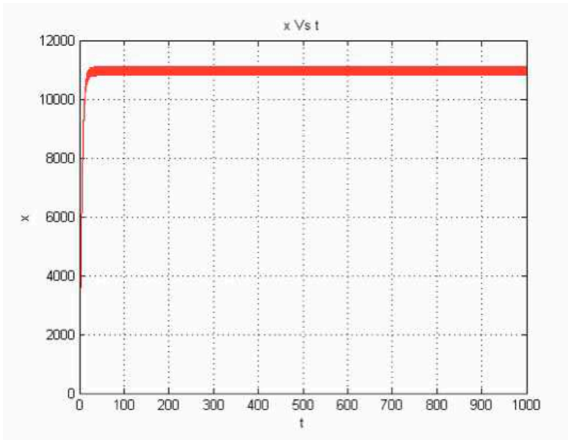

Figure 2: Waveform for the biomass evolution when $D = 5$.

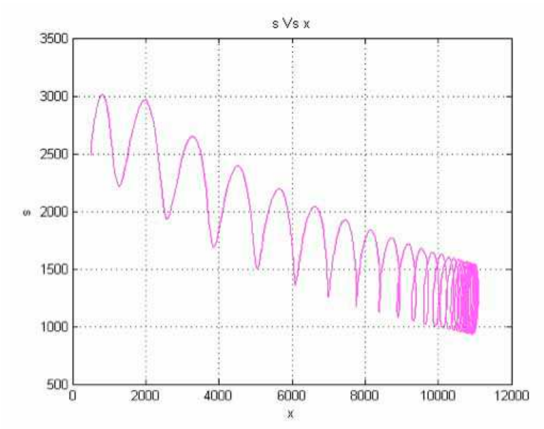

Figure 3: Orbit in the phase space when $D = 5$.

5 Conclusion

Based on dynamic analysis of the model, it was found that in general, the study of an UASB reactor has good stability to different operating conditions, especially when the effluent to be treated is from a leachate. This is because in this case the washout phenomenon can occur.

The system stops working properly under washout conditions. According to the results of different simulations, the parameters that most influence this condition are the dilution factor D (which is related to the speed with which the effluent passes through the reactor) and the solid − liquid − gas separator efficiency parameter α.

Numerical simulations showed that, depending on the biomass which is present in the inflow, washout or good operations are possible. Also, when periodic forcing is considered, periodic orbits can be found, as expected. A non-standard transcritical-like bifurcation of periodic orbits was also found.

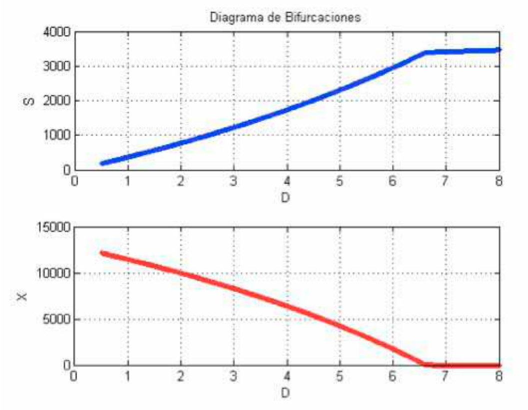

Figure 4: Bifurcation diagrams for the substrate and the biomass as parameter D (dilution time) is varied.

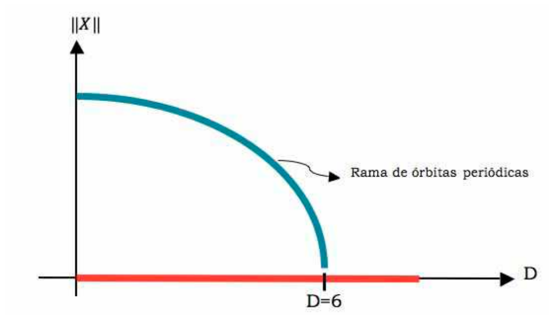

Figure 5: Bifurcation diagram as parameter D is varied. Two periodic orbits get infinitely closer at a transcritical-like bifurcation point.

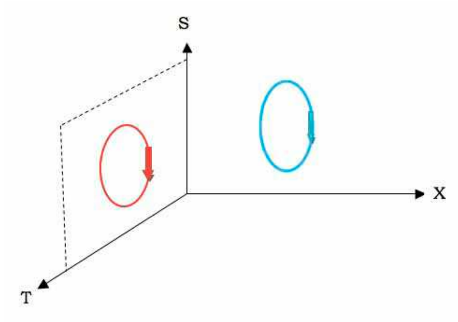

Figure 6: Two periodic orbits which coexist in the phase space. One of them corresponds to washout condition since $X^{in} = 0$.

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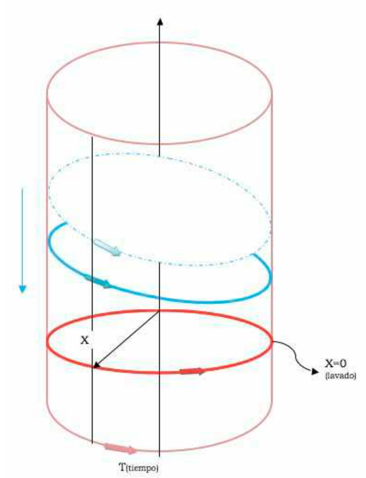

Figure 7: Coexistence of two periodic orbits in the natural space, close to collission.

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