Nonlinear Dynamics and Systems Theory, 15 (4) (2015) 344–359

Fuzzy Modeling and Robust Pole Assignment Control for Difference Uncertain Systems

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Received: June 29, 2015; Revised: November 4, 2015

Abstract: This paper deals with fuzzy modeling and robust control of nonlinear systems affected by bounded uncertainties. The proposed fuzzy model is composed of two parts: a linear uncertain part and a nonlinear one. The linear uncertain part is obtained by the nominal system linearization around some operating points. The nonlinear part is approximated by a Takagi-Sugeno fuzzy system whose parameters are estimated using the descent gradient method. A robust pole assignment called 'pole colouring' is used for the system control. This strategy of control is synthesized based only on the linear uncertain part of the decomposed model. Finally, two simulation examples are treated to illustrate the effectiveness of the proposed fuzzy modeling and control approaches.

Keywords: *uncertain nonlinear system; fuzzy modeling; Takagi-Sugeno system; linearization; robust pole assignment.*

Mathematics Subject Classification (2010): 03B52, 62K25.

1 Introduction

The modeling of an uncertain nonlinear system is an important step for the system analysis and control. It consists in developing a mathematical model ensuring the required accuracy and having a useful structure. In fact, a model must reproduce correctly the dynamics of the considered system even in the presence of nonlinearities, uncertainties and perturbations. These constraints make the classical modeling methods limited. So the evolutionist techniques, such as fuzzy systems [1] and neural networks [2] are considered as potential solutions for this problem. Indeed, they are considered as universal

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approximators [3, 4]. So, they can reproduce any nonlinear dynamics with an arbitrary accuracy.

In this paper, fuzzy systems are considered for nonlinear uncertain systems modeling. They are classified as intelligent modeling tools. A fuzzy system is described by a set of IF-THEN fuzzy rules. According to fuzzy rules conclusions, two types of fuzzy systems are distinguished: Mamdani fuzzy systems [5] and Takagi-Sugeno fuzzy ones [6]. Mamadani fuzzy systems present linguistic conclusions. However, Takagi-Sugeno fuzzy systems possess numerical ones. Two types of fuzzy rules generation approaches are distinguished: manual and analytic ones.

Takagi-Sugeno fuzzy systems are considered as powerful modeling tools [7]. Their parameters are often identified using training algorithms such as descent gradient method [8–10], recursive least square algorithm [11], orthogonal least square algorithm [12], genetic algorithms [13, 14] and robust algorithms [15, 16]. There are several works about fuzzy modeling of nonlinear systems [17–20] and also uncertain ones [21, 22].

A real system is by nature uncertain. So, the use of classical control methods doesn′t guarantee the desired performance indexes. In fact, when the system parameters move from the nominal ones, the desired performances are not satisfied whence the necessity of the use of a robust control where uncertainties are explicitly taken into account. In the literature, there are several researches about the robust control such as the sliding mode [23,24], the gain scheduling [25], the H_2 performance [26], the H_∞ performance [27] and the robust tracking control [28, 29]. Also, there are some researches about robust control for linear uncertain discrete-time systems such as robust pole assignment. It is an interesting control method for linear uncertain systems. It consists of the location of the closed-loop system poles by considering the parameters variations. Nurges [30] proposed the location of the characteristic equation parameters in a stable polytope, also the uncertainties effects on characteristic equation coefficients could be minimized [31–33]. The minimization of the maximum distance between desired poles and obtained ones was proposed by Soylemez and Munro [34]. Discrete-time pole region was approximated by linear matrix inequality for robust pole assignment control design [35, 36].

These robust control techniques could be combined with fuzzy logic tools to benefit from those advantages [37–43]. For example Abid et al [37] used a robust fuzzy sliding mode controller for nonlinear discrete-time systems with parametric uncertainties. Also, Wu [38] proposed a robust H_2 fuzzy controller for the same purpose.

In this paper, fuzzy modeling and robust pole assignment control for uncertain nonlinear systems are considered. The proposed model involves two parts: (1) a linear uncertain one whose parameters are affected by bounded uncertainties and (2) a nonlinear one which is approximated by a Takagi-Sugeno fuzzy system. The linear uncertain part parameters are obtained by the nominal system linearization around some operating points. The Takagi-Sugeno fuzzy system synthesis needs two main phases: (1) the premises variables determination and (2) the conclusions parameters estimation. In fact, the premises variables determination consists essentially in input space partitioning and the conclusions parameters are estimated using the descent gradient method.

The robust pole assignment control proposed by Soylemez and Munro [34] is considered for the control of nonlinear uncertain systems. It is synthesized based only on the linear uncertain part of the developed fuzzy model. It consists in optimizing a cost function by varying the uncertain parameters. The nonlinear part of the model is supposed to be an additive perturbation.

This paper is organized as follows. In Section 2, the problem statement is presented.

The proposed fuzzy modeling approach is explained in Section 3. In Section 4, the used robust pole assignment control is detailed. In Section 5, two simulation examples are presented to illustrate the proposed modeling and control approaches. Finally, concluding remarks are given in Section 6.

2 Problem Statement

Consider the modeling and control problems of the class of nonlinear uncertain systems described by the following expression:

$$y(k+1) = F[y(k), \ldots, y(k-n+1), u(k), \ldots, u(k-m+1), p], \tag{1}$$

where u and y are the system input and the system output, respectively. F is a known nonlinear function and p is a parameters vector affected by additive uncertainties.

$$p = p_0 + \Delta p, \tag{2}$$

where p_0 is the nominal parameters vector and Δp is the uncertainties vector affecting the system.

The proposed modeling approach consists in dividing the behavior of the considered uncertain nonlinear system into two parts: a linear uncertain one y_l and a nonlinear one y_{nl} [44, 45]

$$y^m(k+1) = y_l(k+1) + y_{nl}(k+1), \tag{3}$$

where y^m is the model output.

This modeling approach needs two main steps:

- Step 1: the determination of the linear uncertain part parameters.
- Step 2: the approximation of the nonlinear part y_{nl} by a Takagi-Sugeno fuzzy system.

In this paper, a robust pole assignment control is used for the system control. It is synthesized considering only the linear uncertain part y_l of the model 3. The nonlinear part y_{nl} is considered as an additive perturbation. In the following, the proposed techniques for the model development will be presented. Also, the used approach for robust pole assignment control will be detailed.

3 Fuzzy Model Identification

In this section, the proposed fuzzy modeling approach is detailed. The system dynamics is decomposed into two terms: a linear uncertain expression and a nonlinear one. It will be compared with a global Takagi-Sugeno fuzzy model to demonstrate its interest.

3.1 Decomposed fuzzy model

The decomposed fuzzy model identification consists in determining the linear uncertain part y_l and estimating the nonlinear part y_{nl} by a Takagi-Sugeno fuzzy system. For each part computation, the structure and parameters determinations are necessary.

3.1.1 Linear uncertain part

The first part y_l is a linear expression with uncertain bounded parameters.

$$y_l(k+1) = -\sum_{i=1}^{n} a_{iu}(k)\, y(k-i+1) + \sum_{j=1}^{m} b_{ju}(k)\, u(k-j+1). \tag{4}$$

For a_{iu} and b_{ju} the index u indicates uncertain parameters. a_{iu},$i = \overline{1,n}$ and b_{ju},$j = \overline{1,m}$ are bounded uncertain parameters. It is to be noted that the coefficients a_{iu} and b_{ju} are obtained by the nominal system linearization around some operating points. In fact, around an operating point (U_l,Y_l) , the dynamics of the considered system is described by the expression

$$\delta y(k+1) = -\sum_{i=1}^{n} a_{il}\, \delta y(k-i+1) + \sum_{j=1}^{m} b_{jl}\, \delta u(k-j+1), \tag{5}$$

where

$$a_{il} = -\frac{\partial y(k+1)}{\partial y(k-i+1)}|_{(U_l,Y_l)}, \tag{6}$$

$$b_{jl} = \frac{\partial y(k+1)}{\partial u(k-j+1)}|_{(U_l,Y_l)}, \tag{7}$$

$$\delta y(k-i+1) = y(k-i+1) - Y_l,\ i = \overline{1,n}, \tag{8}$$

$$\delta u(k-j+1) = u(k-j+1) - U_l,\ j = \overline{1,m}, \tag{9}$$

$$l = \overline{1,L},$$

L is the considered operating points number. Using the expressions (8) and (9), the system dynamics is represented as follows:

$$y(k+1) = -\sum_{i=1}^{n} a_{il}\, y(k-i+1) + \sum_{j=1}^{m} b_{jl}\, u(k-j+1) + (Y_l + \sum_{i=1}^{n} a_{il}\, Y_l - \sum_{j=1}^{m} b_{jl}\, U_l). \tag{10}$$

So, the linear part y_l is given by the expression

$$y_l(k+1) = -\sum_{i=1}^{n} a_{il}\, y(k-i+1) + \sum_{j=1}^{m} b_{jl}\, u(k-j+1). \tag{11}$$

The nominal system must be linearized around some operating points to describe the dynamics of the considered nonlinear system for the global operating area. The operating points must be chosen properly. In fact, they have to be distributed on the global operating area. So, the obtained coefficients a_{iu},$i = \overline{1,n}$ and b_{ju},$j = \overline{1,m}$ are bounded uncertain parameters:

$$a_{iu} \in [min_{l=1\cdots L}\, a_{il}\ ;\ max_{l=1\cdots L}\, a_{il}], \quad b_{ju} \in [min_{l=1\cdots L}\, b_{jl}\ ;\ max_{l=1\cdots L}\, b_{jl}].$$

It is to be noted that the static terms $(Y_l + \sum_{i=1}^{n} a_{il}\, Y_l - \sum_{j=1}^{m} b_{jl}\, U_l)$ will be taken into account for the nonlinear part y_{nl} synthesis.

3.1.2 Nonlinear part

The nonlinear part y_{nl} in the expression (3) is approximated by a Takagi-Sugeno fuzzy system. It is described by a set of IF-THEN fuzzy rules having the following form:

$$if\ u(k)\ is\ A_r^1 \cdots and\ u(k-m+1)\ is\ A_r^m\ and\ y(k)\ is\ B_r^1 \cdots and\ y(k-n+1)\ is\ B_r^n,$$
$$then\ y_{nr}(k+1) = -\sum_{i=1}^{n} e_i^r\, y(k-i+1) + \sum_{j=1}^{m} f_j^r\, u(k-j+1), \quad (12)$$

where $r = \overline{1, R}$, R is the rules number. It is fixed after several simulations in order to get a compromise between a minimal error and a reasonable rules number. Consider x the premise variable vector such as: $x = [u(k), \ldots, u(k-m+1), y(k), \ldots, y(k-n+1)]$. The used membership function is the Gaussian

$$\mu_r(x_t) = \exp[-\frac{(x_t - c_t^r)^2}{2(\sigma_t^r)^2}], \quad t = \overline{1, n+m}. \quad (13)$$

The dynamics of the nonlinear part y_{nl} is described by the local models interpolation

$$y_{nl}(k+1) = \frac{\sum_{r=1}^{R} \alpha_r\, y_{nr}(k+1)}{\sum_{r=1}^{R} \alpha_r}, \quad (14)$$

where

$$\alpha_r = \prod_{t=1}^{n+m} \mu_r(x_t). \quad (15)$$

Consider $\theta_p = [c_t^r, \sigma_t^r, r = \overline{1, R}, t = \overline{1, n+m}]$ the vector of the premises parameters of the fuzzy system. c_t^r and σ_t^r are, respectively, the center and the width of the Gaussian function relating to the r^{th} rule and the t^{th} member of the premise variable vector x. They are determined manually. In fact, the centers c_t^r are determined by the operating area partitioning and the widths σ_t^r are fixed such as there is neither discontinuity nor overlapping between the membership functions. However, the vector of the conclusions parameters is noted θ_c such as $\theta_c = [e_i^r,\ f_j^r,\ r = \overline{1, R},\ i = \overline{1, n},\ j = \overline{1, m}]$. The conclusions parameters are determined automatically. Indeed, they are estimated using the descent gradient method. The criterion to minimize is given by the expression (16). It is minimized through the minimization of the error corresponding to each example

$$J_c = \sum_{k=1}^{N} e(k), \quad (16)$$

where

$$e(k) = \frac{1}{2}[y^m(k) - y(k)]^2, \quad (17)$$

N is the size of the training data set.

The conclusions parameters are updated using the following expression

$$\theta_c(\tau) = \theta_c(\tau-1) - \epsilon \frac{\partial e(k)}{\partial\, \theta_c(\tau-1)}, \quad (18)$$

where τ is the iteration counter and ϵ is the learning rate. $\frac{\partial e(k)}{\partial\,\theta_c(\tau-1)}$ is given by the expression

$$\frac{\partial e(k)}{\partial\,\theta_c(\tau-1)} = [y^m(k) - y(k)]\,\frac{\partial y_{nl}(k)}{\partial\,\theta_c(\tau-1)}. \tag{19}$$

It should be noted that the linear part has been designed referring only to the nominal nonlinear system. So, the uncertain parameters must be taken into account for the design of the nonlinear part y_{nl}. It is done by varying these parameters to collect the training and the validation data sets.

3.2 Global Takagi-Sugeno fuzzy model

The Takagi-Sugeno fuzzy systems are usually used for the nonlinear systems description. They are described by a set of IF-THEN fuzzy rules having the following form:

$$if\ u(k)\ is\ A_r^1 \cdots and\ u(k-m+1)\ is\ A_r^m\ and\ y(k)\ is\ B_r^1 \cdots and\ y(k-n+1)\ is\ B_r^n,$$
$$then\ y_r^{mc}(k+1) = -\sum_{i=1}^{n} g_i^r\, y(k-i+1) + \sum_{j=1}^{m} h_j^r\, u(k-j+1) \tag{20}$$

with $r = \overline{1, R}$.

The membership function is the Gaussian (13). The premises variables and the rules number are those used for the decomposed fuzzy model (3). The dynamic of the considered system is approximated by the local models interpolation

$$y^{mc}(k+1) = \frac{\sum_{r=1}^{R}\ \alpha_r\, y_r^{mc}(k+1)}{\sum_{r=1}^{R}\ \alpha_r}, \tag{21}$$

where α_r is given by expression (15) and y^{mc} is the global Takagi-Sugeno fuzzy model output.

The conclusions parameters are adjusted using the descent gradient method. The criterion to minimize is given by the expression (16) where $e(k)$ is the following:

$$e(k) = \frac{1}{2}[y^{mc}(k) - y(k)]^2. \tag{22}$$

The control of nonlinear uncertain systems (1) using the prescribed decomposed fuzzy model (3) is considered. But, the control synthesis will be based only on the linear uncertain part y_l. Otherwise, the nonlinear part y_{nl} will be considered as an additive perturbation. In this case, the linear robust controllers as a robust pole assignment one can be exploited.

4 Robust Pole Assignment Control

The robust pole assignment control proposed by Soylemez and Munro [34] is adopted for the control of linear uncertain systems. It can be used for continuous-time and also discrete-time linear systems affected by bounded uncertainties.

Consider a linear discrete-time system affected by bounded uncertainties and described by the following transfer function

$$G(q^{-1}) = \frac{B_u(q^{-1})}{A_u(q^{-1})} = \frac{b_{1u}q^{-1} + \ \cdots\ + b_{mu}q^{-m}}{1 + a_{1u}q^{-1} + \ \cdots\ + a_{nu}q^{-n}}, \tag{23}$$

where $b_{ju} \in [b_{ju}^- \; ; \; b_{ju}^+]$, $j = \overline{1,m}$ and $a_{iu} \in [a_{iu}^- \; ; \; a_{iu}^+], i = \overline{1,n}$.

The proposed controller is a PID one described by the expression

$$u(k) = u(k-1) + q_0 \, e(k) + q_1 \, e(k-1) + q_2 \, e(k-2), \tag{24}$$

where

$$e(k) = y^d(k) - y(k), \tag{25}$$

y^d is the desired output.

When using a PID controller (24), the closed-loop system has the characteristic equation (26) which is also affected by uncertain parameters

$$W(q^{-1}, p) = \sum_i W_i(p) \, q^{-i}, \tag{26}$$

where p is the vector of uncertain parameters affecting the system.

The controller parameters θ are obtained through the minimization of the following cost function

$$J = \min_p \, (J_p), \tag{27}$$

$$\theta = [q_0; \; q_1; \; q_2]. \tag{28}$$

There are multiple choices for the criterion J_p. It can be related to desired performances like rise time, settling time $\cdots$ The simplest choice is the minimization of the maximum distance between the nominal poles and the corresponding perturbed ones of the closed-loop system. So, every pole takes one place in a disc centered on the corresponding nominal pole

$$J_p = \max_{i=1 \cdots M} (|\lambda_i^0 - \lambda_i^p|), \tag{29}$$

where λ_i^0 and λ_i^p are the nominal pole and its corresponding perturbed one of the closed-loop system, respectively, M is the closed-loop system order. The controller synthesis corresponds to an optimization problem which is solved using the function *fminimax* from the Matlab toolbox.

5 Simulation Results

Two simulation examples are considered to show the effectiveness of the proposed modeling approach and the performances of the suggested control scheme. The first example is a chemical reactor and the second one is an academic system.

5.1 First example: Chemical reactor

Consider the modeling and the control problems of the chemical reactor [46] whose dynamics are described by the expression (30).

$$y(k+1) = A_1 + B_1 \, u(k) + A_2 \, y(k) + q(k) \, B_2 \, u^3(k) + A_3 \, y(k-1) \, u(k-1) \, u(k), \tag{30}$$

where $[A_1, \; A_2, \; A_3, \; B_1, \; B_2] = [0.558, \; 0.116, \; -0.034, \; 0.583, \; -0.127]$, q is an uncertain parameter supposed to be variable and bounded in an interval: $q(k) \in [0.9\,; 1.1]$, u is the input flow of the product A and y is the concentration of the product B.

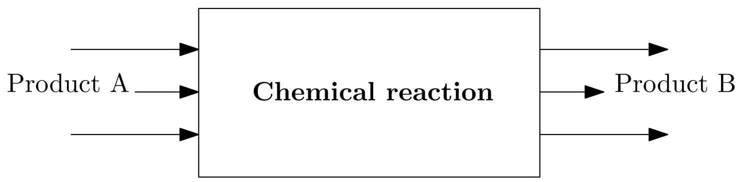

Figure 1: Chemical reactor.

5.1.1 Fuzzy modeling

The dynamics of the chemical reactor is decomposed as in equation (3). The linear uncertain part y_l is presented by the following expression

$$y_l(k+1) = -a_1\, y(k) - a_{2u}(k)\, y(k-1) + b_{1u}(k)\, u(k) + b_{2u}(k)\, u(k-1). \tag{31}$$

Since $\frac{\partial y(k+1)}{\partial y(k)}$ is constant, a_1 is a certain parameter. $a_{2u}(k)$, $b_{1u}(k)$ and $b_{2u}(k)$ are uncertain bounded parameters. They are obtained by the nominal system linearization around two operating points: $a_1 = -0.116$, $a_{2u}(k) \in [0.0014\,;\, 0.0218]$, $b_{1u}(k) \in [0.3103\,;\, 0.5626]$ and $b_{2u}(k) \in [-0.0288\,;\, -0.0052]$.

The nonlinear part y_{nl} in the expression (3) is presented by a set of IF-THEN fuzzy rules:

$$\begin{aligned} &if\ u(k)\ is\ A_r^1\ and\ u(k-1)\ is\ A_r^2\ and\ y(k)\ is\ B_r^1\ and\ y(k-1)\ is\ B_r^2 \\ &\quad then\ y_{nr}(k+1) = -e_1^r\, y(k) - e_2^r\, y(k-1) + f_1^r\, u(k) + f_2^r\, u(k-1). \end{aligned} \tag{32}$$

The obtained modeling results for the training set are given in Figure 2.

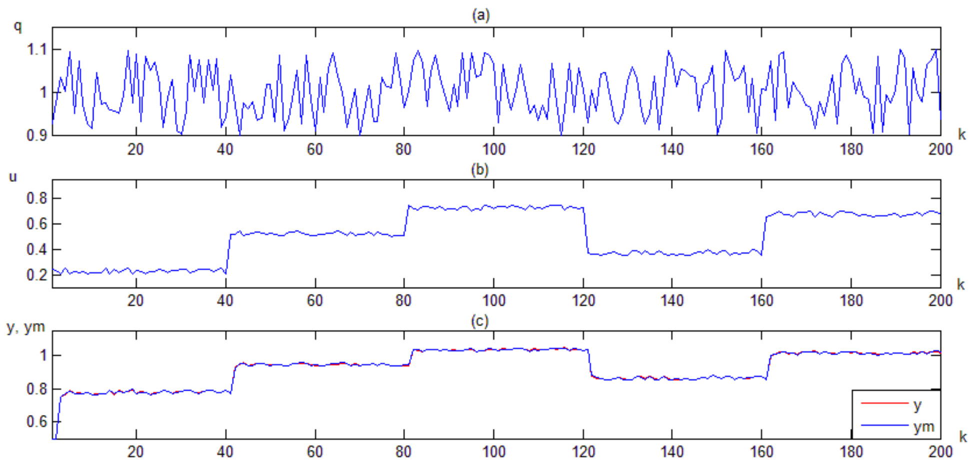

Figure 2: Evolution of the uncertain parameter (a), the input signal (b), the system and the model outputs for the training set.

The obtained results for the validation set are presented in Figure 3.

In order to compare the proposed fuzzy modeling method to the classical one, a global Takagi-Sugeno fuzzy model will be developed. It is described by a set of IF-THEN fuzzy

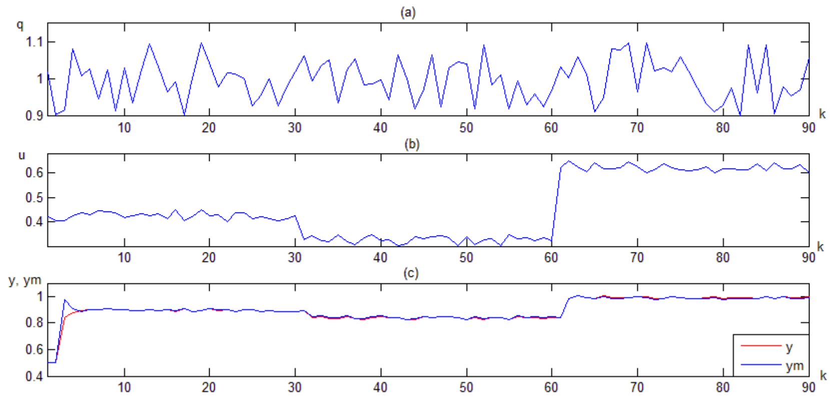

Figure 3: Evolution of the uncertain parameter (a), the input signal (b), the system and the model outputs for the validation set.

rules

$$\begin{aligned}&if\ u(k)\ is\ A_r^1\ and\ u(k-1)\ is\ A_r^2\ and\ y(k)\ is\ B_r^1\ and\ y(k-1)\ is\ B_r^2\\&then\ y_r^{mc}(k+1) = -g_1^r\, y(k) - g_2^r\, y(k-1) + h_1^r\, u(k) + h_2^r\, u(k-1).\end{aligned} \tag{33}$$

The conclusions parameters are estimated using the descent gradient method. The rules number is $R = 16$ and the learning rate is $\epsilon = 0.5$. For both models, the same system input and output partitioning are considered. In addition, the same training and validation sets are used.

The average value of the error committed by each model is evaluated in the validation set to demonstrate the effectiveness of the proposed modeling approaches

$$E = \frac{\sum_{k=1}^{N} |y(k) - y^m(k)|}{N}. \tag{34}$$

	Decomposed fuzzy model	Global Takgi-Sugeno fuzzy model
J (final)	0.0008	0.0008
Iteration number	6462	13740
E	0.0046	0.0049

Table 1: Comparison between the decomposed fuzzy model and the global Takagi-Sugeno fuzzy one.

According to this table, for the same criterion value the decomposed model requires less iterations number than the classical one. It is due to the system dynamics decomposition effect which accelerates the training.

5.1.2 Robust pole assignment control

The chemical reactor is controlled by the PID controller (24) whose parameters are determined using the described robust pole assignment and referring only to the linear uncertain part (31) of the decomposed fuzzy model. The poles are located considering the parameter uncertainties of this part. The objective is to have a double pole $z_1 = z_2 = 0.2$ and two poles such as $z_3 = 0.1$ and $z_4 = 0.3$. The controller parameters are obtained through the minimization of the cost function (27). The results of the optimization problem resolution are the following: $q_0 = 0.2303$, $q_1 = 0.1906$ and $q_2 = 0.0118$.

For the uncertain parameter variations given in Figure 4, the results of the proposed control scheme are illustrated in Figure 5.

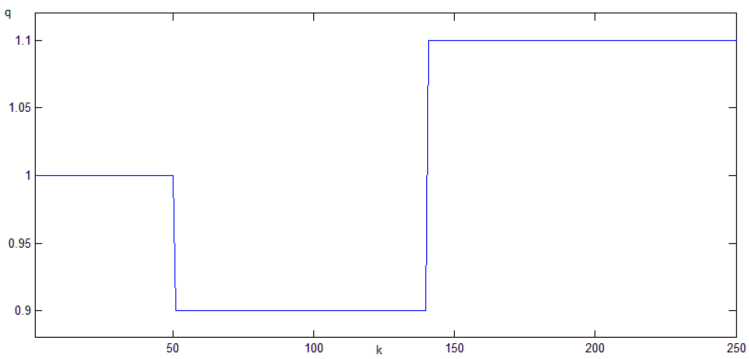

Figure 4: Evolution of the uncertain parameter $q(k)$.

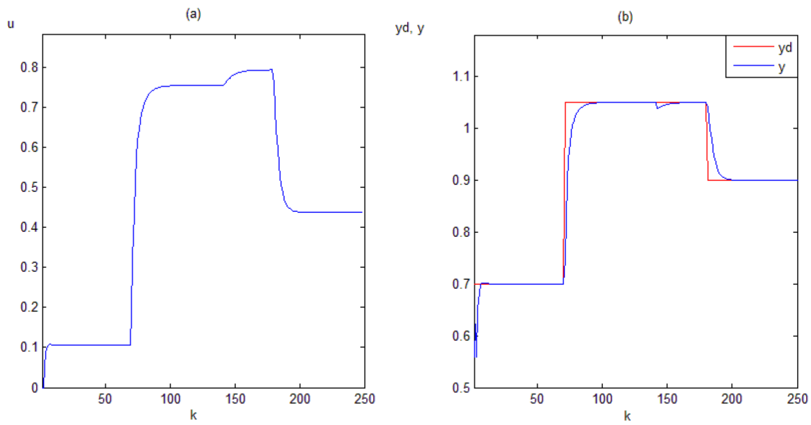

Figure 5: Evolution of the robust PID control action (a), desired output and system output (b).

For the chosen desired signal and uncertain parameter variations, the closed-loop system has acceptable performances.

5.2 Second example

Consider the nonlinear uncertain system described by the following expression [47]:

$$y(k+1) = \frac{y(k)\, y(k-1)\, y(k-2)\, u(k)\, [y(k-2) - 1 - q(k)]}{1 + y^2(k-1) + y^2(k-2)} + \frac{u(k)}{1 + y^2(k-1) + y^2(k-2)}, \tag{35}$$

where q is a bounded uncertain parameter such as: $q(k) \in [0\ ;\ 0.5]$, u and y are the system input and output, respectively.

5.2.1 Fuzzy modeling

The dynamics of the above system is described by the decomposed model (3). The linear uncertain part y_l is presented by the expression

$$y_l(k+1) = -a_{1u}(k)\, y(k) - a_{2u}(k)\, y(k-1) - a_{3u}(k)\, y(k-2) + b_{1u}(k)\, u(k), \tag{36}$$

where $a_{1u}(k)$, $a_{2u}(k)$, $a_{3u}(k)$ and $b_{1u}(k)$ are uncertain bounded parameters. They are obtained by the nominal system linearization around some operating points: $a_{1u}(k) \in [-0.0761\ ;\ 0.4003]$, $a_{2u}(k) \in [-0.4386\ ;\ 0]$, $a_{3u}(k) \in [-0.3516\ ;\ 0.0543]$ and $b_{1u}(k) \in [0.5924\ ;\ 1]$.

The nonlinear part y_{nl} in the expression (3) is described by a set of IF-THEN fuzzy rules

$$\begin{aligned} &if\ u(k)\ is\ A_r^1\ and\ y(k)\ is\ B_r^1\ and\ y(k-1)\ is\ B_r^2\ and\ y(k-2)\ is\ B_r^3 \\ &\quad then\ y_{nr}(k+1) = -e_1^r\, y(k) - e_2^r\, y(k-1) - e_3^r\, y(k-2) + f_1^r\, u(k). \end{aligned} \tag{37}$$

The rules number is $R = 16$ and the learning rate is $\epsilon = 0.2$. The obtained modeling results for the training set are given in Figure 6.

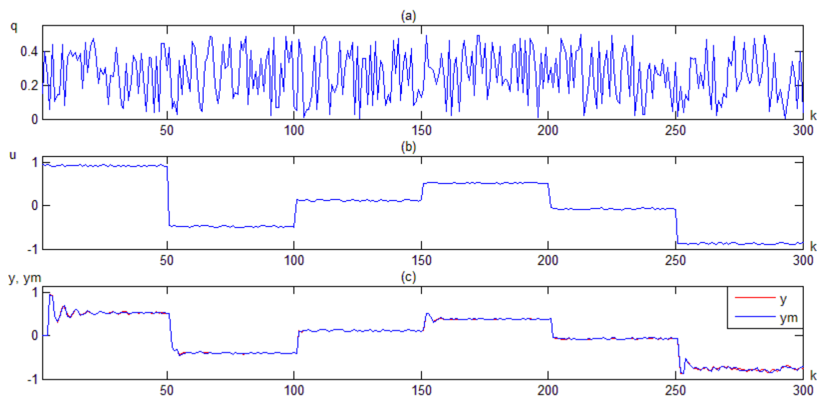

Figure 6: Evolution of the uncertain parameter (a), the input signal (b), the system and the model outputs for the training set.

The modeling results for the validation set are illustrated in Figure 7.

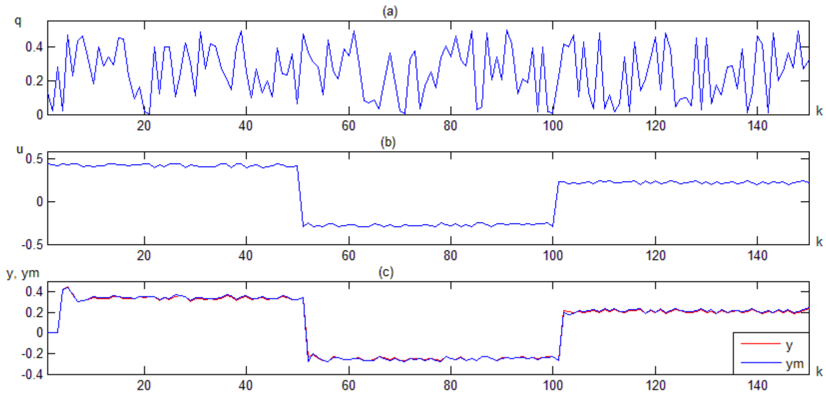

Figure 7: Evolution of the uncertain parameter (a), the input signal (b), the system and the model outputs for the validation set.

This system can be also approximated by a global Takagi-Sugeno fuzzy system composed of a set of IF-THEN fuzzy rules having the following form:

$$\begin{aligned} &if\ u(k)\ is\ A_r^1\ and\ y(k)\ is\ B_r^1\ and\ y(k-1)\ is\ B_r^2\ and\ y(k-2)\ is\ B_r^3 \\ &\quad then\ y_r^{mc}(k+1) = -g_1^r\, y(k) - g_2^r\, y(k-1) - g_3^r\, y(k-2) + h_1^r\, u(k). \end{aligned} \tag{38}$$

The descent gradient method is applied for the estimation of the conclusions parameters. The rules number is $R = 16$ and the learning rate is $\epsilon = 0.2$. For both models, the same membership functions are used.

	Decomposed fuzzy model	Global Takgi-Sugeno fuzzy model
J (final)	0.025	0.025
Iteration number	4121	10999
E	0.0066	0.0079

Table 2: Comparison between the decomposed fuzzy model and the global Takagi-Sugeno fuzzy one.

According to this table, the decomposed fuzzy model is slightly more accurate and requires less time for the parameters training. It is due to the system dynamics decomposition.

5.2.2 Robust pole assignment control

The robust PID controller (24) is applied for the control of the system (35). The PID parameters are computed using the prescribed robust pole assignment and referring only to the linear uncertain part (36) of the decomposed fuzzy model. The objective is to have a double pole $z_1 = z_2 = 0.1$ and a double pole $z_3 = z_4 = 0.2$. The resulted PID parameters are the following ones: $q_0 = -0.5658$, $q_1 = 1.2047$ and $q_2 = -0.5586$.

For the uncertain parameter evolution given in Figure 8, the obtained control results are illustrated in Figure 9.

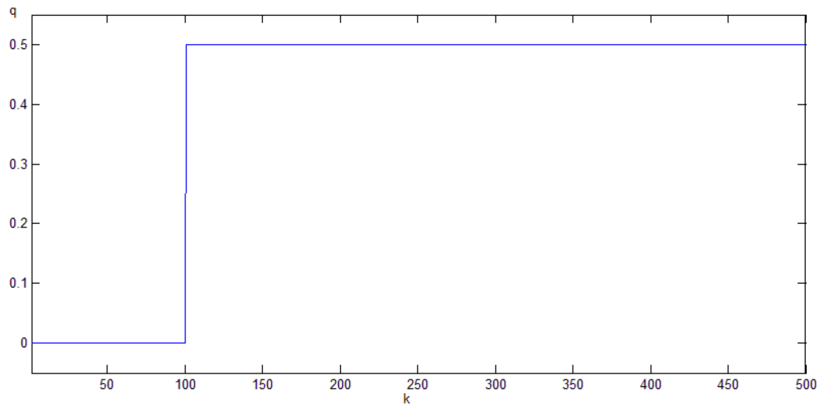

Figure 8: Evolution of the uncertain parameter $q(k)$.

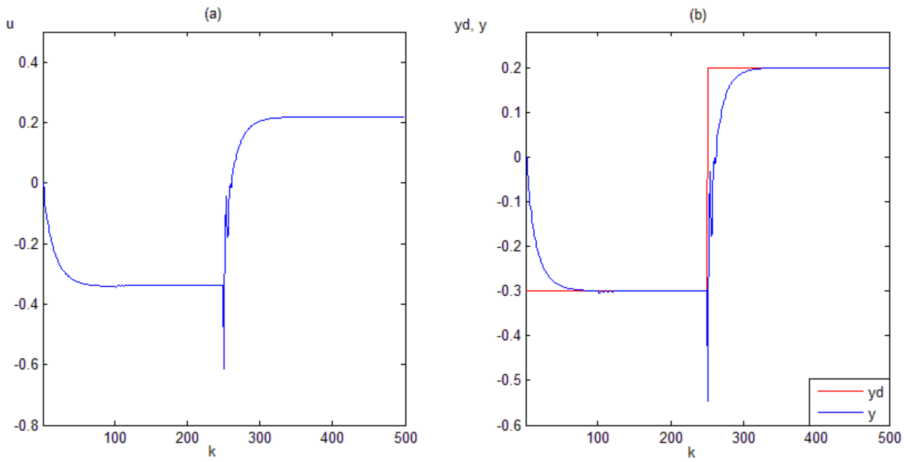

Figure 9: Evolution of the robust PID control (a), desired output and system output (b).

The resulted closed-loop system is stable and the static error is equal to zero for the chosen uncertain parameter values. But, the obtained results for the transient time are poor. So, the proposed control method is limited to the guarantee of desired performances. In addition, there is no guarantee for the closed-loop system stability. This may be caused by neglecting the nonlinear part of the model. So, in future works this control approach must be robustified and a stability study must be done to guarantee the performance and stability robustness of the closed-loop uncertain nonlinear system.

6 Conclusions

This study has developed new modeling and control schemes for nonlinear systems affected by bounded uncertainties. The proposed model consists in dividing the behavior of the considered system into two parts: a linear uncertain part and a nonlinear one. The used techniques for the system modeling have been explained. In fact, the linear uncertain part has been obtained by the nominal system linearization around some operating points and the nonlinear part has been approximated by a Takagi-Sugeno fuzzy system whose parameters are estimated using the descent gradient method. A robust pole assignment control for the considered nonlinear system has been synthesized based only on the linear uncertain part of the decomposed fuzzy model. Two simulation examples have been treated to demonstrate the effectiveness of the suggested modeling approach and to experiment the proposed control scheme.

References

[1] Zadeh, L.A. Fuzzy Sets. *Information and control* **8** (1965) 338–353.

[2] Narendra, K.S. and Parthasarathy, K. Identification and control of dynamical systems using neural networks. *IEEE Transactions on Neural Networks* **1** (1) (1990) 4–27.

[3] Kosko, B. Fuzzy systems as universal approximators. *IEEE Transactions on Computers* **43** (11) (1994) 1329–1333.

[4] Hornik, K. Multilayer feedforward networks are universal approximators. *Neural Networks* **2** (5) (1989) 359–366.

[5] Mamdani, E.H. Application of Fuzzy Algorithms for control of a simple dynamic Plant. *Proc. of the IEE Control and Science* **121** (12) (1974) 1585–1588.

[6] Takagi, T. and Sugeno, M. Fuzzy identification of systems and its applications to modeling and control. *IEEE Transaction on Systems, Man and Cybernetics* **15** (15) (1985) 116–132.

[7] Cai, L., Cui, Z. and Liu, H. Universal approximation of T-S fuzzy systems. In: *Symposium on ICT and Energy Efficiency and Workshop on Information Theory and Security.* Dublin (2012) 166–171.

[8] Chen, M.Y. and Linkens, D.A. Rule-base-self-generation and simplification for data-driven fuzzy models. *Fuzzy Sets and Systems* **142** (2004) 243–265.

[9] Zhao, Z., Xie, W. and Hong, H. Identification of Takagi-Sugeno (TS) fuzzy model with evolutionary parallel gradient search. In: *Annual Meeting of the North American Fuzzy Information Processing Society (NAFIPS).* New York City (2008) 1–6.

[10] Aflab, M.S. and Kadri, M.B. Parameter identification of Takagi-Sugeno fuzzy model of surge tank system. In: 3^{rd} *International Conference on Computer, Control & Communication.* Karachi (2013) 1–4.

[11] Xu, S. and Xuesong, X. Fuzzy identification base on cat swarm optimization algorithm. In: *The* 26^{th} *Chineese Control and Decision Conference.* Changsha (2014) 4264–4269.

[12] Soltani, M., Chaari, A., BenHmida, F. and Gossa, M. A new objective function for fuzzy c-regression model and its application to T-S fuzzy model identification. In: *International Conference on Communications, Computing and Control Applications.* Hammamet, Tunisia (2011) 1–5.

[13] Jin, Y. Fuzzy modeling of high-dimensional systems: complexity reduction and interpretability improvement. *IEEE Transactions on Fuzzy Systems* **8** (2) (2000) 212–221.

[14] Lavygina, A. and Hodashinsky, I. Hybrid algorithm for fuzzy model parameter estimation based on genetic algorithm and derivation based methods. In: *International Conference on Fuzzy Computation Theory and Applications (FCTA)* (2011) 513–515.

[15] Chuang, C.C., Su, S.F. and Chen, S.S. Robust TSK fuzzy modeling for function approximation with outliers. *IEEE Transactions on Fuzzy Systems* **9** (6) (2001) 810–821.

[16] Chuang, C.C., Jeng, J.T. and Tao, C.W. Hybrid robust approach for TSK fuzzy modeling with outliers. *Expert Systems with Applications* **36** (2009) 8925–8931.

[17] Rezaei Sadrabadi, M. and Fazel Zarandi, M.H. Identification of the linear parts of nonlinear systems for fuzzy modeling. *Applied Soft Computing* **11** (1) (2011) 807–819.

[18] Abdelazim, T. and Malik, O.P. Identification of nonlinear systems by Takagi-Sugeno fuzzy logic grey box modeling for real-time control. *Control Engineering Practice* **13** (12) (2005) 1489–1498.

[19] Sonbol, A.H., Fadali, M.S. and Jakarzadeh, S. TSK fuzzy function approximators: design and accuracy analysis. *IEEE Transactions on Systems Man Cybernetics, Part B: Cybernetics* **42** (3) (2012) 702–712.

[20] Hadjili, M.L. and Kara, K. Modelling and control using Takagi-Sugeno fuzzy models. In: *Saudi International Electronis, Communications and Photonics Conference*. Riyadh (2011) 1–6.

[21] Du, H. and Li, W. Model-based Takagi-Sugeno fuzzy approach for vehicle longitudinal velocity estimation during barking. In: *IEEE International Conference on Fuzzy Systems*. Beijing (2014) 1851–1858.

[22] Leite, D., Cominhas, W., Lemos, A. and Palhares, R. Parameter estimation of dynamic fuzzy models from uncertain data streams. In: *IEEE Conference on Norbert Wiener in the* 21^{st} *Century*. Boston MA (2014) 1–7.

[23] Khandekar, A.A., Malwatkar, G.M. and Patre, B.M. Discrete sliding mode control for robust tracking of high order delay time systems with experimental application. *ISA Transactions* **52** (1) (2013) 36–44.

[24] Xu, Q. Discrete-time second-order sliding mode control for a nanopositioning stage. In: *The* 33^{rd} *Chinese Control Conference*. Nanjing (2014) 7976–7981.

[25] Khansah, H., Mahout, V. and Bernussou, J. Scheduled robust control of nonlinear system by norm bounded approximation. In: 4^{th} *IEEE International Multi-Conference on Systems, Signals & Devices*, Hammamet, Tunisia (2007).

[26] Bedioui, N., Salhi, S. and Ksouri, M. H_2 performance via static output feedback for a class of nonlinear systems . In: 3^{rd} *International Conference on Signals, Circuits and Systems*. Medenine (2009) 1–6.

[27] Morais, C.F., Braga, M.F. Oliveira, R.C.L.F. and Peres, P.L.D. H_∞ static output feedback control of discrete-time Markov jump linear systems with uncertain transition probability matrix. In: *American Control Conference*. Portland OR (2014) 489–494.

[28] Yu, Y., Zhang, Q., Wang, J. and Sun, C.Y. Robust tracking control for the hypersonic flight vehicle via backstepping method. In: 33^{rd} *Chinese Control Conference*. Nanjing (2014) 4306–4311.

[29] Yu, J., Zhao, Y. and Wu, Y. Robust tracking control for a class of uncertain nonlinear systems. In: 33^{rd} *Chinese Control Conference*. Nanjing (2014) 2075–2079.

[30] Nurges, U. Robust pole assignment via reflection coefficients of polynomials. *Automatica* **42** (7) (2006) 1223–1230.

[31] Halpern, M.E., Evans, R.J. and Hill, R.D. Pole assignment with robust stability. *IEEE Transactions on Automatic Control* **40** (4) (1995) 725–729.

[32] Lordelo, A.D.S. and Ferreira, P.A.V. Interval analysis and design of robust pole assignment controllers. In: *proceedings of the* 41^{st} *IEEE Conference on Design and Control*. USA (2002) 1461–1466.

[33] Lordelo, A.D.S., Juzzo, E.A. and Ferreira, P.A.V. On the design of robust controller using the interval Diophantine equation. In: *IEEE International Symposium on Computer Aided Control Systems Design*. Taiwan (2004) 173–178.

[34] Soylemez, M.T. and Munro, N. Robust pole assignment in uncertain systems. *IEE Proceedings: Control Theory and Applications* **144** (3) (1997) 217–224.

[35] Risonova, D. and Holic, I. LMI approximation of pole-region for discrete-time linear dynamic systems. In: 15^{th} *International Carpathian Control Conference (ICCC).* Velke Karlovice (2014) 497–502.

[36] Risonova, D. and Valach, P. Switched system robust control: pole placement LMI based approach. In: 15^{th} *International Carpathian Control Conference (ICCC).* Velke Karlovice (2014) 491–496.

[37] Abid, H., Chtourou, M. and Toumi, A. Robust fuzzy sliding mode controller for discrete nonlinear systems. *International Journal of Computers, Communications & Control* **3** (1) (2008) 6–20.

[38] Wu, H.-N. Robust H_2 fuzzy output feedback control for discrete-time nonlinear systems with parametric uncertainties. *International Journal of Approximate Reasoning* **46** (1) (2007) 151–165.

[39] Aydi, A., Zaidi, I., Djemel, M. and Chtourou, M. Robust fuzzy PID controller for discrete-time uncertain nonlinear systems. In: 6^{th} *International Multi-conference on Systems, Signals and Devices, SSD′09.* Djerba, Tunisia (2009) 1–6.

[40] Yang, H., Shi, P., Zhang, J. and Qui, J. Robust H_∞ control for a class of discrete-time fuzzy systems via delta operator approach. *Information Sciences* **184** (1) (2012) 230–245.

[41] Hu, Y., Liu, J. and Lin, Z. LPV T-S fuzzy gain scheduling control of WTGS below rated wind speed . In: *the* 26^{th} *Chinese Control and Decision Conference.* Changsha (2014) 3328–3333.

[42] Chae, S., Ngung, S.K. and Wang, W. Robust H_∞ fuzzy control of discrete nonlinear networked control systems: a SOS approach. *Journal of the Franklin Institute* **351** (8) (2014) 4065–4083.

[43] Patkure, J., More, D.S. and Todkar, M. Fuzzy gain scheduling based control technique for the feed in the tool and cutter grinding machine. In: *International Conference on Circuit, Power and Computing Technologies.* Nagercoil (2014) 1090–1093.

[44] Aydi, A., Djemel, M. and Chtourou, M. On the fuzzy modeling of uncertain nonlinear systems. In: 15^{th} *International Conference on Sciences and Techniques of Automatic Control & Computer Engineering (STA).* Hammamet, Tunisia (2014) 1055–1060.

[45] Chen, L. and Narendra, K.S. Identification and control of a nonlinear discrete-time system based on its linearization: a unified framework *IEEE Transactions on Neural Networks* **15** (3) (2004) 663–673.

[46] Hernandez, E. and Arkun, Y. Stability of nonlinear polynomial ARMA models and their inverse. *International Journal of Control* **63** (5) (1996) 885–906.

[47] Jin, L., Nikiforuk, P.N. and Gupta, M.M. Fast neural learning and control of discrete-time nonlinear systems. *IEEE Transactions on Systems, Man and Cybernetics* **25** (3) (1995) 478–488.