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A New Hyper Chaotic System and Study of Hybrid Projective Synchronization Behavior

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Abstract: In this paper, hybrid projective synchronization (HPS) of two identical new hyper chaotic systems is defined and scheme of HPS is developed by using tracking control method. A new hyper chaotic system has been constructed and then response system. Numerical simulations verify the effectiveness of this scheme, which has been performed by mathematica.

Keywords: *hybrid projective synchronization; chaotic systems and hyper chaos; tracking control method.*

Mathematics Subject Classification (2010): 34D06.

1 Introduction

Chaos is a dynamical regime in which a system becomes extremely sensitive to initial conditions and reveals an unpredictable and random-like behavior, even though the underlying model of a system exhibiting chaos can be deterministic and very simple. Small differences in initial conditions yield widely diverging outcomes for chaotic systems, rendering long term prediction impossible in general. Chaotic behavior can be observed in many natural phenomenon such as weather etc. Pecora and Carroll introduced a paper entitled *Synchronization in Chaotic Systems* in 1990. By that time, if there was a system challenging the capability of synchronizing that was a chaotic one. They demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Chaotic synchronization did not attract much attention until Pecora and Carroll [8] introduced a method to synchronize two identical chaotic systems

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with different initial conditions. From then on, enormous studies have been done by researchers on the synchronization of dynamical systems [5–7,26]. In the last two decades considerable research has been done in non-linear dynamical systems and their various properties. One of the most important properties is synchronization. Synchronization techniques have been improved in recent years and many different methods are applied theoretically as well as experimentally to synchronize the chaotic-systems including adaptive control [9–11], back stepping design [12–14], active control [15–17], nonlinear control [18,19] and observer based control method [20]. Using these methods, numerous synchronization problem of well-known chaotic systems such as Lorenz, Chen, Lü and Rössler system have been worked on by many researchers.

Also, several types of chaos synchronization are well known, which include complete synchronization (CS), antisynchronization (AS), phase synchronization, generalized synchronization (GS), projective synchronization (PS), and modified projective synchronization (MPS). Among all type of synchronization, Projective synchronization (PS) [21, 24, 25] has been extensively considered because it can obtain faster communication. The drive and response system could be synchronized up to a scaling factor in projective synchronization. In this continuation of study, in order to increase the degree of secrecy for secure communications, in hybrid projective synchronization same scaling factor can be replaced by vector function factor. In this paper, we have constructed a new hyper chaotic system and verified the chaotic behavior of this system by time series analysis and drawing chaotic attractors via mathematica. Hyperchaotic behavior of this system is discovered within some system parameter range, which has not yet been reported previously. Since hyperchaotic systems have the characteristics of high capacity, high security and high efficiency, it has been studied with increasing interest in recent years [23, 24] in the fields of non-linear circuits, secure communications, lasers, control, synchronization, and so on. So we have studied Hybrid Projective Synchronization behavior for this new hyper chaotic systems, which is ofcourse more effective and useful in secure communication as HPS is more useful in secure communication as compare to others because of its unpredictability. Here we have used tracking control scheme for HPS. Numerical simulations have been done by using Mathematica.

2 Preliminaries

In this section, we mention some definitions and scheme of the main work.

Definition 2.1 Hybrid Projective Synchronization(HPS) between two chaotic system achieved if there exist an $n \times n$ matrix A such that $\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|Ay - x\| = 0$, where $\|\cdot\|$ is the Euclidean norm.

2.1 Methodology for HPS

In this section, we put a glimpse of methodology and problem formulation for hybrid projective synchronization for identical hyperchaotic systems via tracking control. Consider the following n-dimensional hyperchaotic system as drive (master) system

$$\frac{dx}{dt} = f(x), \tag{2.1}$$

where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a differentiable function. Now construct the following identical response system

$$\frac{dy}{dt} = g(y) + \Psi(y, x), \tag{2.2}$$

where $y \in \mathbb{R}^n$ and $g : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a differentiable function and $\Psi(y, x)$ is vector controller to be designed via tracking control method.

In order to achieve the hybrid projective synchronization between two hyperchaotic systems, we choose the system (2.1) as a drive system and construct a response system as follows:

$$\frac{dy}{dt} = A^{-1}[f(Ay) + \Psi(y, x)], \tag{2.3}$$

where A^{-1} is the inverse matrix of the invertible matrix A and $y \in \mathbb{R}^n$ are state vector of the response system(2.2) and $\Psi(y, x)$ is controller which will be designed. Now define the HPS errors between two given systems (2.1)and (2.3) as

$$e(t) = Ay - x,$$

where e=$(e_1, e_2...e_n)^T$, and $A = \begin{pmatrix} a_{11} & a_{12} & ...a_{1n} \\ a_{21} & a_{22} & ...a_{2n} \\ . & . & . \\ a_{n1} & a_{n2} & ...a_{nn} \end{pmatrix}$.

So,

$$e_i = \left(\sum_{j=1}^{n} a_{ij} y_j \right) - x_i, (i, j = 1, 2, ...n). \tag{2.4}$$

Let

$$f(Ay) - f(x) = F(x, e). \tag{2.5}$$

Now, we assume that the error vectors e can be divided into $\overline{e_k}$=$(e_1, e_2...e_k)^T$ and $\overline{e_{k+1}}$=$(e_{k+1}, e_{k+2}...e_n)^T$ such that $F(x, e)$ has the following form

$$F(x, e) = \begin{pmatrix} B_k\overline{e_k} + h_1(x, \overline{e_k}, \overline{e_{k+1}}) \\ B_{k+1}\overline{e_{k+1}} + h_{21}(x, \overline{e_k}, \overline{e_{k+1}}) + h_{22}(x, \overline{e_k}, \overline{e_{k+1}}) \end{pmatrix}, \tag{2.6}$$

where $h_1(x, \overline{e_k}, \overline{e_{k+1}}) \in \mathbb{R}^k$, $h_{21}(x, \overline{e_k}, \overline{e_{k+1}}) \in \mathbb{R}^{n-k}$, $h_{22}(x, \overline{e_k}, \overline{e_{k+1}}) \in \mathbb{R}^{n-k}$ and $\lim\limits_{\overline{e_k} \to 0} h_{21}(x, \overline{e_k}, \overline{e_{k+1}}) = 0$, respectively and $B_k \in \mathbb{R}^{k \times k}$, $B_{k+1} \in \mathbb{R}^{n-k \times n-k}$ are real constant matrix. Now, following theorem is based on the Lyapunov stability theory, which gives the final destination of the problem formulation.

Theorem 2.1 *If controller $\Psi(y, x)$ in response system (2.3) is*

$$\Psi(y, x) = \begin{pmatrix} \Psi_k(x, y) \\ \Psi_{k+1}(x, y) \end{pmatrix} = \begin{pmatrix} \Lambda_k\overline{e_k} - h_1(x, \overline{e_k}, \overline{e_{k+1}}) \\ \Lambda_{k+1}\overline{e_{k+1}} - h_{22}(x, \overline{e_k}, \overline{e_{k+1}}) \end{pmatrix}, \tag{2.7}$$

where $\Lambda_k \in \mathbb{R}^{k \times k}$ and $\Lambda_{k+1} \in \mathbb{R}^{n-k \times n-k}$ are suitable chosen constant matrices. If all eigenvalues of $B_k + \Lambda_k$ and $B_{k+1} + \Lambda_{k+1}$ have negative real parts, then hybrid projective synchronization between drive and response systems can be achieved.

3 System Description

3.1 Hyper chaotic Rabinovich-Fabrikant system

The Rabinovich-Fabrikant chaotic system is a set of three coupled ordinary differential equations exhibiting chaotic behavior for certain values of parameters. They are named after Mikhail Rabinovich and Anatoly Fabrikant, who described them in 1979 [22]. The equations of system are :

$$\left.\begin{aligned} \dot{x_1} &= x_2(x_3 - 1 + x_1^2) + \gamma x_1, \\ \dot{x_2} &= x_1(3x_3 + 1 - x_1^2) + \gamma x_2, \\ \dot{x_3} &= -2x_3(x_1 x_2 + \alpha), \end{aligned}\right\} \tag{3.1}$$

where α and γ are constant parameters that control the evolution of the system. For some values of α and γ, the system is chaotic but for other it tends to a stable periodic orbit. Now, we construct a new hyper chaotic system by introducing one more differential equation with a new parameter δ in the above system as follow:

$$\left.\begin{aligned} \dot{x_1} &= x_2(x_3 - 1 + x_1^2) + \gamma x_1, \\ \dot{x_2} &= x_1(3x_3 + 1 - x_1^2) + \gamma x_2, \\ \dot{x_3} &= -2x_3(x_1 x_2 + \alpha), \\ \dot{x_4} &= -3x_3(x_2 x_4 + \delta) + x_4^2. \end{aligned}\right\} \tag{3.2}$$

This new system shows hyper chaotic behavior with some values of parameters and tend to stable periodic orbits with other values of parameters. We have investigated system's behavior for different values of δ. Figures are given below:

4 Results and Discussions

In this section, we perform hybrid projective synchronization for hyper chaotic Rabinovich Fabrikant system. If we take this system as a drive system, then according to methodology, response system is

$$\tfrac{dy}{dt} = A^{-1}[f(Ay) + \Psi(y, x)], \tag{4.1}$$

which leads to response system as follows

$$\left(\frac{dy_1}{dt}, \frac{dy_2}{dt}, \frac{dy_3}{dt}, \frac{dy_4}{dt}\right)^T = A^{-1}[f(Ay) + \Psi(x, y)], \tag{4.2}$$

yields

$$A \begin{pmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \\ \frac{dy_3}{dt} \\ \frac{dy_4}{dt} \end{pmatrix} =$$

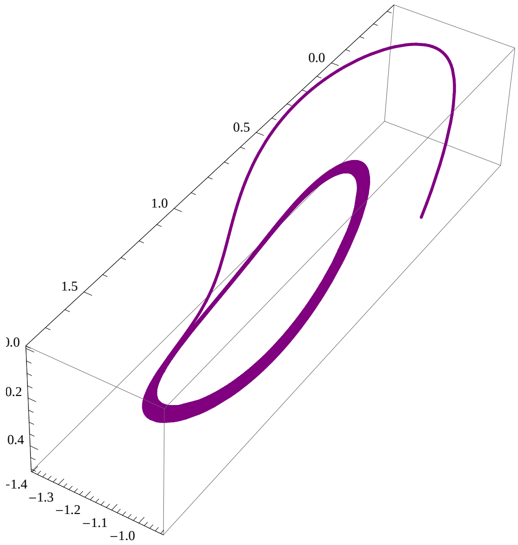

Figure 1: Chaotic behavior of the system (3.2) with $\alpha = 0.14$,$\gamma = 1.1$ and $-0.01 \leq \delta \leq 7650$ tending to stable periodic orbits.

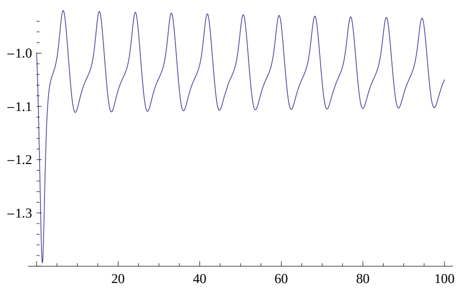

Figure 2: Time series analysis of x1[t] with $\alpha = 0.14$,$\gamma = 1.1$ and $-0.01 \leq \delta \leq 7650$.

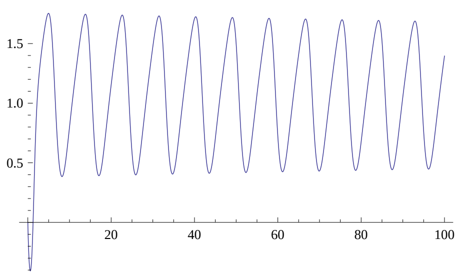

Figure 3: Time series analysis of x2[t] with $\alpha = 0.14$,$\gamma = 1.1$ and $-0.01 \leq \delta \leq 7650$.

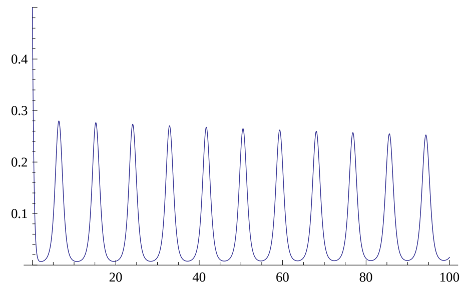

Figure 4: Time series analysis of x3[t] with $\alpha = 0.14$,$\gamma = 1.1$ and $-0.01 \leq \delta \leq 7650$.

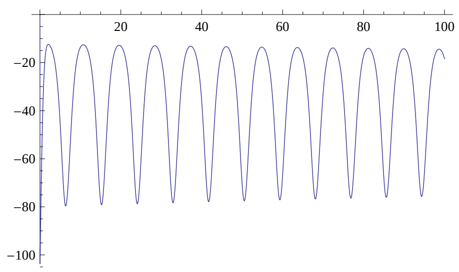

Figure 5: Time series analysis of x4[t] with $\alpha = 0.14$,$\gamma = 1.1$ and $-0.01 \leq \delta \leq 7650$.

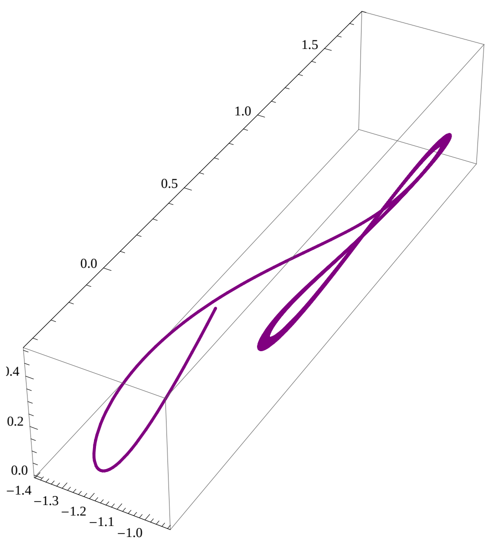

Figure 6: Chaotic Behavior of the system (3.2) with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = 1890$.

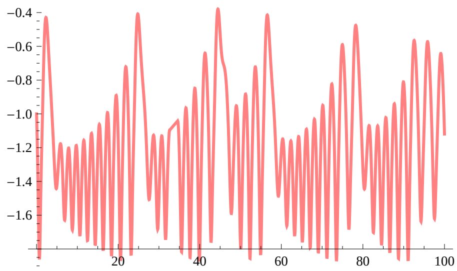

Figure 7: Time series analysis of x1[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = 1890$.

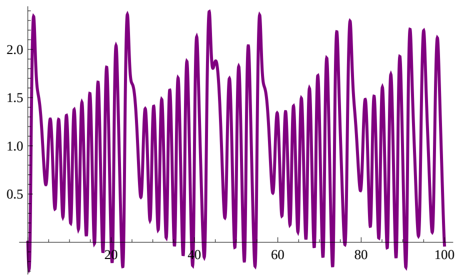

Figure 8: Time series analysis of x2[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = 1890$.

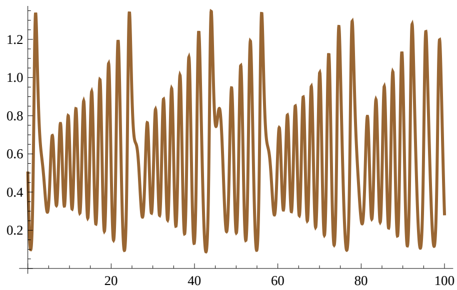

Figure 9: Time series analysis of x3[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = 1890$.

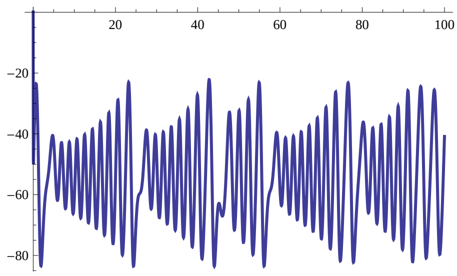

Figure 10: Time series analysis of x4[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = 1890$.

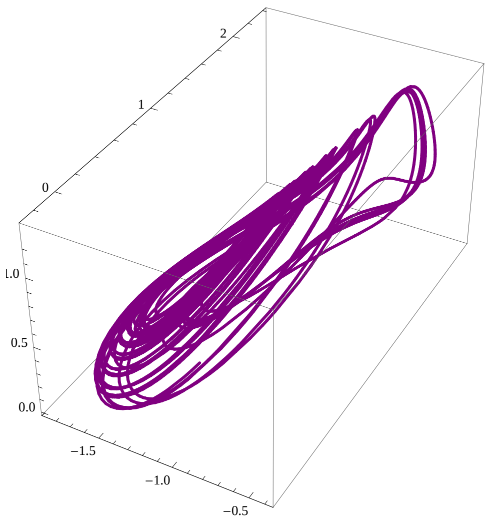

Figure 11: Chaotic Behavior of the system (3.2) with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = -0.2$.

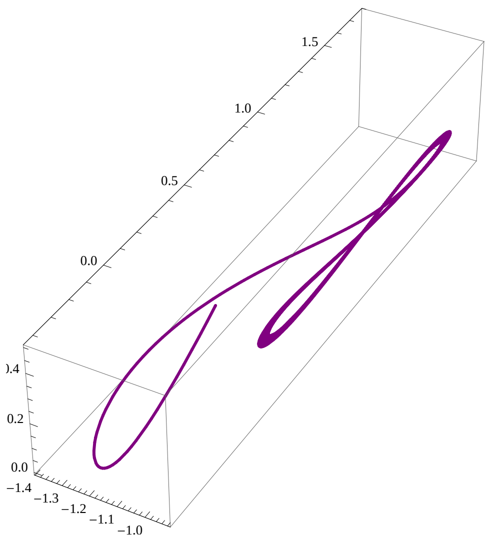

Figure 12: Time series analysis of x1[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = -0.2$.

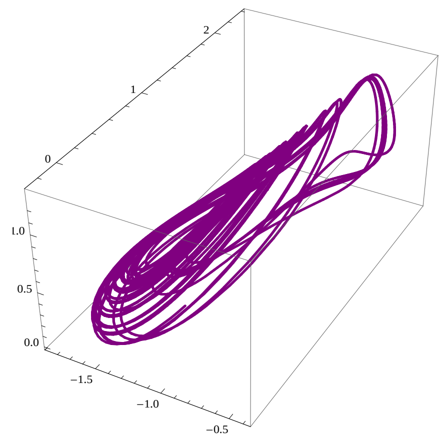

Figure 13: Time series analysis of x2[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = -0.2$.

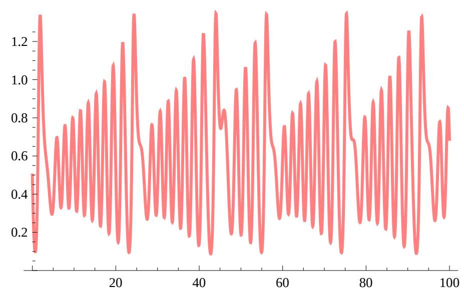

Figure 14: Time series analysis of x3[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = -0.2$.

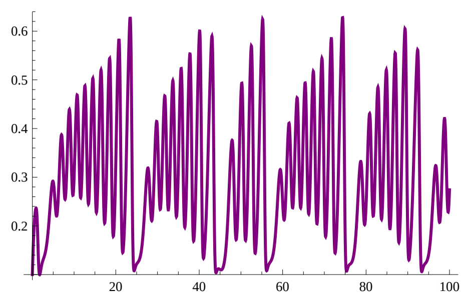

Figure 15: Time series analysis of x4[t] with $\alpha = 0.87$,$\gamma = 1.1$ and $\delta = -0.2$.

$$\begin{pmatrix} \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{3j}y_j - \sum_{j=1}^{4} a_{2j}y_j + \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{1j}^2 y_j^2 + \gamma \sum_{j=1}^{4} a_{1j}y_j \\ 3\sum_{j=1}^{4} a_{1j}y_j \sum_{j=1}^{4} a_{3j}y_j + \sum_{j=1}^{4} a_{1j}y_j - \sum_{j=1}^{4} a_{1j}^3 y_j^3 + \gamma \sum_{j=1}^{4} a_{2j}y_j \\ -2\sum_{j=1}^{4} a_{3j}y_j \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{3j}y_j - 2\alpha \sum_{j=1}^{4} a_{3j}y_j \\ -3\sum_{j=1}^{4} a_{3j}y_j \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{4j}y_j - 3\delta \sum_{j=1}^{4} a_{3j}y_j + \sum_{j=1}^{4} a_{4j}^2 y_j^2 \end{pmatrix} + \Psi(x,y). \tag{4.3}$$

Now, according to definition of HPS error dynamics we have,

$$\frac{de}{dt} = A\frac{dy}{dt} - \frac{dx}{dt} = f(Ay) - f(x) + \Psi(x,y). \tag{4.4}$$

Let

$$f(Ay) - f(x) = F(x,e). \tag{4.5}$$

From equation (4.4) and (4.5), we have following

$$\frac{de}{dt} = F(x,e) + \Psi(x,y). \tag{4.6}$$

Our goal is to find $F(x,e)$ and to design controller $\Psi(x,y)$ to achieve the HPS.

Equation (4.5) gives $F(x,e) =$

$$\begin{pmatrix} \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{3j}y_j - \sum_{j=1}^{4} a_{2j}y_j + \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{1j}^2 y_j^2 + \gamma \sum_{j=1}^{4} a_{1j}y_j \\ 3\sum_{j=1}^{4} a_{1j}y_j \sum_{j=1}^{4} a_{3j}y_j + \sum_{j=1}^{4} a_{1j}y_j - \sum_{j=1}^{4} a_{1j}^3 y_j^3 + \gamma \sum_{j=1}^{4} a_{2j}y_j \\ -2\sum_{j=1}^{4} a_{3j}y_j \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{3j}y_j - 2\alpha \sum_{j=1}^{4} a_{3j}y_j \\ -3\sum_{j=1}^{4} a_{3j}y_j \sum_{j=1}^{4} a_{2j}y_j \sum_{j=1}^{4} a_{4j}y_j - 3\delta \sum_{j=1}^{4} a_{3j}y_j + \sum_{j=1}^{4} a_{4j}^2 y_j^2 \end{pmatrix}$$

$$- \begin{pmatrix} x_2x_3 - x_2 + x_2x_1^2 + \gamma x_1 \\ 3x_1x_3 + x_1 - x_1^3 + \gamma x_2 \\ -2x_1x_2x_3 - 2\alpha x_3 \\ -3x_2x_3x_4 - 3\delta x_3 + x_4^2 \end{pmatrix}$$

which yields, $F(x,e)$

$$= \begin{pmatrix} e_2e_3 + e_2e_1^2 + e_2x_3 + e_3x_2 - e_2 + e_2x_1^2 + 2e_1e_2x_1 + x_2e_1^2 + 2e_1x_1x_2 + \gamma e_1 \\ e_1 - e_1^3 + 3e_1e_3 + 3x_3e_1 + 3x_1e_3 - 3x_1^2e_1 - 3x_1e_1^2 + \gamma e_2 \\ -2x_1x_3e_2 - 2x_2x_3e_1 - 2x_3e_1e_2 - 2x_1x_2e_3 - 2x_1e_2e_3 - 2e_1e_3x_2 - 2e_1e_2e_3 - 2e_3\alpha \\ -3x_4x_3e_2 - 3x_2x_3e_4 - 3x_3e_2e_4 - 3x_2x_4e_3 - 3x_2e_4e_3 - 3e_2e_3x_4 - 3e_2e_3e_4 - 3e_3\delta + e_4^2 + 2e_4x_4 \end{pmatrix}.$$

So, after putting all above values, we have

$$F(x,e) = \begin{pmatrix} B_1\overline{e_1} + h_1(x,\overline{e_1},\overline{e_2}) \\ B_2\overline{e_2} + h_{21}(x,\overline{e_1},\overline{e_2}) + h_{22}(x,\overline{e_1},\overline{e_2}) \end{pmatrix}. \tag{4.7}$$

Obviously, $\lim_{e_1\to 0} h_{21}(x, \overline{e_1}, \overline{e_2}) = 0$. Now, according to theorem (2.1), we define feedback controller $\Psi(x, y)$ as,

$$\Psi(y,x) = \begin{pmatrix} \Psi_1(x,y) \\ \Psi_2(x,y) \end{pmatrix} = \begin{pmatrix} \Lambda_1\overline{e_1} - h_1(x,\overline{e_1},\overline{e_2}) \\ \Lambda_2\overline{e_2} - h_{22}(x,\overline{e_2},\overline{e_2}) \end{pmatrix}. \tag{4.8}$$

So from equations (4.7) and (4.8) error dynamical system (4.6) can be rewritten as,

$$\left.\begin{array}{l} \frac{d\overline{e_1}}{dt} = (B_1+\Lambda_1)\overline{e_1}, \\ \frac{d\overline{e_2}}{dt} = (B_2+\Lambda_2)\overline{e_2} + h_{21}(x,\overline{e_1},\overline{e_2}). \end{array}\right\} \tag{4.9}$$

So we choose now suitable $B_1 + \Lambda_1 \in \mathbb{R}^1$ and $B_2 + \Lambda_2 \in \mathbb{R}^{3\times 3}$, for which eigen values are negative. As Eq.(4.9) is asymptotically stable with equilibrium point $e_1 = 0$ and $\overline{e_2} = 0$. Obviously $\lim_{t\to\infty} \|e_1\| = 0$ and $\lim_{e_1\to 0} h_{21}(x, \overline{e_1}, \overline{e_2}) = 0$, then the hybrid projective synchronization between response system and master system can be achieved.

5 Numerical Simulations

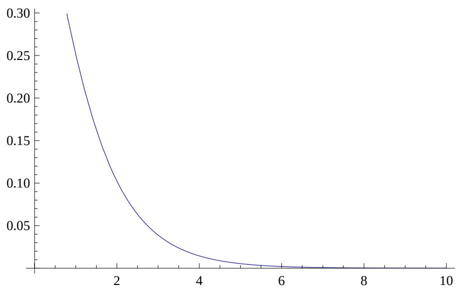

Figure 16: Convergence of error e_1, $t \in [0, 10]$.

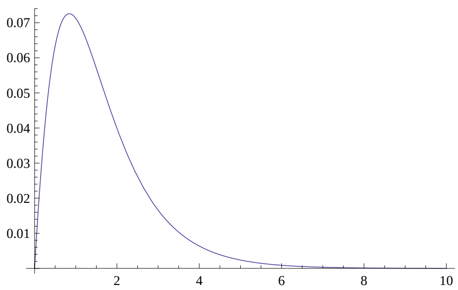

Figure 17: Convergence error of e_2, $t \in [0, 10]$.

Parameters of the system are $-0.01 \le \delta \le 7650$ with $\alpha = 0.14$, $\gamma = 1.1$ and $-0.2 \le \delta \le 1890$ with $\alpha = 0.87$,$\gamma = 1.1$ for which the systems are chaotic. In (4.9),

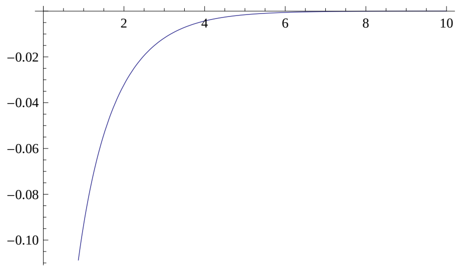

Figure 18: Convergence of error e_3, $t \in [0, 10]$.

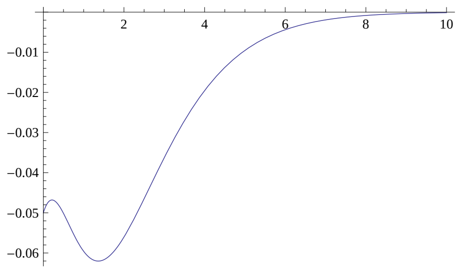

Figure 19: Convergence of error e_4, $t \in [0, 10]$.

we have chosen $\Lambda_1 = (-2)$ and $\Lambda_2 = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 3 & -1 \end{pmatrix}$, which leads to stability conditions as eigenvalues of $B_1+\Lambda_1$ and $B_2+\Lambda_2$ are negative. The initial conditions for master and slave systems $[x_1(0), x_2(0), x_3(0), x_4(0)] = [8, 3, 1, 4]$ and $[y_(0), y_2(0), y_3(0), y_4(0)] = [0.1, 0.41, 0.31, 0.51]$, respectively, and $A = \begin{pmatrix} -1 & 0 & -1 & 0 \\ 0 & -1 & -2 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 \end{pmatrix}$. Then for $[e_1(0), e_2(0), e_3(0), e_4(0)] = [-8.41, -4.03, -0.51, -2.15]$ diagrams of convergence of errors are the witness of achieving hybrid projective synchronization between master and slave system.

6 Conclusion

In this paper, we have investigated hybrid projective synchronization behavior of a new hyper chaotic Rabinovich-Fabrikant system. The results are validated by numerical simulations using mathematica. It has more advantage over other synchronization to enhance security of communication as hybrid projective synchronization is more unpredictable and moreover it is performed for hyperchaotic system, which makes it more useful.

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