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Approximation of the Optimal Control Problem on an Interval with a Family of Optimization Problems on Time Scales

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Abstract: In this paper we consider a family of optimization problems defined on variable time scales $\mathbb{T}_\lambda$, which depend on the parameter λ We prove that the family of value functions $V_\lambda(t_0, x)$ of the optimal control problem on $[t_0, t_1]_{\mathbb{T}_\lambda}$ converges locally uniformly in $\mathbb{R}^d$ to the value function $V(t_0, x)$ of the optimal control problem on $[t_0, t_1]$, provided $\sup_{t\in[t_0,t_1]_{\mathbb{T}_\lambda}} \mu_\lambda(t) \to 0$ as $\lambda \to 0$, where $\mu_\lambda(t)$ is the graininess function of $\mathbb{T}_\lambda$.

Keywords: *time scale; value function; right-scattered point; right-dense point; graininess function.*

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1 Introduction

This work is devoted to the study of the limiting behavior of the optimal control problem for dynamic equations, defined on a family of time scales $\mathbb{T}_\lambda$, in the regime when the graininess function μ_λ converges to zero as $\lambda \to 0$. At the same time the segment of the time scale $[t_0, t_1]_{\mathbb{T}_\lambda} = [t_0, t_1] \cap \mathbb{T}_\lambda$ approaches $[t_0, t_1]$ e.g. in the Hausdorff metric. The natural question that arises is how the optimal control problem on the time scale is related to the corresponding control problem on the interval $[t_0, t_1]$.

The answer to the above question is well understood for Eulerian time scales (according to classification [6]) that is, if $\mathbb{T}_\lambda = \lambda\mathbb{Z}_+$, $\lambda > 0$, and the equation on time scales becomes a difference equation.

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