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Application of Extended Fan Sub-Equation Method to Generalized Zakharov Equation

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Abstract: In this paper, the extended Fan sub-equation method is applied to obtain exact solutions of the generalized Zakharov equation. Applying this method, we obtain various solutions which are benefit to further understand the concepts of the complicated nonlinear physical phenomena. This method is straightforward, and it can be applied to many nonlinear equations. In this work, we use Mathematica for computations and programming.

Keywords: *extended Fan sub-equation method; generalized Zakharov equation; solitary wave solution.*

Mathematics Subject Classification (2010): 35-XX, 35Qxx.

1 Introduction

Nonlinear partial differential equations (PDEs) appear in many fields, such as fluid mechanics, solid state physics, plasma physics, chemical physics, nonlinear optics , and so on. Thus, nonlinear PDEs play an important role in the study of nonlinear science, especially in the study of nonlinear physical science. Exact solutions of nonlinear PDEs can provide much physical information to understand the mechanism that governs these physical models or provide better knowledge of the physical problems and possible applications [2]. For example, the wave phenomena observed in fluid dynamics, plasma and elastic media are often modeled by the bell-shaped sech solutions and the kink-shaped tanh solutions. Therefore, finding exact solutions of nonlinear PDEs has been of great significance. In the past decades, many researchers have paid more attention to various powerful methods for obtaining exact solutions to nonlinear PDEs. Some of the most

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important methods are the Jacobi elliptic method [4], Taylor-series expansion method [6], simplest equation method [9], the transformed rational function method [11], variational iteration method [12], tanh-sech method [14], sine-cosine method [1, 15], $\frac{G'}{G}$-expansion method [17], exp function method [7], homotopy analysis method [8], and so on.

Yomba [16] demonstrated that the F-expansion method, the tanh and the extended tanh function method belonged to a class of methods called the sub-equation methods, because we can obtain exact solutions of the complicated nonlinear PDEs in use and study some simple nonlinear ordinary differential equations. These methods consist of solving the nonlinear PDEs under a suggestion that a polynomial in a variable satisfies an equation (named the sub-equation). Fan [5] recently developed a new algebraic method, called the Fan sub-equation method, for obtaining exact analytical solutions to nonlinear equations. These solutions include polynomial solutions, trigonometric periodic wave solutions, exponential solutions, rational solutions, hyperbolic and solitary wave solutions. The powerful Fan sub-equation method is widely applied by many scientists, see [3] and the references therein. In this paper, the extended Fan sub-equation method will be used to find exact solutions for the generalized Zakharov equation. We show the extended Fan sub-equation method is a very powerful mathematical technique for finding exact solutions of nonlinear differential equations. Here the exact solutions of the nonlinear PDEs can be expressed as a polynomial and the degree of this polynomial can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms in the considered equation. The aim of this paper is to find exact solutions of the generalized Zakharov equation by using the extended Fan sub-equation method as follows.

The form of the generalized Zakharov equation is [10]

$$\begin{cases} iu_t + u_{xx} - 2\alpha|u|^2u + 2uv = 0, \\ v_{tt} - v_{xx} + (|u|^2)_{xx} = 0. \end{cases} \tag{1}$$

Here the coefficient α is a real arbitrary constant. The nonlinear self-interaction in the high-frequency subsystem, such as a term corresponding to a self-focusing effect in plasma physics can be described via the third term of the first equaton in (1). The rest of this paper is organized as follows. In Section 2, we describe the extended Fan sub-equation method for solving nonlinear PDEs. In Section 3, we give an application of the proposed method to the generalized Zakharov equation. In Section 4, some conclusions are given.

2 Extended Fan Sub-Equation Method for Finding the Exact Solutions of Nonlinear PDEs

In this section, we illustrate the basic idea of the extended Fan sub-equation method for solving nonlinear differential equations. We consider a nonlinear PDE in two independent variables x, t and dependent variable u. Then by means of an appropriate transformation, it can be reduced to a nonlinear ordinary differential equation(ODE) as follows:

$$P(u, u', u'', u''', ...) = 0. \tag{2}$$

Here prime denotes the derivative with respect to ξ. Exact solution for this equation can be constructed as follows:

$$u(\xi) = \frac{A_{-n}}{\psi(\xi)^n} + ... + \frac{A_{-1}}{\psi(\xi)} + A_0 + A_1\psi(\xi) + ... + A_n\psi(\xi)^n; \quad A_n \neq 0. \tag{3}$$

Here A_i $(i = 0, 1, 2, \cdots, n)$ are constants to be determined later. Also, $\psi = \psi(\xi)$ satisfies the following ODE:

$$\psi'(\xi) = \epsilon\sqrt{\sum_{i=0}^{4}\omega_i\psi^i}, \tag{4}$$

where $\epsilon = \pm 1$ and ω_i are constants. Thus the derivatives with respect to ξ can be calculated with respect to the variable ψ as follows:

$$\frac{du}{d\xi} = \epsilon\sqrt{\sum_{i=0}^{4}\omega_i\psi^i}\,\frac{du}{d\psi}, \tag{5}$$

$$\frac{d^2u}{d\xi^2} = \frac{1}{2}\sum_{i=0}^{4} i\omega_i\psi^{i-1}\frac{du}{d\psi} + \sum_{i=0}^{4}\omega_i\psi^i\frac{d^2u}{d\psi^2}, \ldots. \tag{6}$$

The solutions of equation (4) are:

- Case 1. When $\omega_0 = \omega_1 = \omega_3 = 0$, we have the following solutions

$$\psi = \sqrt{-\frac{\omega_2}{\omega_4}}\,sech(\sqrt{\omega_2}\xi); \quad \omega_2 > 0,\ \omega_4 < 0, \tag{7}$$

$$\psi = \sqrt{-\frac{\omega_2}{\omega_4}}\,\sec(\sqrt{-\omega_2}\xi); \quad \omega_2 < 0,\ \omega_4 > 0, \tag{8}$$

$$\psi = -\frac{\epsilon}{\sqrt{\omega_4}\xi}; \quad \omega_2 = 0,\ \omega_4 > 0. \tag{9}$$

- Case 2. When $\omega_1 = \omega_3 = 0$, $\omega_0 = \frac{\omega_2^2}{4\omega_4}$, we have the following solutions

$$\psi = \epsilon\sqrt{-\frac{\omega_2}{2\omega_4}}\,\tanh(\sqrt{-\frac{\omega_2}{2}}\xi); \quad \omega_2 < 0,\ \omega_4 > 0, \tag{10}$$

$$\psi = \epsilon\sqrt{\frac{\omega_2}{2\omega_4}}\,\tan(\sqrt{\frac{\omega_2}{2}}\xi); \quad \omega_2 > 0,\ \omega_4 < 0. \tag{11}$$

- Case 3. When $\omega_1 = \omega_3 = 0$, we have the following solutions

$$\psi = \sqrt{-\frac{\omega_2 m^2}{\omega_4(2m^2-1)}}\,cn(\sqrt{\frac{\omega_2}{2m^2-1}}\xi, m);\ \omega_2 > 0, \omega_4 < 0, \omega_0 = \frac{1-m^2}{(2m^2-1)^2}, \tag{12}$$

$$\psi = \epsilon\sqrt{-\frac{\omega_2 m^2}{\omega_4(m^2+1)}}\,sn(\sqrt{-\frac{\omega_2}{m^2+1}}\xi, m);\ \omega_2 < 0, \omega_4 > 0, \omega_0 = \frac{\omega_2^2 m^2}{2\omega_4(m^2+1)}, \tag{13}$$

where m is the modulus. In limiting cases, the Jacobi elliptic function solutions can degenerate to hyperbolic function solutions and trigonometric function solutions, for example, $sn(\xi) \to tanh(\xi)$ as $m \to 1$, and $sn(\xi) \to sin(\xi)$ as $m \to 0$.

- Case 4. When $\omega_0 = \omega_1 = \omega_4 = 0$, we have the following solutions

$$\psi = -\frac{\omega_2}{\omega_3}\, sech^2(\frac{\sqrt{\omega_2}}{2}\xi); \quad \omega_2 > 0, \tag{14}$$

$$\psi = -\frac{\omega_2}{\omega_3}\, \sec^2(\frac{\sqrt{-\omega_2}}{2}\xi); \quad \omega_2 < 0, \tag{15}$$

$$\psi = \frac{1}{\omega_3 \xi^2}; \quad \omega_2 = 0. \tag{16}$$

Substituting (3)-(6) into equation (2) and collecting all terms with the same powers of ψ together, the left-hand side of equation (2) is converted into a polynomial. After setting each coefficients of this polynomial to zero, we obtain a set of algebraic equations in terms of A_n (n=0,1,2,...,n). Solving the system of algebraic equations and then substituting the results and the general solutions of (7)-(16) into equation (3), gives solutions of equation (2).

3 Application of the Extended Fan Sub-Equation Method

In this section, we apply the extended Fan sub-equation method for solving the generalized Zakharov equation as follows.

Example 3.1 We consider the generalized Zakharov equation in the form

$$iu_t + u_{xx} - 2\alpha|u|^2 u + 2uv = 0, \tag{17}$$

$$v_{tt} - v_{xx} + (|u|^2)_{xx} = 0. \tag{18}$$

For obtaining exact solutions of (17) and (18), we use

$$u(x,t) = \rho(x,t)\, e^{i(kx+\lambda t)}, \tag{19}$$

where k, λ are constants which should to be determined later. Substituting equation (19) into equations (17) and (18), we get

$$i(\rho_t + 2k\rho_x) + \rho_{xx} - (\lambda + k^2)\rho - 2\alpha\rho^3 + 2\rho v = 0, \tag{20}$$

$$v_{tt} - v_{xx} + \rho^2_{xx} = 0. \tag{21}$$

We take the traveling wave transformation

$$\rho = \rho(\xi), \quad v = v(\xi), \quad \xi = \omega(x - 2kt), \tag{22}$$

here ω is a constant which should be determined later. Then equations (20) and (21) are reduced into two nonlinear ODEs

$$\omega\rho'' - (\lambda + k^2)\rho - 2\alpha\rho^3 + 2\rho v = 0, \tag{23}$$

$$(4k^2 - 1)v'' + (\rho^2)'' = 0, \tag{24}$$

integrating equation (24) with respect to ξ, we have

$$v = \frac{\rho^2}{1 - 4k^2}. \tag{25}$$

Substituting equation (25) into equation (23) yields

$$\omega^2\rho'' - (\lambda + k^2)\rho - 2\alpha\rho^3 + \frac{2}{1-4k^2}\rho^3 = 0. \tag{26}$$

Balancing ρ'' with ρ^3 in (26) gives n=1. Thus the extended Fan sub-equation method admits the following solution

$$\rho(\xi) = \frac{A_{-1}}{\psi(\xi)} + A_0 + A_1\psi(\xi), \tag{27}$$

where A_{-1}, A_0, A_1 are constants to be determined and ψ satisfies equation (4).

By substituting equations (27) and (4) into equation (26), collecting the coefficients of ψ^i and setting them to be zero, a set of algebraic equations is obtained. Solving this set of algebraic equations using *Mathematica* [13], we get

- $$A_0 = 0,\ A_1 = \frac{\omega\sqrt{\omega_4\beta}}{\sqrt{1+\alpha\beta}},\ A_{-1} = \frac{[(\lambda+k^2) - \omega^2\omega_2]\sqrt{\beta}}{6\,\omega\sqrt{\omega_4(1+\alpha\beta)}},\ \beta = -1+4k^2, \tag{28}$$
  $\omega_0 = \omega_0,\ \omega_1 = \omega_3 = 0,\ \omega_2 = \omega_2,\ \omega_4 \neq 0.$

- $$A_0 = \frac{\sqrt{\beta\gamma}}{4\sqrt{3}},\ A_1 = \frac{\sqrt{3\beta\gamma}\omega^2\omega_3}{2[5\omega^2\omega_2 - 2(\lambda+k^2)]},\ A_{-1} = \frac{\sqrt{\beta\gamma}[-2(\lambda+k^2) - \omega^2\omega_2]}{24\sqrt{3}\omega^2\omega_3}, \tag{29}$$
  $\gamma = 10\omega^2\omega_2 - (1+4k),\ \omega_0 = \omega_0,\ \omega_1 = 0,\ \omega_2,\ \omega_3 \neq 0,\ \omega_4 = \omega_4.$

By using (28), (27) and cases (7)-(13) respectively, we get

$$\rho_1(x,t) = \frac{[(\lambda+k^2) - \omega^2\omega_2]\sqrt{\beta}}{6\omega\sqrt{-\omega_2(1+\alpha\beta)}} cosh[\sqrt{\omega_2}(\omega(x-2kt))] + \frac{\omega\sqrt{-\omega_2\beta}}{\sqrt{1+\alpha\beta}} sech[\sqrt{\omega_2}(\omega(x-2kt))], \tag{30}$$

$$\rho_2(x,t) = \frac{[(\lambda+k^2) - \omega^2\omega_2]\sqrt{\beta}}{6\omega\sqrt{-\omega_2(1+\alpha\beta)}} cos[\sqrt{-\omega_2}(\omega(x-2kt))] + \frac{\omega\sqrt{-\omega_2\beta}}{\sqrt{1+\alpha\beta}} sec[\sqrt{\omega_2}(\omega(x-2kt))], \tag{31}$$

$$\rho_3(x,t) = -\frac{\sqrt{\beta}}{\sqrt{1+\alpha\beta}}\left\{\frac{[(\lambda+k^2) - \omega^2\omega_2](\omega(x-2kt))}{6\,\epsilon\,\omega} + \frac{\epsilon\,\omega}{\omega(x-2kt)}\right\}, \tag{32}$$

$$\rho_4(x,t) = \frac{[(\lambda+k^2) - \omega^2\omega_2]\sqrt{2\beta}}{6\,\epsilon\,\omega\sqrt{-\omega_2(1+\alpha\beta)}} coth[\sqrt{-\frac{\omega_2}{2}}(\omega(x-2kt))] + \frac{\epsilon\,\omega\sqrt{-\omega_2\beta}}{\sqrt{2(1+\alpha\beta)}} tanh[\sqrt{-\frac{\omega_2}{2}}(\omega(x-2kt))], \tag{33}$$

$$\rho_5(x,t) = \frac{[(\lambda+k^2) - \omega^2\omega_2]\sqrt{2\beta}}{6\,\epsilon\,\omega\sqrt{-\omega_2(1+\alpha\beta)}} cot[\sqrt{\frac{\omega_2}{2}}(\omega(x-2kt))] + \frac{\epsilon\,\omega\sqrt{-\omega_2\beta}}{\sqrt{2(1+\alpha\beta)}} tan[\sqrt{\frac{\omega_2}{2}}(\omega(x-2kt))], \tag{34}$$

$$\rho_6(x,t) = \frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta(2m^2-1)}}{6\,\omega\sqrt{-\omega_2\, m^2(1+\alpha\beta)}}\frac{1}{cn(\sqrt{\frac{\omega_2}{2m^2-1}}\,\omega(x-2kt),m)},$$

$$\frac{\omega\sqrt{-\omega_2 m^2\beta}}{\sqrt{(2m^2-1)(1+\alpha\beta)}}\,cn(\sqrt{\frac{\omega_2}{2m^2-1}}\,\omega(x-2kt),m), \tag{35}$$

$$\rho_7(x,t) = \frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta(m^2+1)}}{6\,\epsilon\,\omega\sqrt{-\omega_2\, m^2(1+\alpha\beta)}}\frac{1}{sn(\sqrt{-\frac{\omega_2}{m^2+1}}\,\omega(x-2kt),m)},$$

$$\frac{\epsilon\,\omega\sqrt{-\omega_2 m^2\beta}}{\sqrt{(m^2+1)(1+\alpha\beta)}}\,sn(\sqrt{-\frac{\omega_2}{m^2+1}}\,\omega(x-2kt),m). \tag{36}$$

Substituting (30)-(36) into (19) and (25) respectively, we have

$$u_1(x,t) = \{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta}}{6\omega\sqrt{-\omega_2(1+\alpha\beta)}}cosh[\sqrt{\omega_2}(\omega(x-2kt))] + \frac{\omega\sqrt{-\omega_2\beta}}{\sqrt{1+\alpha\beta}}$$
$$sech[\sqrt{\omega_2}(\omega(x-2kt))]\}e^{i(kx+\lambda t)},$$
$$v_1(x,t) = \frac{1}{1-4k^2}\{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta}}{6\omega\sqrt{-\omega_2(1+\alpha\beta)}}cosh[\sqrt{\omega_2}(\omega(x-2kt))] + \frac{\omega\sqrt{-\omega_2\beta}}{\sqrt{1+\alpha\beta}}$$
$$sech[\sqrt{\omega_2}(\omega(x-2kt))]\}^2,$$

$$u_2(x,t) = \{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta}}{6\omega\sqrt{-\omega_2(1+\alpha\beta)}}cos[\sqrt{-\omega_2}(\omega(x-2kt))] + \frac{\omega\sqrt{-\omega_2\beta}}{\sqrt{1+\alpha\beta}}$$
$$sec[\sqrt{\omega_2}(\omega(x-2kt))]\}e^{i(kx+\lambda t)},$$
$$v_2(x,t) = \frac{1}{1-4k^2}\{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta}}{6\omega\sqrt{-\omega_2(1+\alpha\beta)}}cos[\sqrt{-\omega_2}(\omega(x-2kt))] + \frac{\omega\sqrt{-\omega_2\beta}}{\sqrt{1+\alpha\beta}}$$
$$sec[\sqrt{\omega_2}(\omega(x-2kt))]\}^2,$$

$$u_3(x,t) = \{-\frac{\sqrt{\beta}}{\sqrt{1+\alpha\beta}}\left\{\frac{[(\lambda+k^2)-\omega^2\omega_2](\omega(x-2kt))}{6\,\epsilon\,\omega} + \frac{\epsilon\,\omega}{\omega(x-2kt)}\right\}\}e^{i(kx+\lambda t)},$$
$$v_3(x,t) = \frac{1}{1-4k^2}\{-\frac{\sqrt{\beta}}{\sqrt{1+\alpha\beta}}\left\{\frac{[(\lambda+k^2)-\omega^2\omega_2](\omega(x-2kt))}{6\,\epsilon\,\omega} + \frac{\epsilon\,\omega}{\omega(x-2kt)}\right\}\}^2,$$

$$u_4(x,t) = \{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{2\beta}}{6\,\epsilon\,\omega\sqrt{-\omega_2(1+\alpha\beta)}}coth[\sqrt{-\frac{\omega_2}{2}}(\omega(x-2kt))] + \frac{\epsilon\,\omega\sqrt{-\omega_2\beta}}{\sqrt{2(1+\alpha\beta)}}$$
$$tanh[\sqrt{-\frac{\omega_2}{2}}(\omega(x-2kt))]\}e^{i(kx+\lambda t)},$$
$$v_4(x,t) = \frac{1}{1-4k^2}\{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{2\beta}}{6\,\epsilon\,\omega\sqrt{-\omega_2(1+\alpha\beta)}}coth[\sqrt{-\frac{\omega_2}{2}}(\omega(x-2kt))] + \frac{\epsilon\,\omega\sqrt{-\omega_2\beta}}{\sqrt{2(1+\alpha\beta)}}$$
$$tanh[\sqrt{-\frac{\omega_2}{2}}(\omega(x-2kt))]\}^2.$$

$$u_5(x,t) = \{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{2\beta}}{6\,\epsilon\,\omega\sqrt{-\omega_2(1+\alpha\beta)}} cot[\sqrt{\frac{\omega_2}{2}}(\omega(x-2kt))] + \frac{\epsilon\,\omega\sqrt{-\omega_2\beta}}{\sqrt{2(1+\alpha\beta)}}$$
$$tan[\sqrt{\frac{\omega_2}{2}}(\omega(x-2kt))]\}e^{i(kx+\lambda t)},$$
$$v_5(x,t) = \frac{1}{1-4k^2}\{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{2\beta}}{6\,\epsilon\,\omega\sqrt{-\omega_2(1+\alpha\beta)}} cot[\sqrt{\frac{\omega_2}{2}}(\omega(x-2kt))] + \frac{\epsilon\,\omega\sqrt{-\omega_2\beta}}{\sqrt{2(1+\alpha\beta)}}$$
$$tan[\sqrt{\frac{\omega_2}{2}}(\omega(x-2kt))]\}^2,$$

$$u_6(x,t) = \{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta(2m^2-1)}}{6\,\omega\sqrt{-\omega_2\,m^2(1+\alpha\beta)}} \frac{1}{cn(\sqrt{\frac{\omega_2}{2m^2-1}}\,\omega(x-2kt),m)}$$
$$\frac{\omega\sqrt{-\omega_2 m^2\beta}}{\sqrt{(2m^2-1)(1+\alpha\beta)}}\,cn(\sqrt{\frac{\omega_2}{2m^2-1}}\,\omega(x-2kt),m)\}e^{i(kx+\lambda t)},$$
$$v_6(x,t) = \frac{1}{1-4k^2}\{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta(2m^2-1)}}{6\,\omega\sqrt{-\omega_2\,m^2(1+\alpha\beta)}} \frac{1}{cn(\sqrt{\frac{\omega_2}{2m^2-1}}\,\omega(x-2kt),m)}$$
$$\frac{\omega\sqrt{-\omega_2 m^2\beta}}{\sqrt{(2m^2-1)(1+\alpha\beta)}}\,cn(\sqrt{\frac{\omega_2}{2m^2-1}}\,\omega(x-2kt),m)\}^2,$$

$$u_7(x,t) = \{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta(m^2+1)}}{6\,\epsilon\,\omega\sqrt{-\omega_2\,m^2(1+\alpha\beta)}} \frac{1}{sn(\sqrt{-\frac{\omega_2}{m^2+1}}\,\omega(x-2kt),m)}$$
$$\frac{\epsilon\,\omega\sqrt{-\omega_2 m^2\beta}}{\sqrt{(m^2+1)(1+\alpha\beta)}}\,sn(\sqrt{-\frac{\omega_2}{m^2+1}}\,\omega(x-2kt),m)\}e^{i(kx+\lambda t)},$$
$$v_7(x,t) = \frac{1}{1-4k^2}\{\frac{[(\lambda+k^2)-\omega^2\omega_2]\sqrt{\beta(m^2+1)}}{6\,\epsilon\,\omega\sqrt{-\omega_2\,m^2(1+\alpha\beta)}} \frac{1}{sn(\sqrt{-\frac{\omega_2}{m^2+1}}\,\omega(x-2kt),m)}$$
$$\frac{\epsilon\,\omega\sqrt{-\omega_2 m^2\beta}}{\sqrt{(m^2+1)(1+\alpha\beta)}}\,sn(\sqrt{-\frac{\omega_2}{m^2+1}}\,\omega(x-2kt),m)\}^2.$$

By using (29), (27) and cases (14) and (15) respectively, we get

$$\rho_8(x,t) = \frac{[2(\lambda+k^2)+\omega^2\omega_2]\sqrt{\beta\gamma}}{24\sqrt{3}\,\omega^2\omega_2} cosh^2[\frac{\sqrt{\omega_2}}{2}\omega(x-2kt)] + \frac{\sqrt{\beta\gamma}}{4\sqrt{3}}$$
$$-\frac{\sqrt{3\beta\gamma}\,\omega^2\omega_2}{2(5\omega^2\omega_2-2(\lambda+k^2))} sech^2[\frac{\sqrt{\omega_2}}{2}\omega(x-2kt)], \tag{37}$$
$$\rho_9(x,t) = \frac{[2(\lambda+k^2)+\omega^2\omega_2]\sqrt{\beta\gamma}}{24\sqrt{3}\,\omega^2\omega_2} cos^2[\frac{\sqrt{-\omega_2}}{2}\omega(x-2kt)] + \frac{\sqrt{\beta\gamma}}{4\sqrt{3}}$$
$$-\frac{\sqrt{3\beta\gamma}\,\omega^2\omega_2}{2(5\omega^2\omega_2-2(\lambda+k^2))} sec^2[\frac{\sqrt{-\omega_2}}{2}\omega(x-2kt)]. \tag{38}$$

Substituting (37)-(38) into (19) and (25) respectively, we have

$$u_8(x,t) = \{\frac{[2(\lambda+k^2)+\omega^2\omega_2]\sqrt{\beta\gamma}}{24\sqrt{3}\,\omega^2\omega_2}cosh^2[\frac{\sqrt{\omega_2}}{2}\omega(x-2kt)] + \frac{\sqrt{\beta\gamma}}{4\sqrt{3}} - \frac{\sqrt{3\beta\gamma}\,\omega^2\omega_2}{2(5\omega^2\omega_2-2(\lambda+k^2))}sech^2[\frac{\sqrt{\omega_2}}{2}\omega(x-2kt)]\}e^{i(kx+\lambda t)},$$

$$v_8(x,t) = \frac{1}{1-4k^2}\{\frac{[2(\lambda+k^2)+\omega^2\omega_2]\sqrt{\beta\gamma}}{24\sqrt{3}\,\omega^2\omega_2}cosh^2[\frac{\sqrt{\omega_2}}{2}\omega(x-2kt)] + \frac{\sqrt{\beta\gamma}}{4\sqrt{3}} - \frac{\sqrt{3\beta\gamma}\,\omega^2\omega_2}{2(5\omega^2\omega_2-2(\lambda+k^2))}sech^2[\frac{\sqrt{\omega_2}}{2}\omega(x-2kt)]\}^2,$$

$$u_9(x,t) = \{\frac{[2(\lambda+k^2)+\omega^2\omega_2]\sqrt{\beta\gamma}}{24\sqrt{3}\,\omega^2\omega_2}cos^2[\frac{\sqrt{-\omega_2}}{2}\omega(x-2kt)] + \frac{\sqrt{\beta\gamma}}{4\sqrt{3}} - \frac{\sqrt{3\beta\gamma}\,\omega^2\omega_2}{2(5\omega^2\omega_2-2(\lambda+k^2))}sec^2[\frac{\sqrt{-\omega_2}}{2}\omega(x-2kt)]\}e^{i(kx+\lambda t)},$$

$$v_9(x,t) = \frac{1}{1-4k^2}\{\frac{[2(\lambda+k^2)+\omega^2\omega_2]\sqrt{\beta\gamma}}{24\sqrt{3}\,\omega^2\omega_2}cos^2[\frac{\sqrt{-\omega_2}}{2}\omega(x-2kt)] + \frac{\sqrt{\beta\gamma}}{4\sqrt{3}} - \frac{\sqrt{3\beta\gamma}\,\omega^2\omega_2}{2(5\omega^2\omega_2-2(\lambda+k^2))}sec^2[\frac{\sqrt{-\omega_2}}{2}\omega(x-2kt)]\}^2.$$

4 Conclusion

We have applied the extended Fan sub-equation method to solve nonlinear partial differential equations. As an application of the proposed method, some exact analytical solutions of the generalized Zakharov equation are successfully obtained. These solutions include hyperbolic function solutions, trigonometric function solutions and rational function solutions. Moreover, the proposed method is shown to be a simple, yet powerful algorithm for handling the systems of PDEs. *Mathematica* has been used for computations and programming in this paper.

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