Degenerate Bogdanov-Takens Bifurcations in the Gray-Scott Model

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Abstract: In this paper, we show that for a wide range of parameter values, the Gray-Scott model of families of traveling wave solutions posses two degenerate Bogdanov-Takens points. Furthermore, we explicitly define a unique compact form for the critical normal form coefficients of order 3 and 4. This is guaranteed by applying suitable solvability conditions to singular linear systems coming from the center manifold reduction combined with a normalization technique.

Keywords: *Gray-Scott model; travelling waves; degenerate Bogdanov-Takens bifurcation.*

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1 Introduction

One of the most important contributions to the bifurcation theory has been developed independently and simultaneously by Bogdanov [3,4] and Takens [19], where the topological normal form of the so-called "Bogdanov-Takens (BT) bifurcation" is derived. This bifurcation plays an important role in the analysis of dynamical systems because it gives the appearance of local bifurcations (Saddle-node bifurcation and Hopf bifurcation) and global bifurcations (homoclinic orbits to saddle equilibria) near the critical parameter values [12].

The exact bifurcation scenario near a BT point is determined by an unfolding of the critical ODE on the 2D center manifold, with as many unfolding parameters as the co-dimension of the bifurcation. More precisely, the bifurcation diagram of the unfolding depends on the coefficients of the critical normal form on the center manifold. The

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