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Existence of Weak Solutions for Nonlinear p-Elliptic Problem by Topological Degree

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Abstract: In this work, we establish the existence results on weak solutions via the recent Berkovits topological degree for the following nonlinear p-elliptic problems :

$$-div(|\nabla u|^{p-2}\nabla u) = \lambda |u|^{q-2}u + f(x,\ u,\ \nabla u)$$

in a bounded set $\Omega \in \mathbb{R}^N$, where the vector field f is a Carathéodory function.

Keywords: *weighted Sobolev spaces; Hardy inequality; topological degree; Berkovits topological degree; p-elliptic problems.*

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1 Introduction

Let Ω be a bounded open set of $\mathbb{R}^N$ $(N \geq 1)$ with a Lipschitz boundary if $N \geq 2$, and let p, q be real numbers such that $2 < q < p < \infty$, and $w = \{w_i(x),\ 0 \leq i \leq N\}$ be a vector of weight functions on Ω, i.e., each $w_i(x)$ is measurable a.e. positive on Ω. Let $W_0^{1,p}(\Omega, w)$ be the weighted Sobolev space associated with the vector w. Our objective is to prove the existence of weak solutions to the following nonlinear p-elliptic problem :

$$\begin{cases} -div(a(x,\nabla u)) = \lambda |u|^{q-2}u + f(x,u,\nabla u) & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $a(x,\nabla u) = |\nabla u|^{p-2}\nabla u$. We shall suppose that the following degenerate ellipticity condition is satisfied for all $\xi \in \mathbb{R}^N$ and almost every $x \in \Omega$:

$$a(x,\xi)\cdot\xi \geq \gamma \sum_{i=1}^{N} w_i |\xi_i|^p, \tag{2}$$

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