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Fixed Point Regions, Unified Construction of Fixed Point Mappings for Integral, Quadratic, and Fractional Equations

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Abstract: This paper is a study of integral equations by means of mapping a closed bounded convex nonempty set G into its interior. This tells us that all possible fixed points reside in G which we then call a fixed point region. The study is restricted to convolution kernels $A(t-s)$ for which there is a transformation yielding an equivalent equation. We then devise a method whereby we can often find the above mentioned set G. This leads us to globally stable fixed points. The term which makes the equation of quadratic type is added in after the transformation, whereas existing theory along these lines adds it in directly to the Volterra equation. That method produces difficulties with compactness of the mapping. In our work compactness is never an issue.

Keywords: *fixed point regions; integral equations; quadratic equations; fractional equations; fixed points; transformations.*

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1 Introduction

This paper was motivated by the fact that many fixed point theorems begin with an integral equation and the preemptive assumption that there is a closed convex nonempty bounded set G in a Banach or normed space of bounded continuous functions $\phi : [0, \infty) \to \Re$ with the supremum norm, together with a continuous mapping of $G \to G$. Often the first mapping which comes to mind is the natural one defined by the integral equation.

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