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Modification of the Trajectory Following Method for Asymptotic Stability in a System Nonlinear Control

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Abstract: In this paper, we present the asymptotic stability for a class of nonlinear control systems. To achive the asymptotic stability, we will design a dynamic feedback control. The design of the dynamic feedback control is based on the modification of the trajectory following method. To apply the modification of the trajectory following method, the system will be transformed through the input state linearization.

Keywords: *relative degree of system; input state linearization; zero dynamics; modified trajectory following method.*

Mathematics Subject Classification (2010): 93C10, 93D20.

1 Introduction

In the analysis for nonlinear control systems, there is no general method which can be applied to any nonlinear control system in designing the control input for solving the stability problems. Therefore, in general, the researchers describe some particular nonlinear classes only. Recently, stability problems for nonlinear control systems have been intensively investigated. Daizhan Cheng [1] has discussed the stability problem for a nonlinear system, where the zero dynamic has a multiplicity eigenvalue of 2. Zhengtao Ding [2] has discussed the stability of a nonlinear system through backstepping, where the backstepping design starts from the estimation of the output transformation. In 2004, Chen P *et al.* [3] and Diao L *et al.* [4] introduced the problem of stability through the system transformation, where the transformation of the system is made through dynamic feedback. In 2019, Erkan Kayacan [5] has discussed the Sliding Mode Learning Control (SMLC) of uncertain nonlinear systems with the Lyapunov stability analysis.

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One of popular methods for solving stability problems is the input-output linearization method. Some research on the stability problems of a nonlinear control system using the input-output linearization method was carried out by: Ricardo Marino and Patrizio Tomei [6], who discussed the stability of lower triangular nonlinear control system. Its stabiity control was the dynamic feedback of order $n + 2(r-1)$ (n is the system order, r is the relative degree). Results on stabilization of nonlinear lower triangular systems with uncertainties in the output feedback form have been presented in [7] and [8]. In [9], Naiborhu J. *et al.* discussed the asymptotic stability problem for a nonlinear class, where its control design used the exact linearization. Furthermore, Firman *et al.* [10] have introduced the problem of stabilization for a class of nonlinear systems with uncertainty. Then, in [11], Firman *et al.* have introduced the problem of stabilization for some class of affine nonlinear control systems with the relative degrees of the system being 1 and n-1. For the design of input controls, the system will be transformed through the partial feedback linerization. Naiborhu J. and Shimizu K. [12] have proposed a dynamic feedback control for the asymptotic stability of a nonlinear class where its unforced dynamic is asymptotically stable.

In this paper, we will propose a dynamic feedback control for asymptotic stability in a system nonlinear control, even though its unforced dynamic is not asymptotically stable. The proposed dynamic feedback control is a modification of the trajectory following method.

2 Problem Formulation

Consider the affine nonlinear control system

$$\dot{x}(t) = f(x(t)) + g(x(t))u, \tag{1}$$

$$y(t) = h\left(x(t)\right), \tag{2}$$

where $x(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}$. $f : D \to \mathcal{R}^n$, $f(\vec{0}) = \vec{0}$ and $g : D \to \mathcal{R}^n$ are sufficiently smooth in a domain $D \subset \mathcal{R}^n$. Let a state $y(t) = h(x(t))$, $h : D \to \mathcal{R}$ is sufficiently smooth in a domain $D \subset \mathcal{R}^n$, $h(\vec{0}) = 0$.

Our objective is to make the output $y(t)$ go to zero as $t \to \infty$. The main task is to design the input control u such that the system (1) has an asymptotically stable equilibrium at $x = 0$.

For designing the control input u, we need a system transformation based on the relative degree of the system. In the following, we present the method of the input state linearization by Isidori [13].

Let the relative degree of the system (1) with respect to the state y be r, $r \leq n$. If the relative degree of the system (1)-(2) is n, the system (1) with respect to the state y can be transformed to

$$\dot{z}_k = z_{k+1}, \; k = 1, 2, \cdots, n-1, \tag{3}$$

$$\dot{z}_n = f(z) + g(z)u, \tag{4}$$

$$y = z_1.$$

If $g(z) \neq 0$, $\forall t$, then the relative degree of the system with respect to the state y is well defined.

Let the relative degree of the system (1)-(2) be r, $r < n$, the system (1) with respect to the state y can be transformed to

$$\begin{aligned} \dot{z}_k &= z_{k+1}, k = 1, 2, \cdots, r-1, && (5)\\ \dot{z}_r &= f(z, \eta) + g(z, \eta)u, && (6)\\ \dot{\eta} &= q(z, \eta), && (7)\\ y &= z_1 \end{aligned}$$

with the internal dynamic

$$\dot{\eta} = q(z, \eta), \tag{8}$$

where $(z, \eta) = (z_1, z_2, \cdots, z_r, \eta_1, \eta_2, \cdots, \eta_{n-r})$. If $g(z, \eta) \neq 0$, $\forall t$, then the relative degree of the system with respect to the state y is well defined. Then if $z_1 = 0$, for all t, the system (8) is said to be zero dynamic with respect to the state $y = z_1$.

Consider a function $G : \mathcal{R}^{r+1} \to \mathcal{R}$, where $G = G\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right)$ is a positive definite function and $\frac{\partial G}{\partial x_i}$ exits for $i = 1, 2, \cdots, n$. Our objective is to find a dynamic feedback control $\dot{u}(t)$, for all t such that the function G becomes minimum. In this case, if the function G becomes minimum, then the state $y(t)$ goes to zero. The main task is to design the control input u such that $y(t) \to \infty 0$ as $t \to \infty$. Then our problem is formulated as follows:

$$min\ G\left(z_1, \dot{z}_1, \cdots, z_1^{(r)}\right), \tag{9}$$

$$\begin{aligned} subj.\, to\ \dot{x}(t) &= f(x(t) + g(x(t))u(t), && (10)\\ y(t) &= h(x(t)). && (11) \end{aligned}$$

The dynamic feedback control is designed based on the trajectory following method [14] as follows:

$$\dot{u} = -\frac{\partial G}{\partial u}. \tag{12}$$

When using the dynamic feedback control (12), the value of time derivative of function G along the trajectory of the system can not be guaranteed to be less than zero, $\forall t \geq 0$.

In this paper, we present the asymptotic stability of some class of affine nonlinear control systems by modifying the dynamic feedback control (12), i.e., by adding an artificial input.

3 Main Results

Consider the system (1)-(2). Let the relative degree of the system (1)-(2) be r, $r \leq n$. We design an input control u through the properties of the solution of a higher order ordinary differential equation. Consider a differential equation

$$a_r y^{(r)}(t) + a_{r-1} y^{(r-1)}(t) + \cdots + a_1 \dot{y} + a_0 y(t) = 0, \tag{13}$$

with $y^{(i)} = \frac{d^i y}{dt^i}$, $i = 1,\ 2,\ \cdots,\ r$, where r is the relative degree of the system (1)-(2), $r \leq n$. From equation (13), let $\omega_1 = y$, $\omega_2 = \dot{y}$, $\cdots$, $\omega_r = y^{(r-1)}$, then the equation

(13) becomes $\dot{\omega} = B\omega$, with $B = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ \frac{-a_0}{a_r} & \frac{-a_1}{a_r} & \dots & \frac{-a_{(r-1)}}{a_r} \end{pmatrix}$. If all the roots of the polynomial

$$p(\lambda) = a_r\lambda^r + a_{r-1}\lambda^{r-1} + \ldots + a_1\lambda + a_0 \tag{14}$$

have negative real part, then a solution of differential equation (13) tends to zero as $t \to \infty$. Constans a_i, $i = 0.1, \cdots, r$ can be chosen such that all the roots of the polynomial (14) have negative real part.

Define a function

$$G(z_1, \dot{z_1}, \cdots, z_1^{(r)}) = \Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)^2, \quad r \leq n. \tag{15}$$

The main task is to design the control $u(t)$ such that the function G becomes minimum. If $r < n$, then the function G (15) contains the internal dynamic solution variable. Therefore, if the zero dynamic of the system (1) with respect to the state $y = z_1$ is not asymptotically stable, then the function G (15) becomes unbounded. Furthermore, to get dynamic feedback control, assume as follows.

Assumption 3.1 *The zero dynamic of the system (1) with respect to the state $y = z_1$ is asymptotically stable.*

From equation (12), the dynamic feedback control

$$\dot{u} = -2a_r\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\frac{\partial z_1^{(r)}}{\partial u}. \tag{16}$$

Consider the extended system

$$\dot{x} = f_1(x) + f_2(x)u, \tag{17}$$

$$\dot{u} = -2a_r\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\frac{\partial z_1^{(r)}}{\partial u}. \tag{18}$$

The derivative of function G (15) along the trajectory of the system (17)-(18) is given by

$$\dot{G}\Big(z_1, \dot{z_1}, \cdots, z_1^{(r)}\Big) = 2\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\Big(\sum_{j=0}^{r-1} a_j z_1^{(j+1)}\Big) + 2a_r\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\Big(\frac{\partial f(z,\eta)}{\partial t} + \frac{\partial g(z,\eta)}{\partial t}u\Big) - \Big(\frac{\partial G}{\partial u}\Big)^2. \tag{19}$$

From equation (19), the value of the derivative of function G (15) along the trajectory of the system (17)-(18) can not be guaranteed to be less than zero for $0 \leq t$. For this, the dynamic feedback control in equation (16) will be modified by adding an input ν. Then the extended system (17)-(18) becomes

$$\dot{x} = f_1(x) + f_2(x)u, \tag{20}$$

$$\dot{u} = -2a_r\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\frac{\partial z_1^{(r)}}{\partial u} + \nu. \tag{21}$$

In the same way, the derivative of function G (16) along the trajectory of the system (20)-(21) is given by

$$\dot{G}\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right) = 2\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\Big(\sum_{j=0}^{r-1} a_j z_1^{(j+1)}\Big) + 2a_r\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\Big(\frac{\partial f(z,\eta)}{\partial t} + \frac{\partial g(z,\eta)}{\partial t}u\Big) - \Big(\frac{\partial G}{\partial u}\Big)^2 + \frac{\partial G}{\partial u}\nu. \tag{22}$$

Suppose equation (22) is written as follows:

$$\dot{G}\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right) = \phi\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right) + \frac{\partial G}{\partial u}\nu - \left(\frac{\partial G}{\partial u}\right)^2, \tag{23}$$

where

$$\begin{aligned}\phi\Big(z_1, \dot{z_1}, \cdots, z_1^{(r)}\Big) \;&=\; 2\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\Big(\sum_{j=0}^{r-1} a_j z_1^{(j+1)}\Big)\\ &\quad + 2a_r\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\Big(\frac{\partial f(z,\eta)}{\partial t} + \frac{\partial g(z,\eta)}{\partial t}u\Big).\end{aligned} \tag{24}$$

If we take

$$\nu = \frac{1}{\frac{\partial G}{\partial u}}\left(-\phi\left(z_1, \dot{z_1}, \dots, z_1^{(r)}\right)\right), \tag{25}$$

then

$$\dot{G}\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right) = -\left(\frac{\partial G}{\partial u}\right)^2, \tag{26}$$

with $\frac{\partial G}{\partial u} \neq 0$.

Consider the function G (15) and its time derivative (26). Adding the artificial input ν into dynamic controller (16) is used to guarantee the function G (15) will decrease until $\left(\sum_{j=0}^{r} a_j z_1^{(j)}\right)$ becomes zero. Furthermore, if $\frac{\partial G}{\partial u} = 0$, then we have $2a_r\left(\sum_{j=0}^{r} a_j z_1^{(j)}\right)\frac{\partial z_1^{(r)}}{\partial u} = 0$. Therefore $\left(\sum_{j=0}^{r} a_j z_1^{(j)}\right)$ becomes zero if $\frac{\partial z_1^{(r)}}{\partial u} \neq 0$. In this case, the relative degree of the system (1)-(2) is well defined.

Theorem 3.1 *Consider system (1)-(2). Let the relative degree of the system (1)-(2) be r, $r \leq n$, with the relative degree of the system (1)-(2) being well defined. Especially if $r < n$ satisfies Assumptions 1. Choose constans a_i such that all the roots of the polynomial*

$$p(\lambda) = a_r\lambda^r + a_{r-1}\lambda^{r-1} + \ldots + a_1\lambda + a_0 \tag{27}$$

have negative real part. Then, when using the dynamic feedback control

$$\dot{u} = -2a_r\Big(\sum_{j=0}^{r} a_j z_1^{(j)}\Big)\frac{\partial z_1^{(r)}}{\partial u} + \nu, \tag{28}$$

with ν as in (25), $y = z_1$ tends to zero as $t \to \infty$. Furthermore, the system (1) has an asymptotically stable equilibrium at $x = 0$.

Proof. Let the relative degree of the system (1)-(2) be r, $r < n$ satisfies Assumption 1, then the function G becomes bounded if $\frac{\partial G}{\partial u} \neq 0$. From equation (26), then $\dot{G}\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right) < 0$, with $\left(\sum_{j=0}^{r} a_j z_1^{(j)}\right) \neq 0$. Let $\left(\sum_{j=0}^{r} a_j z_1^{(j)}\right) = 0$. From equation (26), $\dot{G}\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right) = 0$. Thus, the function G (15) becomes minimum, where the minimum value is zero. Therefore, if $G\left(z_1, \dot{z_1}, \cdots, z_1^{(r)}\right) = 0$, then $\left(\sum_{j=0}^{r} a_j z_1^{(j)}\right) = 0$.
Furthermore, $\frac{\partial G}{\partial u} = 0$. Because the relative degree of the system (1)-(2) is well defined, then $\frac{\partial z_1^{(r)}}{\partial u} \neq 0$, $\forall t$. Then $\left(\sum_{j=0}^{r} a_j z_1^{(j)}\right) = 0$. Thus, if we choose a_j, $j = 0,\ 1,\ \cdots,\ r$ such that all the roots of polynomial (27) have negative real part, then $y = z_1$ goes to zero as $t \to \infty$. Furthermore, x goes to zero as $t \to \infty$. Thus the system (1) has an asymptotically stable equilibrium at $x = 0$.

Example 3.1 Consider the nonlinear system

$$\begin{aligned} \dot{x_1} &= x_2 + 2x_1^2, \\ \dot{x_2} &= x_3 + u, \\ \dot{x_3} &= x_1 + x_3. \end{aligned} \tag{29}$$

If we choose the state $y = x_3$, then the relative degree of the system (29) with respect to x_3 is 3. Thus the system transformation with respect to the state x_3 is

$$\begin{aligned} \dot{z_1} &= z_2, \\ \dot{z_2} &= z_3, \\ \dot{z_3} &= a(z) + u, \end{aligned}$$

where $z_1 = x_3$, $a(z) = z_1 + z_2 + (2(z_2 - z_1) + 1)(z_3 - z_2 - 2)(z_2 - z_1)^2 + 2(z_2 - z_1)^2$.
Define a function as follows:

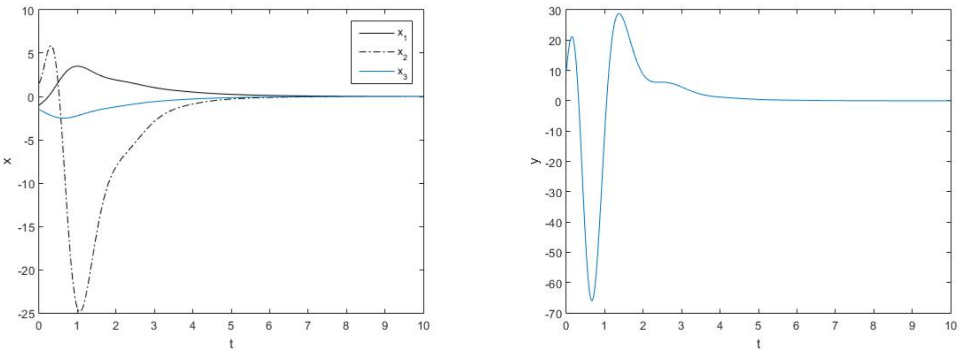
$$G\left(z_1, \dot{z_1}, \ddot{z_1}, z_1^{(3)}\right) = \Big(\sum_{j=0}^{3} a_j (z_1)^{(j)}\Big)^2. \tag{30}$$

With the above equation, the dynamic feedback control is

$$\dot{u} = -2a_3\Big(\sum_{j=0}^{3} a_j (z_1)^{(j)}\Big) + v, \tag{31}$$

with v as in equation (25).

Simulation results are shown in Figs.1a) and 1b) for constants $a_0 = 15$, $a_1 = 23$, $a_2 = 4$, $a_3 = 1$. The initial value $x_1(0) = -1$, $x_2(0) = 1,5$, $x_3(0) = -1.5$, $u(0) = 10$. In Fig.1a), with the application of the control as in equation (31), the system (29) is asymptotically stable at the equilibrium point $x = (0, 0, 0)$. In Fig.1b), the response curve of the control input is presented.



a b

Figure 1: a) the simulation result for Example 3.1, b) the response curve of the input.

Example 3.2 Consider the nonlinear system

$$\begin{aligned} \dot{x_1} &= -x_1 + x_2, \\ \dot{x_2} &= 3x_2 + x_1^3 + \left(2 + \sin^2(x_4)\right) u, \\ \dot{x_3} &= x_1 - 2x_3, \\ \dot{x_4} &= -x_4 + x_3^2. \end{aligned} \tag{32}$$

If we choose the state $y = x_4$, then the relative degree of the system (29) with respect to x_3 is 4. Thus the system transformation with respect to the state x_3 is

$$\begin{aligned} \dot{z_1} &= z_2, \\ \dot{z_2} &= z_3, \\ \dot{z_3} &= z_4, \\ \dot{z_4} &= a(z) + b(z)u, \end{aligned} \tag{33}$$

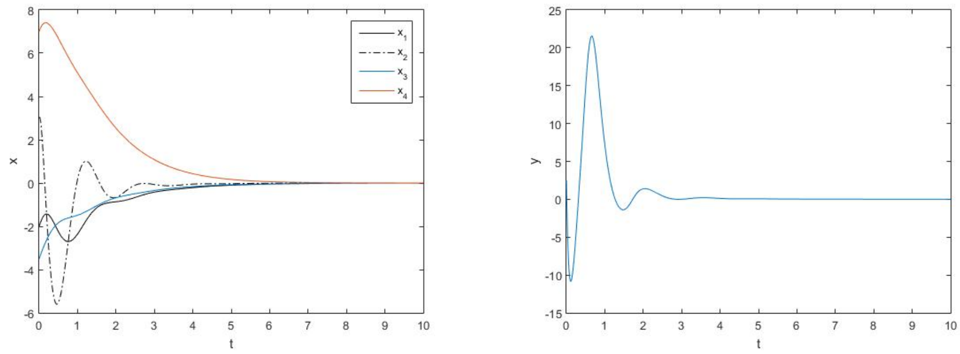
where $z_1 = x_4$, $b(z) = 2x_3\left(2 + \sin^2(x_4)\right)$.

From the system transformation (33), we see that the relative degree of the system (33) with respect to the state $y = x_4$ is not well defined. So the input control as in equation (28) cannot be used to make the state $y = x_4 \to 0$, $t \to \infty$. In this case, the system (32) can not be achieved. The problem is how to choose such a state that the transformation of the system with respect to that state has an asymptotically stable zero dynamic.

Choose the state $y = x_3$. Then the system transformation with respect to the state x_3 is

$$\begin{aligned} \dot{z_1} &= z_2, \\ \dot{z_2} &= z_3, \\ \dot{z_3} &= a(z,\eta) + b(z,\eta), \\ \dot{\eta} &= -\eta + x_1^3, \end{aligned}$$

where $z_1 = x_3$, $\eta = x_4$, $a(z,\eta) = 6z_1 + 7z_2 + (2z_1 + z_2)^3$, $z_2 = x_1 - 2x_3$, $b(z,\eta) = \left(2 + \sin^2(\eta)\right)$. So the zero dynamic of the system (29) with respect to the output x_3 is asymptotically stable, with the relative degree of the system being well defined.



a b

Figure 2: a) the simulation result for Example 3.2, b) the response curve of the input.

Define a function as follows:

$$G\big(z_1, \dot{z_1}, \ddot{z}_1, z_1^{(3)}\big) = \Big(\sum_{j=0}^{3} a_j (z_1)^{(j)}\Big)^2. \tag{34}$$

With the above equation, the dynamic feedback control is

$$\dot{u} = \big(-4 - \sin^2(\eta)\big)\, a_3 \Big(\sum_{j=0}^{3} a_j (z_1)^{(j)}\Big) + v, \tag{35}$$

where $v = \dfrac{1}{2a_3\,(a_0 z_1 + a_1 \dot{z_1} + \dot{z_2} + a_3 \dot{z_3})\,(2 + \sin^2(\eta))}\,(k\,(z_1, \dot{z_1}, \dot{z_2}, \dot{z_3}))$, with

$$\begin{aligned}(k\,(z_1, \dot{z_1}, \dot{z_2}, \dot{z_3})) &= 2\Big(\sum_{j=0}^{3} a_j (z_1)^{(j)}\Big)\Big(\sum_{j=0}^{2} a_j (z_1)^{(j+1)}\Big) \\ &\quad + 2a_3\Big(\sum_{j=0}^{3} a_j (z_1)^{(j)}\Big)\left(\frac{\partial a(z,\eta)}{\partial t} + \frac{\partial b(z,\eta)}{\partial t} u\right).\end{aligned}$$

Simulation results are shown in Figs.2a) and 2b) for constants $a_0 = 15$, $a_1 = 13$, $a_2 = 9$, $a_3 = 1$. Initial value $x_1(0) = -2$, $x_2(0) = 3$, $x_3(0) = -3.5$, $x_4(0) = -7$, $u(0) = 2.5$. In Fig.2a), with the application of the control as in equation (35), the system (32) is asymptotically stable at the equilibrium point $x = (0, 0, 0, 0)$. In Fig.2b), the response curve of the control input is shown.

4 Conclusion

In this paper, we have investigated the asymptotic stability for a class of nonlinear control systems, with the relative degree of the system being well defined. The dynamic feedback control has been designed for asymptotic stability problems. The design of the dynamic feedback control is based on the modification of the trajectory following method. To apply the modification of the trajectory following method, the system will be transformed through the input state linearization. If the relative degree of the system is smaller than the dimensions of the system, then the requirement to design a dynamic feedback control is that the zero dynamic of the system must be asymptotically stable.

From the results obtained, the modification of the trajectory following method can be an alternative control design for the asymptotic stability, even though its unforced system is not asymptotically stable.

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