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Active Fault Tolerant Synchronization of Two Hyper Chaos Lu Systems with Disturbance Input and Parametric Uncertainty

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Abstract: In this paper, a robust controller is proposed to synchronize two hyperchaos Lu systems with unwanted factors such as uncertainty and disturbance. In this research, first, an integral sliding mode control to synchronize two hyperchaos Lu systems with known parameters and known bound of uncertainty and disturbance is proposed. In the second part of the paper, a controller for synchronization of two hyperchaos systems with unknown parameters and unknown bound of uncertainty and disturbance is designed, and unknown parameters are estimated by an adaptation rule. The stability of the control is proved using the Lyapunov stability method in the corresponding cases. The simulation results with MATLAB software show that the designed controller is able to synchronize two systems, although they have uncertain parameters.

Keywords: *integral sliding mode; adaptive control; hyperchaos; uncertainty; parameter estimation.*

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1 Introduction

Nonlinear dynamics is widely used in engineering, physics, biology and many other scientific areas. The interest in nonlinear dynamics and chaotic dynamics has grown rapidly since 1963, when Edward Lorenz, an American meteorologist, discovered a classic chaotic system, and the phenomenon of chaos was gradually being considered by many scholars in various fields. Given the vast applications of chaos phenomena in various sciences such as secure communications [1], nonlinear circuits [2], chemical reactions [3], power electronics [4], lasers [5], encryption [6], study and research on the inherent characteristics of this phenomenon and its control has become of importance in sciences. Due to the introduction of new chaotic systems, the problem of controlling chaos in these types of systems was considered by scientists and researchers in order to control chaos for different purposes such as removing chaos, behavior and anti-chaos control (chaos for a system), bipolar control and synchronization of two chaotic systems. A chaotic system with more than one positive Lyapunov exponent is known as a hyperchaotic system which means that its dynamics extends simultaneously in several different directions. Hyperchaos systems in the presence of more than one positive Lyapunov exponent due to more complex dynamics, which improves applications in secure communications, encryption and decryption, have attracted the attention of many researchers in recent years. Lately, several supercharged systems have been discovered with high-level dynamics. For example, Chua hyperchaos [7], Rossler system [8], Lorenz hyperchaos system [9]. In 2002, Levechin found a new chaotic system known as the Lu system which is the bridge between Lorenz's chaotic system and Chen's chaotic system. The Lu hyperchaos system is based on the chaos Lu system and state feedback [10].

One of the important applications of the hyperchaotic Lu system similar to most of the other hyperchaotic systems mentioned above in the field of secure communications is the use of hyperchaotic systems to increase the level of information security. Because of the noise-like and complex behaviors, chaotic systems have the ability to cover information with a high degree of reliability. The general idea for transmitting information by chaotic systems is based on the fact that the embedding of information in the transmitter system produces a chaotic signal.

In recent years, chaos and synchronization control have been investigated, for example, synchronization with adaptive control [11], in which the problem of synchronizing two hyperchaos systems with an adaptive controller is investigated, active control [12], fuzzy sliding mode control [13], impulsive synchronization [14], active backstepping synchronization [15], nonlinear schemes [16], [17], hybrid projective synchronization [18] and so on.

Synchronization of chaos systems has been widely discussed in recent decades, and attracted the attention of many researchers in controlling chaos. As a general synchronization definition, it is possible to synchronize the variables of a chaotic system with another chaotic system, when the primary system is called master, and the second system is slave. The first method of synchronizing two chaotic systems was proposed in [19].

In this paper, synchronization of hyperchaos systems, despite the uncertainties, disturbance and different initial conditions, was investigated. A sliding-adaptive control, regarding its advantages such as simple and easy realization, quick answer, good transient performance, and robustness against system uncertainties and disturbances, is designed as a control method for synchronization. The stability of the chaotic system has been proved by controllers designed using the Lyapunov theorem, and it is shown that the slave system states asymptotically track the states of the master system. One of the most important applications of the presented method in this paper in nonlinear systems and systems theory is a secure communications system, where synchronization between the transmitter and the receiver is a vital problem. The overall structure of this paper is as follows. In the second section, the dynamical model of the hyperchaos system is introduced. The third part of the paper describes the issue of synchronization between two hyperchaos systems, with known bound of uncertainties and disturbances. Furthermore in this section, estimation of the unknown parameters of the hyperchaos system is investigated in spite of uncertainty and disturbance. Finally, simulation results of the proposed controller are presented in Section 4.

2 Introduction of Dynamic Model

Elabbasy et al. [20] represented dynamic equations of the hyperchaos system as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = cx_2 - x_1x_3 + x_4, \\ \dot{x}_3 = x_1x_2 - bx_3, \\ \dot{x}_4 = x_3 - dx_4, \end{cases} \tag{1}$$

where the fourth state is a simple feedback, that is added to the second state, and $a = 20,\ b = 5,\ c = 10,\ d = 1.5$, and $X = [x_1, x_2, x_3, x_4]$ is the vector of the state variables of the master system. Both master and slave systems follow the same dynamical equations as equation (1) with different initial conditions, but the main difference is that all states of the master system should be followed by a slave system using a controller. Therefore, the slave system, with the disturbance and parametric uncertainty, is expressed as follows:

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + \Delta f_1 + w_1 + u_1, \\ \dot{y}_2 = cy_2 - y_1y_3 + y_4 + \Delta f_2 + w_2 + u_2, \\ \dot{y}_3 = y_1y_2 - by_3 + \Delta f_3 + w_3 + u_3, \\ \dot{y}_4 = y_3 - dy_4 + \Delta f_4 + w_4 + u_4, \end{cases} \tag{2}$$

in which $u = [u_1, u_2, u_3, u_4]$ is the control vector, and $Y = [y_1, y_2, y_3, y_4]$ is the vector of states of the slave system, $\|\Delta f_i\| \leq \alpha_i, i = 1, .., 4$, is the parametric uncertainty with known bound and $\|w_i\| \leq \beta_i, i = 1, .., 4$, is the disturbance input with known bound. In Figure 1, the hyperchaos system is shown with a parametric set of $a = 20, b = 5, c = 10, d = 1.5$ [14]. These parameters, with the Lyapunov exponent $0.75, 0.03, -1.55, -15.73$ calculated in [21], cause a hyperchaos system.

3 Synchronization of Two Hyperchaos Lu Systems

In the real world, all or some of the system's parameters are unknown or uncertain. So, the synchronization issue may fail. In this section, a synchronization method for two same hyperchaos Lu systems is mentioned. Consider the master and slave systems (1) and (2). Due to the definition of the error as $e_i = y_i - x_i, \quad i = 1, 2, 3, 4$, we have

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + \Delta f_1 + w_1 + u_1, \\ \dot{e}_2 = ce_2 + e_4 - e_1e_3 - x_1e_3 - x_3e_1 + \Delta f_2 + w_2 + u_2, \\ \dot{e}_3 = -be_3 + e_1e_2 + x_2e_1 + x_1e_2 + \Delta f_3 + w_3 + u_3, \\ \dot{e}_4 = e_3 - de_4 + \Delta f_4 + w_4 + u_4. \end{cases} \tag{3}$$

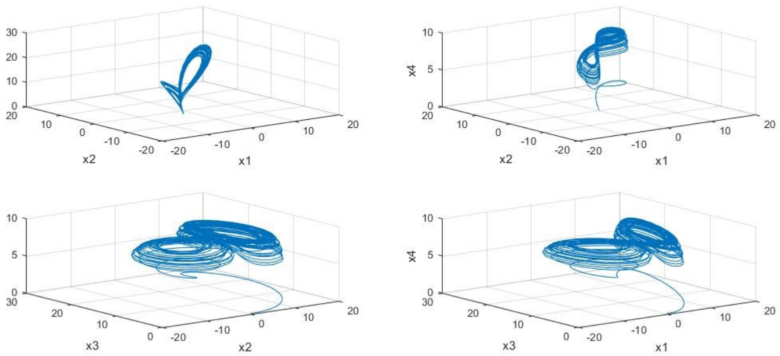

Figure 1: Images of attractor of hyperchaos Lu system.

3.1 Synchronization of two hyperchaos Lu systems with uncertainty and disturbance input

First, the problem of synchronizing two same hyperchaos systems, with known parameters is considered and the sliding mode controller is designed. The sliding mode control is a nonlinear control method that guarantees control strategy over uncertainties. In this way, stability is obtained by keeping system modes on the sliding surface.

In general, the sliding mode controller design consists of two steps:

- A. Sliding surface design that reduces the order of the closed loop system, and provides a resilient bed in the movement of the system towards the equilibrium point.
- B. Choosing the right control policy to move the system to this level and ensure that it stays on it.

Now, with the sliding surface definition, we have the following:

$$s_i = e_i + \int k_i e_i \qquad i = 1, 2, 3, 4, \tag{4}$$

$$\begin{cases} \dot{s}_1 = a(e_2 - e_1) + k_1 e_1 + \Delta f_1 + w_1 + u_1, \\ \dot{s}_2 = ce_2 + e_4 - e_1 e_3 - x_1 e_3 - x_3 e_1 + k_2 e_2 + \Delta f_2 + w_2 + u_2, \\ \dot{s}_3 = -be_3 + e_1 e_2 + x_2 e_1 + x_1 e_2 + \Delta f_3 + k_3 e_3 + w_3 + u_3, \\ \dot{s}_4 = e_3 - de_4 + k_4 e_4 + \Delta f_4 + w_4 + u_4, \end{cases} \tag{5}$$

and considering $\dot{s} = 0$, we have

$$u_{eq} = \begin{cases} u_1 = -a(e_2 - e_1) - k_1 e_1, \\ u_2 = -ce_2 - e_4 + e_1 e_3 + x_1 e_3 + x_3 e_1 - k_2 e_2, \\ u_3 = be_3 - e_1 e_2 - x_2 e_1 - x_1 e_2 - k_3 e_3, \\ u_4 = -e_3 + de_4 - k_4 e_4. \end{cases} \tag{6}$$

On the other hand, the control signal of the proposed controller is considered as follows:

$$u_i = u_{eq_i} - (rs_i + \rho \operatorname{sgn}(s_i)) - (\alpha_i + \beta_i), \tag{7}$$

in which ρ and r are greater than zero.

Theorem 3.1 *If in the control signal (7), the parameters are positive and certain, then all system states of (2) will tend to the states of system (1).*

Proof. Suppose that the Lyapunov function is considered as (8), which is a positive definite function. Given the Lyapunov stability theorem, to prove the stability of the sliding mode dynamic (5), we need to show that the derivative of the Lyapunov function is negative, so, according to the selective S, u proves the asymptotic stability by using the Lyapunov stability.

The proposed Lyapunov function is as follows:

$$V = \frac{1}{2}\sum_{i=1}^{4} s_i^2 \tag{8}$$

and its derivative is as follows:

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{4} s_i\dot{s}_i = s_1\dot{s}_1 + s_2\dot{s}_2 + s_3\dot{s}_3 + s_4\dot{s}_4\\ &= s_1(a(e_2 - e_1) + k_1e_1 + \Delta f_1 + w_2 + u_1)\\ &+s_2(ce_2 + e_4 - e_1e_3 - x_1e_3 - x_3e_1 + k_2e_2 + \Delta f_2 + w_2 + u_2)\\ &+s_3(-be_3 + e_1e_2 + x_2e_1 + x_1e_2 + \Delta f_3 + k_3e_3 + w_3 + u_3)\\ &+s_4(e_3 - de_4 + k_4e_4 + \Delta f_4 + w_4 + u_4),\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{V} \leq\ & s_1(a(e_2 - e_1) + k_1e_1 + \alpha_1 + \beta_1 + u_1)\\ &+s_2(ce_2 + e_4 - e_1e_3 - x_1e_3 - x_3e_1 + k_2e_2 + \alpha_2 + \beta_2 + u_2)\\ &+s_3(-be_3 + e_1e_2 + x_2e_1 + x_1e_2 + k_3e_3 + \alpha_3 + \beta_3 + u_3)\\ &+s_4(e_3 - de_4 + k_4e_4 + \alpha_4 + \beta_4 + u_4),\end{aligned}$$

$$\begin{aligned}\dot{V} \leq\ & s_1\left(-rs_1 - \rho\,\mathrm{sgn}\left(s_1\right)\right) + s_2\left(-rs_2 - \rho\,\mathrm{sgn}\left(s_2\right)\right)\\ &+ s_3\left(-rs_3 - \rho\,\mathrm{sgn}\left(s_3\right)\right) + s_4\left(-rs_4 - \rho\,\mathrm{sgn}\left(s_4\right)\right),\end{aligned}$$

$$\dot{V} \leq \left(-rs_1^2 - \rho\left|s_1\right|\right) + \left(-rs_2^2 - \rho\left|s_2\right|\right) + \left(-rs_3^2 - \rho\left|s_3\right|\right) + \left(-rs_4^2 - \rho\left|s_4\right|\right)$$

$$for\ \ r \geq 0, \quad \rho \geq 0 \quad \Rightarrow \quad \dot{V} \leq 0. \tag{10}$$

By choosing ρ, r greater than zero, $\dot{V}$ becomes negative, and Lyapunov's stability condition will be established. □

3.2 Synchronization of hyperchaos Lu systems with disturbance input and unknown system parameters

Here is an estimate of the system's uncertain parameters synchronizing two same hyperchaos systems despite the uncertainty. The master systems in the form of Equation (1) and the slave system are defined as follows:

$$\begin{cases}\dot{y}_1 = \bar{a}(y_2 - y_1) + \Delta f_1 + w_1 + u_1,\\ \dot{y}_2 = \bar{c}y_2 - y_1y_3 + y_4 + \Delta f_2 + w_2 + u_2,\\ \dot{y}_3 = y_1y_2 - \bar{b}y_3 + \Delta f_3 + w_3 + u_3,\\ \dot{y}_4 = y_3 - \bar{d}y_4 + \Delta f_4 + w_4 + u_4,\end{cases} \tag{11}$$

where $u_i, i = 1, ..., 4$, are the control signals used to synchronize two same hyperchaos systems and $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are unknown parameters, and to estimate them, an adaptive rule is suggested in the synchronization process.

Due to the definition of the error in the form $e_i = y_i - x_i, \quad i = 1, 2, 3, 4,$ we have

$$\begin{cases} \dot{e}_1 = \bar{a}(y_2 - y_1) - a(x_2 - x_1) + \Delta f_1 + w_1 + u_1, \\ \dot{e}_2 = \bar{c}y_2 + e_4 - e_1e_3 - x_1e_3 - x_3e_1 - cx_2 + \Delta f_2 + w_2 + u_2, \\ \dot{e}_3 = -\bar{b}y_3 + e_1e_2 + x_2e_1 + x_1e_2 + bx_3 + \Delta f_3 + w_3 + u_3, \\ \dot{e}_4 = e_3 - \bar{d}y_4 + dx_4 + \Delta f_4 + w_4 + u_4. \end{cases} \tag{12}$$

Now, by defining the sliding surface as (4), we have

$$\begin{cases} \dot{s}_1 = \bar{a}\,(y_2 - y_1) - a\,(x_2 - x_1) + k_1e_1 + \Delta f_1 + w_1 + u_1, \\ \dot{s}_2 = \bar{c}y_2 - cx_2 + e_4 - e_1e_3 - x_1e_3 - x_3e_1 + k_2e_2 + \Delta f_2 + w_2 + u_2, \\ \dot{s}_3 = bx_3 - \bar{b}y_3 + e_1e_2 + x_2e_1 + x_1e_2 + k_3e_3 + \Delta f_3 + w_3 + u_3, \\ \dot{s}_4 = e_3 - \bar{d}y_4 + dx_4 + k_4e_4 + \Delta f_4 + w_4 + u_4. \end{cases} \tag{13}$$

The proposed controller is presented as follows:

$$u_i = u_{eq_i} - (rs_i + \rho\,\mathrm{sgn}(s_i)), \tag{14}$$

where

$$u_{eq} = \begin{cases} u_1 = -(\tilde{a} + a)\,(e_2 - e_1) - k_1e_1 - \left(\tilde{\alpha}_1 + \alpha_1 + \tilde{\beta}_1 + \beta_1\right), \\ u_2 = -(\tilde{c} + c)e_2 - e_4 + e_1e_3 + x_1e_3 + x_3e_1 + k_2e_2 - \left(\tilde{\alpha}_2 + \alpha_2 + \tilde{\beta}_2 + \beta_2\right), \\ u_3 = (\tilde{b} + b)e_3 - e_1e_2 - x_2e_1 - x_1e_2 - k_3e_3 - \left(\tilde{\alpha}_3 + \alpha_3 + \tilde{\beta}_3 + \beta_3\right), \\ u_4 = (\tilde{d} + d)y_4 - e_3 - k_4e_4 - \left(\tilde{\alpha}_4 + \alpha_4 + \tilde{\beta}_4 + \beta_4\right) \end{cases} \tag{15}$$

and $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$ are the estimation of the adaptation error.

Theorem 3.2 *If the control signal is the relation (14) with the adaptation rules of relation (22), then all system states (11) will tend to the states of system (1).*

Proof. Using the Lyapunov stability theorem, we consider the Lyapunov candidate function as (16) which is a positive definite function

$$V = \frac{1}{2}\left(\sum_{i=1}^{4} s_i^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{\alpha}_1^2 + \tilde{\beta}_1^2 + \tilde{\alpha}_2^2 + \tilde{\beta}_2^2 + \tilde{\alpha}_3^2 + \tilde{\beta}_3^2 + \tilde{\alpha}_4^2 + \tilde{\beta}_4^2\right) \tag{16}$$

with derivation, we have

$$\dot{V} = \sum_{i=1}^{4} s_i\dot{s}_i + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{d}\dot{\tilde{d}} + \tilde{\alpha}_1\dot{\tilde{\alpha}}_1 + \tilde{\beta}_1\dot{\tilde{\beta}}_1 + \tilde{\alpha}_2\dot{\tilde{\alpha}}_2 + \tilde{\beta}_2\dot{\tilde{\beta}}_2 + \tilde{\alpha}_3\dot{\tilde{\alpha}}_3 + \tilde{\beta}_3\dot{\tilde{\beta}}_3 + \tilde{\alpha}_4\dot{\tilde{\alpha}}_4 + \tilde{\beta}_4\dot{\tilde{\beta}}_4, \tag{17}$$

where

$$\begin{array}{l} \tilde{a} = \bar{a} - a, \quad \tilde{b} = \bar{b} - b, \quad \tilde{c} = \bar{c} - c, \quad \tilde{d} = \bar{d} - d, \\ \tilde{\alpha}_1 = \bar{\alpha}_1 - \alpha_1, \quad \tilde{\beta}_1 = \bar{\beta}_1 - \beta_1, \;\; \tilde{\alpha}_2 = \bar{\alpha}_2 - \alpha_2, \quad \tilde{\beta}_2 = \bar{\beta}_2 - \beta_2, \\ \tilde{\alpha}_3 = \bar{\alpha}_3 - \alpha_3, \quad \tilde{\beta}_3 = \bar{\beta}_3 - \beta_3, \;\; \tilde{\alpha}_4 = \bar{\alpha}_4 - \alpha_4, \quad \tilde{\beta}_4 = \bar{\beta}_4 - \beta_4, \\ \dot{\tilde{a}} = \dot{\bar{a}}, \quad \dot{\tilde{b}} = \dot{\bar{b}}, \quad \dot{\tilde{c}} = \dot{\bar{c}}, \quad \dot{\tilde{d}} = \dot{\bar{d}}, \quad \dot{\tilde{\alpha}}_1 = \dot{\bar{\alpha}}_1, \quad \dot{\tilde{\beta}}_1 = \dot{\bar{\beta}}_1, \\ \dot{\tilde{\alpha}}_2 = \dot{\bar{\alpha}}_2, \quad \dot{\tilde{\beta}}_2 = \dot{\bar{\beta}}_2, \quad \dot{\tilde{\alpha}}_3 = \dot{\bar{\alpha}}_3, \quad \dot{\tilde{\beta}}_3 = \dot{\bar{\beta}}_3, \quad \dot{\tilde{\alpha}}_4 = \dot{\bar{\alpha}}_4, \quad \dot{\tilde{\beta}}_4 = \dot{\bar{\beta}}_4. \end{array} \tag{18}$$

By replacing (13) in (17), we have

$$\begin{aligned}\dot{V} &= s_1\left(\bar{a}(y_2-y_1)-a\left(x_2-x_1\right)+k_1e_1+\Delta f_1+w_1+u_1\right)\\ &+\ s_2\left(\bar{c}y_2-cx_2+e_4-e_1e_3-x_1e_3-x_3e_1+k_2e_2+\Delta f_2+w_2+u_2\right)\\ &+\ s_3\left(-\bar{b}y_3+bx_3+e_1e_2+x_2e_1+x_1e_2+k_3e_3+\Delta f_3+w_3+u_3\right)\\ &+s_4\ \left(e_3-\bar{d}y_4+dx_4+k_4e_4+\Delta f_4+w_4+u_4\right)\\ &+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{\alpha}_1\dot{\tilde{\alpha}}_1+\tilde{\beta}_1\dot{\tilde{\beta}}_1+\tilde{\alpha}_2\dot{\tilde{\alpha}}_2+\tilde{\beta}_2\dot{\tilde{\beta}}_2+\tilde{\alpha}_3\dot{\tilde{\alpha}}_3+\tilde{\beta}_3\dot{\tilde{\beta}}_3+\tilde{\alpha}_4\dot{\tilde{\alpha}}_4+\tilde{\beta}_4\dot{\tilde{\beta}}_4,\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{V} &= s_1\left(\bar{a}(y_2-y_1)-a\left(x_2-x_1\right)+k_1e_1+\Delta f_1+w_1+u_1+\bar{a}(x_2-x_1)-\bar{a}(x_2-x_1)\right)\\ &+\ s_2\left(\bar{c}y_2-cx_2+e_4-e_1e_3-x_1e_3-x_3e_1+k_2e_2+\Delta f_2+w_2+u_2+\bar{c}x_2-\bar{c}_2x_2\right)\\ &+\ s_3\left(-\bar{b}y_3+bx_3+e_1e_2+x_2e_1+x_1e_2+k_3e_3+\Delta f_3+w_3+u_3+\bar{b}x_3-\bar{b}x_3\right)\\ &+s_4\ \left(e_3-\bar{d}y_4+dx_4+k_4e_4+\Delta f_4+w_4+u_4+\bar{d}x_4-\bar{d}x_4\right)\\ &+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{\alpha}_1\dot{\tilde{\alpha}}_1+\tilde{\beta}_1\dot{\tilde{\beta}}_1+\tilde{\alpha}_2\dot{\tilde{\alpha}}_2+\tilde{\beta}_2\dot{\tilde{\beta}}_2+\tilde{\alpha}_3\dot{\tilde{\alpha}}_3+\tilde{\beta}_3\dot{\tilde{\beta}}_3+\tilde{\alpha}_4\dot{\tilde{\alpha}}_4+\tilde{\beta}_4\dot{\tilde{\beta}}_4,\end{aligned}$$

$$\begin{aligned}\dot{V} &= s_1\left(\bar{a}(e_2-e_1)+k_1e_1+\Delta f_1+w_1+u_1-a(x_2-x_1)+\bar{a}(x_2-x_1)\right)\\ &+\ s_2\left(\bar{c}e_2-cx_2+e_4-e_1e_3-x_1e_3-x_3e_1+k_2e_2+\Delta f_2+w_2+u_2+\bar{c}x_2\right)\\ &+\ s_3\left(-\bar{b}e_3+bx_3+e_1e_2+x_2e_1+x_1e_2+k_3e_3+\Delta f_3+w_3+u_3-\bar{b}x_3\right)\\ &+s_4\ \left(e_3-\bar{d}e_4+dx_4+k_4e_4+\Delta f_4+w_4+u_4-\bar{d}x_4\right)\\ &+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{d}\dot{\tilde{d}}+\tilde{\alpha}_1\dot{\tilde{\alpha}}_1+\tilde{\beta}_1\dot{\tilde{\beta}}_1+\tilde{\alpha}_2\dot{\tilde{\alpha}}_2+\tilde{\beta}_2\dot{\tilde{\beta}}_2+\tilde{\alpha}_3\dot{\tilde{\alpha}}_3+\tilde{\beta}_3\dot{\tilde{\beta}}_3+\tilde{\alpha}_4\dot{\tilde{\alpha}}_4+\tilde{\beta}_4\dot{\tilde{\beta}}_4.\end{aligned} \tag{20}$$

By replacing (14) and (18) in (20), one gets

$$\begin{aligned}\dot{V} &\le s_1\begin{pmatrix}\bar{a}(e_2-e_1)+ke_1+\alpha_1+\beta_1-a(x_2-x_1)+\bar{a}(x_2-x_1)\\ -\bar{a}(e_2-e_1)-ke_1-(\bar{\alpha}_1+\bar{\beta}_1)-rs_1-\rho\operatorname{sgn}(s_1)\end{pmatrix}\\ &+\ s_2\begin{pmatrix}\bar{c}e_2-cx_2+e_4-e_1e_3-x_1e_3-x_3e_1+ke_2+\alpha_2+\beta_2+\bar{c}x_2\\ -\bar{c}e_2-e_4+e_1e_3+x_1e_3+x_3e_1-ke_2-(\bar{\alpha}_2+\bar{\beta}_2)-rs_2-\rho\operatorname{sgn}(s_2)\end{pmatrix}\\ &+\ s_3\begin{pmatrix}-\bar{b}e_3+bx_3+e_1e_2+x_2e_1+x_1e_2+ke_3+\alpha_3+\beta_3+u_3-\bar{b}x_3\\ +\bar{b}e_3-e_1e_2-x_2e_1-x_1e_2-ke_3-(\bar{\alpha}_3+\bar{\beta}_3)-rs_3-\rho\operatorname{sgn}(s_3)\end{pmatrix}\\ &+s_4\begin{pmatrix}e_3-\bar{d}e_4+dx_4+ke_4+\alpha_4+\beta_4+u_4-\bar{d}x_4\\ -e_3+\bar{d}e_4-ke_4-(\bar{\alpha}_4+\bar{\beta}_4)-rs_4-\rho\operatorname{sgn}(s_4)\end{pmatrix}\\ &+(\bar{a}-a)\dot{\tilde{a}}+\left(\bar{b}-b\right)\dot{\tilde{b}}+(\bar{c}-c)\dot{\tilde{c}}+\left(\bar{d}-d\right)\dot{\tilde{d}}+(\bar{\alpha}_1-\alpha_1)\dot{\tilde{\alpha}}_1+\left(\bar{\beta}_1-\beta_1\right)\dot{\tilde{\beta}}_1\\ &+(\bar{\alpha}_2-\alpha_2)\dot{\tilde{\alpha}}_2+\left(\bar{\beta}_2-\beta_2\right)\dot{\tilde{\beta}}_2+(\bar{\alpha}_3-\alpha_3)\dot{\tilde{\alpha}}_3+\left(\bar{\beta}_3-\beta_3\right)\dot{\tilde{\beta}}_3+(\bar{\alpha}_4-\alpha_4)\dot{\tilde{\alpha}}_4\\ &+\left(\bar{\beta}_4-\beta_4\right)\dot{\tilde{\beta}}_4.\end{aligned} \tag{21}$$

The adaptation rules are given as follows:

$$\begin{array}{llll}\dot{\tilde{a}}=(x_1-x_2)\,s_1, & \dot{\tilde{d}}=x_4s_4, & \dot{\tilde{\alpha}}_2=s_2, & \dot{\tilde{\beta}}_3=s_3,\\ \dot{\tilde{b}}=x_3s_3, & \dot{\tilde{\alpha}}_1=s_1, & \dot{\tilde{\beta}}_2=s_2, & \dot{\tilde{\alpha}}_4=s_4,\\ \dot{\tilde{c}}=-x_2s_2, & \dot{\tilde{\beta}}_1=s_1, & \dot{\tilde{\alpha}}_3=s_3, & \dot{\tilde{\beta}}_4=s_4,\end{array} \tag{22}$$

therefore from (21) and (22), we have

$$\begin{gathered}\dot{V}\le s_1\left(-rs_1-\rho\operatorname{sgn}(s_1)\right)+s_2\left(-rs_2-\rho\operatorname{sgn}(s_2)\right)\\ +s_3\left(-rs_3-\rho\operatorname{sgn}(s_3)\right)\ +s_4\left(-rs_4-\rho\operatorname{sgn}(s_4)\right),\\ \dot{V}\le\left(-rs_1^2-\rho\left|s_1\right|\right)+\left(-rs_2^2-\rho\left|s_2\right|\right)+\left(-rs_3^2-\rho\left|s_3\right|\right)+\left(-rs_4^2-\rho\left|s_4\right|\right),\\ for\qquad r\ge 0\quad,\quad\rho\ge 0\qquad\Rightarrow\qquad\dot{V}\le 0.\end{gathered} \tag{23}$$

The hyperchaos system (11) with the initial conditions of $y_i(0)\in R^4$, by the control rules in (14), where $r,\rho>0$, and with the adaptation rules (22), follows the trajectory of the master system. □

4 Simulation

In this section, simulations show the effectiveness of the proposed scheme for synchronizing hyperchaos Lu systems. Simulation results are obtained using the MATLAB software.

Example 4.1 In this part, simulation with the initial conditions $[x_1, x_2, x_3, x_4]^T = [0.1, 0.1, 0.1, 0.1]^T$ and $[y_1, y_2, y_3, y_4]^T = [-9.9,\ -4.9,\ 5.1,\ 10.1]^T$ and $a = 20,\ b = 5,\ c = 10,\ d = 1.5$ is done. Parameters used in the design are $k_i = 15, r = 5, \rho = 10$. Uncertainty and bounded disturbance applied to the system are $\Delta f_i = A \sin(x_1) \cos(x_2),\ \ w_i = A \sin(t), 0.1 < A < 1$, respectively, in which $\Delta f_i \leq \alpha_i = 1,\ \ w_i \leq \beta_i = 1$.

Figures 2 and 3 show the states and error synchronization of hyperchaos system before applying the controller to the slave system. Figures 4 and 5 show the synchronization of the two systems after applying the controller of equation (7) which represents the performance of the proposed controller.

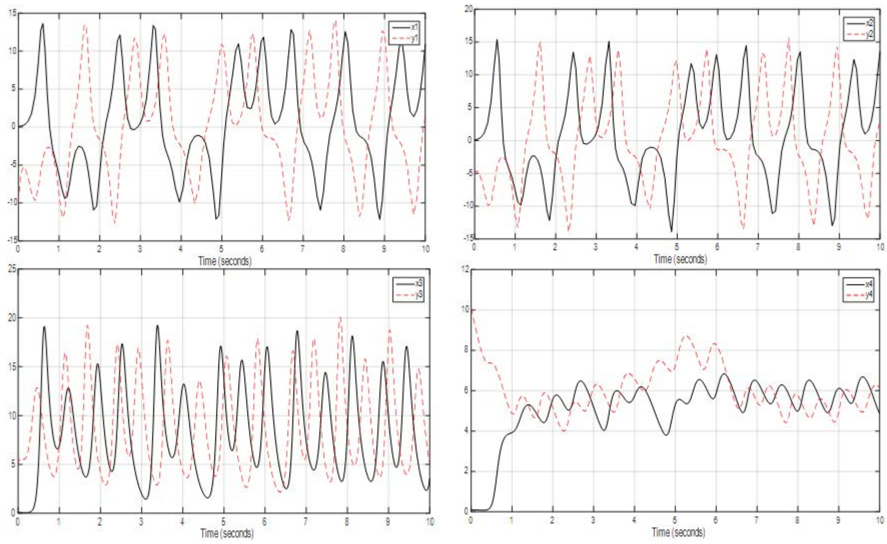

Figure 2: Master and slave system states before applying the controller.

Example 4.2 In this part, we assume unknown slave system parameters. Simulation with the initial conditions $[x_1, x_2, x_3, x_4]^T = [0.1, 0.1, 0.1, 0.1]^T$ and $[y_1, y_2, y_3, y_4]^T = [-9.9, -4.9, 5.1, 10.1]^T$ is performed. Parameters used in the design are $k_i = 15, r = 5, \rho = 10$. Uncertainty and disturbance input applied to the system are in the form of $\Delta f_i = A \sin(x_1) \cos(x_2), w_i = A \sin(t), 0.1 < A < 1$, respectively. By applying the control and estimation parameter rules of (14) and (22), respectively, and applying $\bar{\alpha}_0 = \bar{\beta}_0 = \bar{a}_0 = \bar{b}_0 = \bar{c}_0 = \bar{d}_0 = 1$, the simulation results are shown in Figures 6 and 7. Figure 6 shows the states of the master and slave systems. In Figure 7, the tendency of synchronization error to zero is depicted over time. Figure 8 also shows the estimated unknown parameters $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ of the slave system.

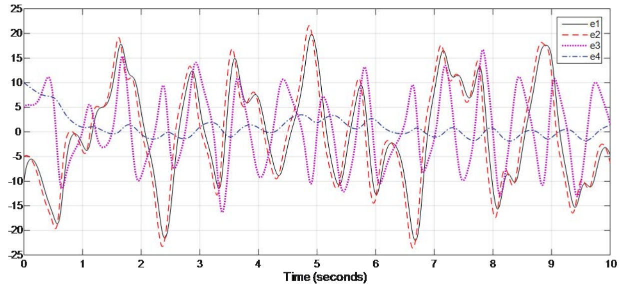

Figure 3: Synchronization error before applying the controller.

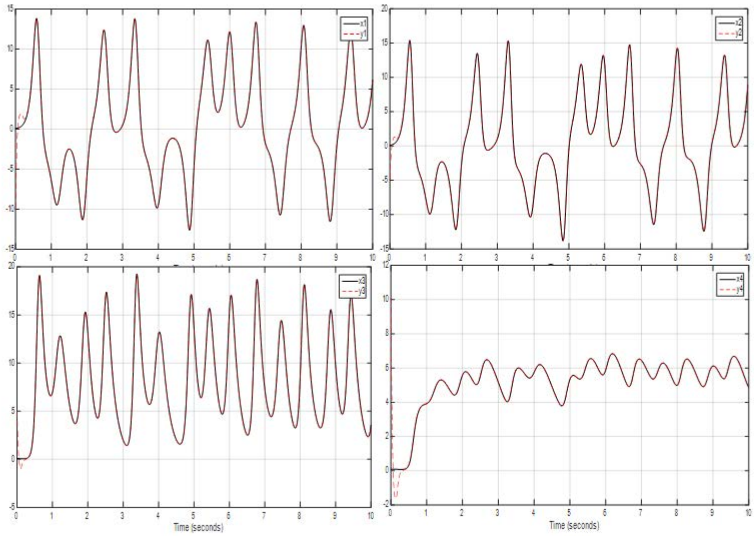

Figure 4: Master and slave system states after applying the controller equation (7).

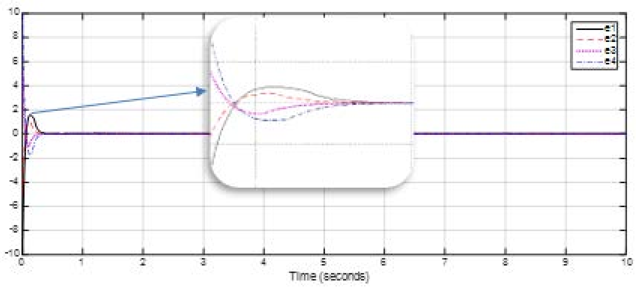

Figure 5: Synchronization error after applying the controller equation (7).

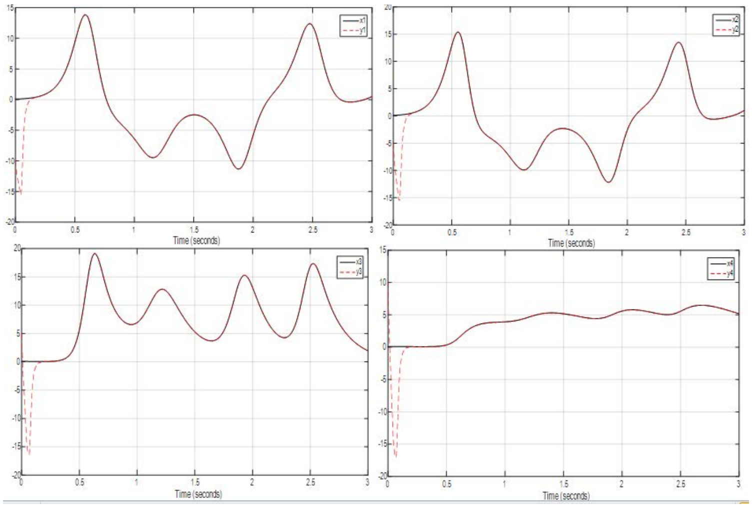

Figure 6: Master and slave system states after applying the controller equation (14).

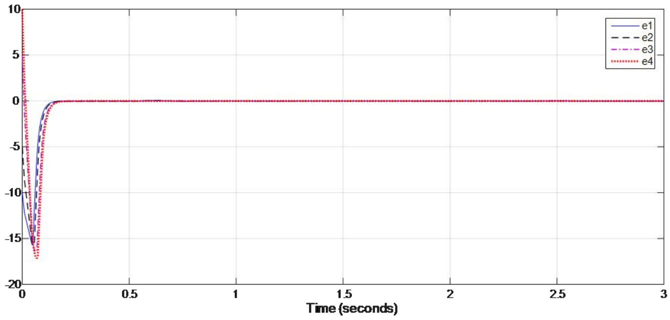

Figure 7: Synchronization error after applying the controller equation (14).

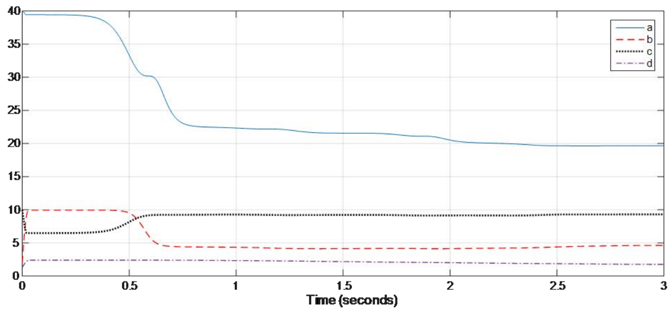

Figure 8: Estimation of $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ parameters.

5 Conclusion

The main objective of this paper is to design the adaptive controller for a hyperchaos system with unknown parameters in the presence of parametric uncertainty and disturbance input. To reach this goal, the combination of the two sliding mode control and the adaptive control methods is proposed to synchronize hyperchaos Lu systems. The stability of the chaotic system is proved using the Lyapunov theorem. To achieve synchronization, the sliding mode control method, which is a robust control against uncertainty, was used. Also, adaptive rules are used to identify the unknown slave system parameters. The results of simulation with MATLAB software showed the well-designed controllers performance in two ways.

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