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Design of Navigation and Guidance Control System of Mobile Robot with Position Estimation Using Ensemble Kalman Filter (EnKF) and Square Root Ensemble Kalman Filter (SR-EnKF)

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Abstract: A mobile robot is one of the unmanned land vehicles which can be controlled and whose position can be detected when it is equipped with a Global Positioning System (GPS). A mobile robot aims to automate some tasks that were usually done manually by a human. To gain an accurate detection of the mobile robot position, the mobile robot must follow the existing trajectory with the right position. Therefore, we need a method to estimate the mobile robot trajectory in order to easily detect its position. In this paper, we propose two trajectory estimation methods, i.e., the Ensemble Kalman Filter (EnKF) and the Square Root Ensemble Kalman Filter (SR-EnKF). Furthermore, we also compare the performance of the two methods on the mobile robot equation. The simulation results showed that the EnKF method has a higher accuracy compared with the SR-EnKF method. The mobile position error of the two methods was less than 2% in the case of 100 and 200 ensembles. The smallest error was obtained when generating 100 ensembles, where the position error w.r.t. the X-axis was 0.02 m, the position error w.r.t. the Y-axis was 0.02 m, and the angle position error was 0.003 rad.

Keywords: *mobile robot; EnKF; SR-EnKF; trajectory estimation.*

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1 Introduction

Estimation is made to solve a problem which requires prior information such that the next step of the problem solving can be determined. Estimation is conducted since some problems can be addressed using preceding information or data associated with the problem [1]. In the literature, there are numerous methods that can be used for estimation. The Kalman filter is one of the methods for estimating state variables in a discrete linear dynamic system such that the estimation error covariance is minimized [2]. The Kalman filter was originally proposed by Rudolph E. Kalman in 1960 for the solution of the linear data-discrete filtering problem. However, in many real-world problems, the models are continuous nonlinear dynamic systems. Such systems cannot be estimated accurately by using the Kalman filter. In this case, we can employ some alternative approaches for nonlinear systems, for example, the Ensemble Kalman Filter (EnKF) and the Square Root Ensemble Kalman Filter (SR-EnKF). In the literature, there are many approaches for modeling, analysis, estimation and control design of linear and nonlinear systems, see [3–12].

The EnKF method uses a certain number of ensembles to represent the underlying probability distribution of state variables. The mean and covariance of the probability distribution are approximated by the mean and covariance of the generated ensembles [1]. The Square Root Ensemble Kalman Filter method (SR-EnKF) is the development of the EnKF method where there are some decomposition matrix operations in the correction stage. This method was developed to reduce the computational time and to improve the accuracy of the estimation results so that the need for fast and accurate navigation and guidance can be satisfied [13]. The development of the application of trajectory estimation techniques in the the field of robotics will be very beneficial to Indonesia because unmanned vehicles have been widely used for civil and military purposes such as missions of spying, surveillance and exploration of places considered dangerous to humans.

A mobile robot is one of the unmanned vehicles that can be driven and whose position can be tracked or detected when it has a Global Positioning System (GPS). Mobile robots are used to replace human functions in doing dangerous work because they have the advantage of being able to move freely. For that purpose, the mobile robot must follow the existing path with the right position. To do so, a method is required to estimate the mobile robot trajectory.

This paper is a study on the implementation of the EnKF and SR-EnKF methods in mobile robot motion equations applied to estimate the mobile robot path, then both methods are simulated by using Matlab software so that the error between estimated and actual trajectories could be obtained. The focus of this paper is the comparison of two position estimation methods: the EnKF and SR-EnKF for mobile robot motion. The paper provides an analysis of numerical study on the performance of both methods.

2 Mathematical Model of Mobile Robot

A mobile robot or car robot is a robot construction that has a wheel actuator to move the whole body of the robot so that the robot can change the position from one point to another. The mobile robot used in the study was a mobile robot operating on land and using the rear wheels to move and transfer position. In other words, the mobile robot system was driven by the rear wheels. Figure 1 shows the position and dimensions of the mobile robot.

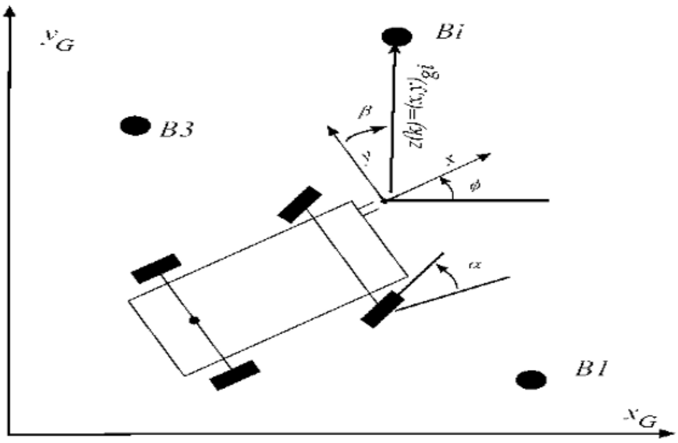

Figure 1: Dynamic model of the mobile robot.

The GPS is mounted right at the midpoint of the car. The steering and front corner systems are shown in Figure 1. In this case, the data are discrete, and the system is nonlinear. The dynamic system equation of a car robot is defined as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} v_c cos(\phi) - \frac{v_c}{L}(asin(\phi) + bcos(\phi))tan(\alpha) \\ v_c sin(\phi) + \frac{v_c}{L}(acos(\phi) - bsin(\phi))tan(\alpha) \\ \frac{v_c}{L}tan(\alpha) \end{bmatrix}, \tag{1}$$

where
x, y : position of the mobile robot in GPS coordinates;
ϕ : position angle of the mobile robot;
v_c : speed of the mobile robot;
α : steering angle of the mobile robot;
L : distance between the front wheel and the rear wheels;
a : distance between the midpoint of the rear car and the GPS position;
b : distance between the center of the car and the GPS position.

3 Square Root Ensemble Kalman Filter (SR-EnKF)

The Square Root Ensemble Kalman Filter algorithm (SR-EnKF) is the development of the EnKF algorithm. The correction stage of the SR-EnKF consists of a Singular Value Decomposition (SVD) and a square root matrix. The SVD is a matrix decomposition method which produces a diagonal matrix containing its singular values and another matrix that contains corresponding singular vectors [14]. The singular value decomposition has been widely used in many theoretical and practical applications. The Ensemble Kalman Filter and the Square Root Ensemble Kalman Filter (SR-EnKF) algorithms are summarized in Table 1.

EnKF	**EnKF-SR**
System Model and Measurement Model	
$x_{k+1} = f(u_k, x_k) + w_k, w_k \sim N(0, Q_k)$ $z_k = Hx_k + v_k, v_k \sim N(0, R_k)$	$x_{k+1} = f(u_k, x_k) + w_k, w_k \sim N(0, Q_k)$ $z_k = Hx_k + v_k, v_k \sim N(0, R_k)$
Initialization	
Generate N *ensemble* in accordance with initial estimate $\overline{x}_0$ $x_{0,i} = [x_{0,1}\ x_{0,2}\ x_{0,3}\ ...\ x_{0,Ne}]$ Determine initial value : $\widehat{x}_0 = \frac{1}{N_e}\sum_{i=1}^{N} X_{0,i}$	Generate N *ensemble* in accordance with initial estimate $\overline{x}_0$ $x_{0,i} = [x_{0,1}\ x_{0,2}\ x_{0,3}\ ...\ x_{0,Ne}]$ Initial Mean *Ensemble* : $\overline{x}_{0,i} = x_{0,i}1_N$ *Ensemble* initial error : $\widetilde{x}_{0,i} = x_{0,i} - \overline{x}_{0,i} = x_{0,i}(I - 1_N)$
Prediction Stage	
$\widehat{x}^-_{k,i} = f(\widehat{x}_{k-1,i}, u_{k-1,i}) + w_{k,i}$ with $w_{k,i} \sim N(0, Q_k)$ Estimate : $\widehat{x}^-_k = \frac{1}{N_e}\sum_{i=1}^{N} \widehat{x}^-_{k,i}$ Covariance error : $P^-_k = \frac{1}{N_e-1}\sum_{i=1}^{N}(\widehat{x}^-_{k,i} - \widehat{x}^-_k)(\widehat{x}^-_{k,i} - \widehat{x}^-_k)^T$	$\widehat{x}^-_{k,i} = f(\widehat{x}_{k-1,i}, u_{k-1,i}) + w_{k,i}$ of which $w_{k,i} \sim N(0, Q_k)$ *Ensemble Mean* : $\overline{x}^-_{k,i} = \widehat{x}^-_{k,i}1_N$ *Ensemble Error* : $\widetilde{x}^-_{k,i} = \widehat{x}^-_{k,i} - \overline{x}^-_{k,i} = \widehat{x}^-_{k,i}(I - 1_N)$
Correction Stage	
$z_{k,i} = z_k + v_{k,i}$ with $v_{k,i} \sim N(0, R_k)$ Kalman gain : $K_k = P^-_k H^T(HP^-_k H^T + R_k)^{-1}$ Estimate : $\widehat{x}_{k,i} = \widehat{x}^-_{k,i} + K_k(z_{k,i} - H\widehat{x}^-_{k,i})$ $\widehat{x}_k = \frac{1}{N_e}\sum_{i=1}^{N} \widehat{x}_{k,i}$	$z_{k,i} = z_k + v_{k,i}$ of which $v_{k,i} \sim N(0, R_k)$ $S_k = H\widetilde{x}^-_{k,i}, E_k = (v_1, v_2, ..., v_N)$, and $C_k = S_kS^T_k + E_kE^T_k$ *Ensemble Mean* : $\bar{x}_{k,i} = \bar{x}^-_{k,i} + \widetilde{x}^-_{k,i}S^T_kC^{-1}_k(\bar{z}_{k,i} - H\bar{x}^-_{k,i})$ Square root schema: - decompose eigenvalue of $C_k = U_k\Lambda_kU^T_k$ - compute matrices $M_k = \Lambda^{-1/2}_k U^T_k S^-_k$ - determine SVD from $M_k = Y_kL_kV^T_k$ *Ensemble Error* : $\widetilde{x}_{k,i} = \widetilde{x}^-_{k,i}V_k(I - L^T_kL_k)^{1/2}$ *Ensemble Estimate* : $\widehat{x}_{k,i} = \widetilde{x}_{k,i} + \bar{x}_{k,i}$

Table 1: EnKF and EnKF-SR algorithms [13].

4 Simulation and Analysis Results

In this study, the navigation system and mobile robot guidance used the EnKF and SR-EnKF methods by generating 100 and 200 ensembles on two paths. The comparison of the two methods is either by generating 100 or 200 ensembles. The starting point is given on each path $x(0) = 0$, $y(0) = 0$, and $z(0) = 0$. In the first trajectory, we obtained the result of path estimation in the XY field by using the EnKF and SR-EnKF and generating 200 ensembles as in Figure 4. In addition, Table 2 shows the average RMSE value by generating 100 and 200 ensembles.

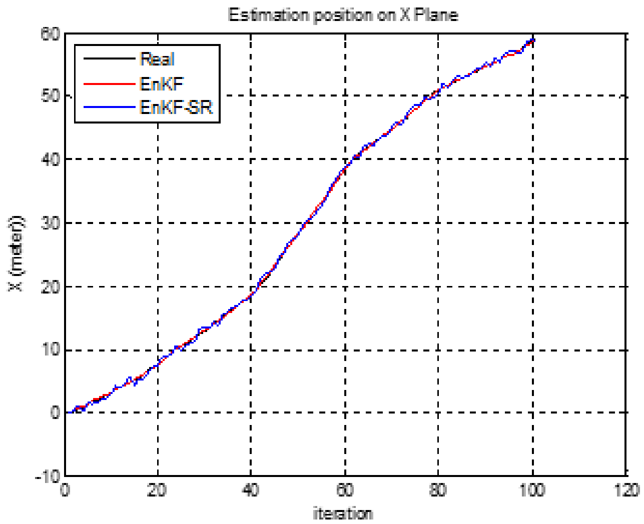

Figure 2: Estimation of position in the first trajectory on the X plane.

In Figure 2, it appears that the EnKF method is more accurate than the EnKF-SR method, where the EnKF method (red line) is smoother following the specified trajectory. Judging from the first iteration to the 60th iteration, it can be seen that the EnKF method is more accurate than the EnKF-SR method with a difference of 2-5% in accuracy.

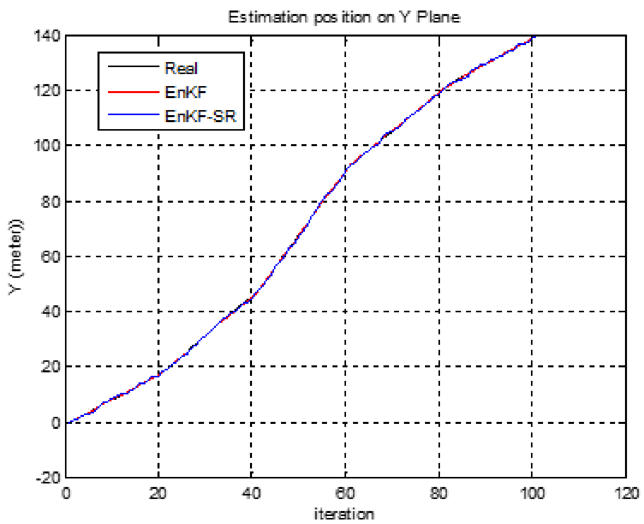

Figure 3: Estimation of position in the first trajectory on the Y plane.

In Figure 3, it appears that the EnKF method is more accurate than the EnKF-SR method, where the EnKF method (red line) is smoother following the specified trajectory. Judging from the first iteration to the 40th iteration, it can be seen that the EnKF method is more accurate than the EnKF-SR method with a difference of 2-3% in accuracy. However, after the 40th iteration, the EnKF and EnKF-SR methods have almost the same level of accuracy.

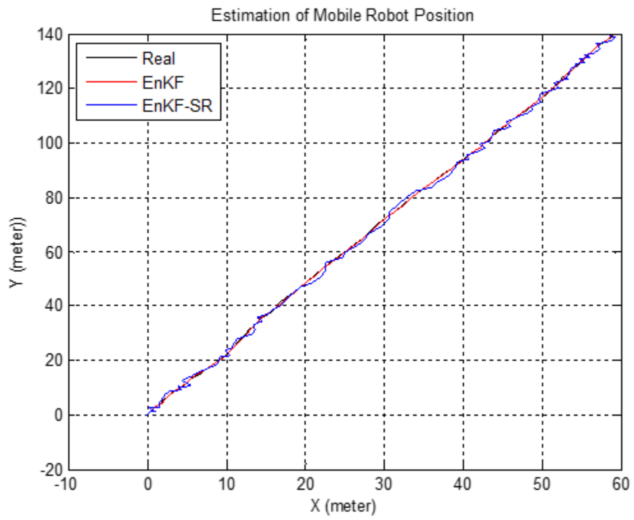

Figure 4: Estimation of the trajectory on the first trajectory in the XY plane.

Figure 4 shows that the system is able to follow the desired path in the XY plane, with the trajectory estimation results obtained by using the EnKF and SR-EnKF methods resulted in an accurate estimation with a position error of less than 2%. The error is obtained when the X position is 1.8 m and the Y position is 2 m. The errors obtained in the simulation when generating 100 and 200 ensembles are shown in Table 2.

In Table 2, notice that the EnKF method was more accurate than the SR-EnKF method in the case of 100 and 200 ensembles. The error of the X and Y positions indicated the deviation of the position as it moved along the path, while the angular position error was the error occurring during the turning movement, and this also affected the error of the X and Y position.

	N = 100		**N = 200**	
	EnKF	**SR-EnKF**	**EnKF**	**SR-EnKF**
X position	**0.085328 m**	0.45222 m	**0.15692 m**	0.70455 m
Y position	**0.084488 m**	0.51156 m	**0.096092 m**	0.72752 m
Angular position	**0.029393 m**	0.031061 m	0.072941 m	**0.072514 m**
Simulation time	1.7031 s	1.8281 s	3.6250 s	3.7813 s

Table 2: The comparison of RMSE values with the EnKF and SR-EnKF methods on the first trajectory in the case of 100 and 200 ensembles.

This simulation used $\Delta t = 0.1$ by generating 100 and 200 ensembles. The third trajectory was the result of the path estimation in the XY plane which was obtained by using the EnKF and SR-EnKF with the starting point given on each path $x(0) = 0$, $y(0) = 0$ and $z(0) = 0$ and generating 200 ensembles as shown in Figure 7. In addition, Table 3 shows the average value of RMSE by generating 100 and 200 ensembles.

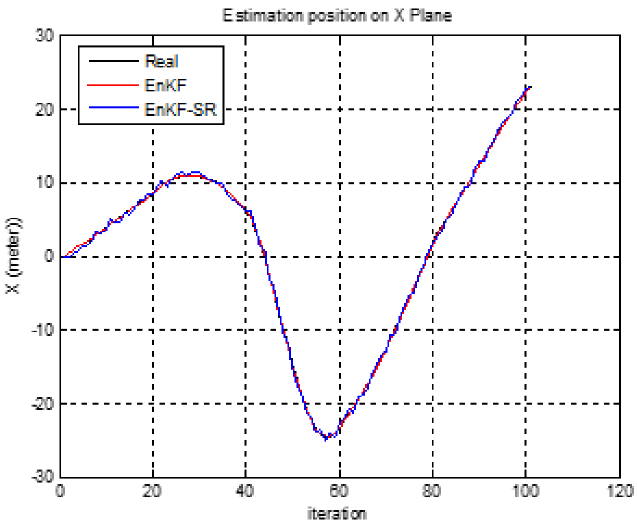

Figure 5: Estimation of position in the second trajectory on the X plane.

In Figure 5, it appears that the EnKF method is more accurate than the EnKF-SR method, where the EnKF method (red line) is smoother following what has been determined. Judging from the first iteration to the 40th iteration and the 40th to the 70th iteration, it can be seen that the EnKF method is more accurate than the EnKF-SR method with a difference of about 3-5% in accuracy.

In Figure 6, it appears that the EnKF method is more accurate than the EnKF-SR method, where the EnKF method (red line) is smoother following the specified trajectory. Judging from the first iteration to the 50th iteration, both methods have the same good accuracy, but from the 51st to the 100th iteration, it can be seen that the EnKF method is more accurate than the EnKF-SR method with a difference of about 2-4% accuracy rate.

Based on Figure 7, the mobile robot followed the desired path in the XY plane, where the trajectory estimations using the EnKF and SR-EnKF methods are very accurate with a position error of less than 2%. The error of 2% is obtained when the X position is 0.7 m and the Y position is 0.8 m.

From the analysis of results of the first, second and third trajectory simulation, it was found that the EnKF method has a higher accuracy compared with the SR-EnKF method either by generating 100 or 200 ensembles. However, the EnKF and SR-EnKF methods both had position errors of less than 2%, so the SR-EnKF method could be used as a method of navigation system and mobile robot guidance.

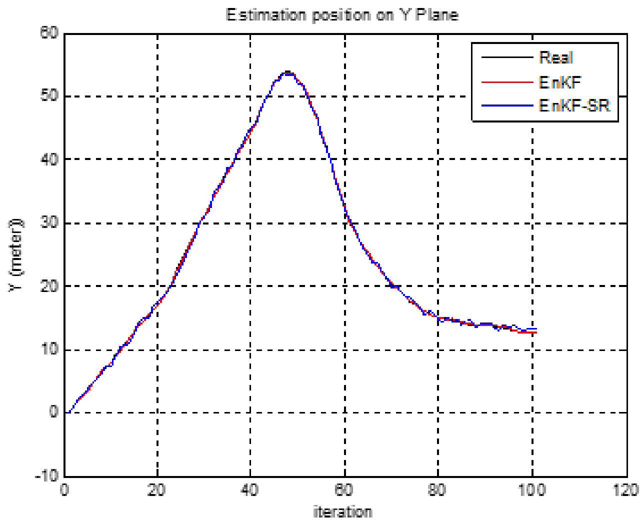

Figure 6: Estimation of position in the second trajectory on the Y plane.

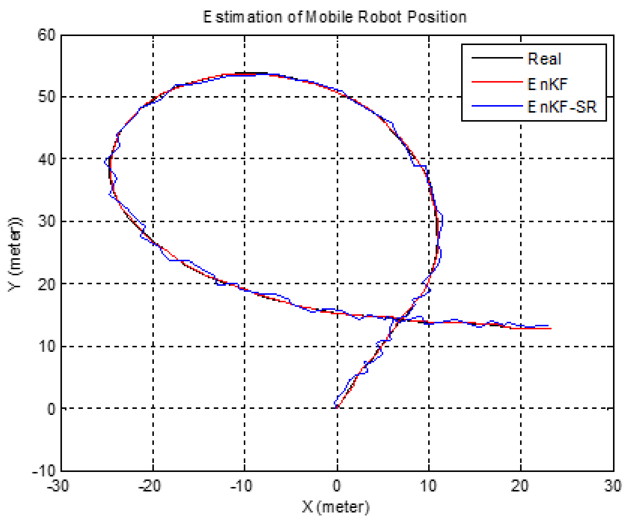

Figure 7: Estimation of trajectory on the third trajectory on the XY field.

5 Conclusions

According to the results of the study on mobile robot mathematical models as well as navigation systems, both the Ensemble Kalman Filter (EnKF) and the Square Root Ensemble Kalman Filter (SR-EnKF) methods could be effectively used as navigation systems and guidance with trajectory estimates with a position error of less than 2%.

	N = 100		**N = 200**	
	EnKF	**SR-EnKF**	**EnKF**	**SR-EnKF**
X position	**0.071713 m**	0.2168 m	**0.10462 m**	0.31981 m
Y position	**0.088223 m**	0.22584 m	**0.18555 m**	0.32489 m
Angular position	**0.0082395 m**	0.010071 m	**0.012895 m**	0.014751 m
Simulation time	1.6875 s	1.8281 s	3.0156 s	3.3281 s

Table 3: The comparison between the RMSE value of the EnKF method and that of the SR-EnKF method on the third trajectory in the case of 100 and 200 ensembles

Viewed from the generation of ensembles, the generation of 100 ensembles resulted in a more accurate result than that of 200 ensembles.

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