Nonlinear Dynamics and Systems Theory, **23** (1) (2023) 14–23

Harvesting Strategies in the Migratory Prey-Predator Model with a Crowley-Martin Type Response Function and Constant Efforts

Didiharyono, Syamsuddin Toaha *, Jeffry Kusuma and Kasbawati

Department of Mathematics, Hasanuddin University, Makassar, 90245, Indonesia

Received: October 10, 2022; Revised: December 21, 2022

Abstract: This paper deals with the dynamics of prey and predator populations in the permitted and prohibited areas of harvesting with a Crowley-Martin response function. The predator can migrate easily into both areas. The prey and predator populations in the permitted area are harvested with constant efforts. The existence and stability of the interior equilibrium point are studied. The stable interior equilibrium point is connected with maximum profit. The stability of the interior equilibrium point is analysed locally using the linearization method and eigenvalues. Due to the complexity, the simulation is carried out using the relevant parameter values to determine the existence of a stable interior equilibrium point and profit function. From simulation, there exists an ordered pair of harvesting efforts that gives a stable interior equilibrium point and also maximizes the profit function. Harvesting in prey and predator populations in the permitted area can prevent the populations from extinction and also provide maximum sustainable profit. The trajectories of prey and predator populations are plotted to visualize the dynamical behaviour for a given span of time. The surface of profit function is also plotted to view the maximum profit.

Keywords: *prey-predator; harvesting effort; migration; stability; Crowley-Martin; maximum profit.*

Mathematics Subject Classification (2010): 34Dxx, 70Kxx, 93-XX.

* Corresponding author: mailto:syamsuddint@gmail.com

1 Introduction

Mathematical modelling has been applied in various fields of studies including biology, ecology, epidemiology, economics, and many other fields, see [1], [2], [3], [4]. The model aims to explain the real phenomena from the mathematical aspect [5] and is also used to make prediction for the future, see [6]. A dynamical population is one of the research objects in the modelling. The growth rates of populations and their interaction are still a concern of researchers, including the dynamics of prey-predator populations living at the same area. The dynamics of populations were not only affected by their growth rate and interaction but also by some other factors like the death rate, competition, predation response, harvesting, and migration, see [7]. The dynamical population analysis not only studies and predicts sustainability but also considers social and economic aspects, see [8].

In the prey-predator model, one thing that is very important is the form of interaction between the prey and the predator, known as the predation function. Some of the predation functions often used in prey-predator models are of the Holling type, Holling-Tanner type, Mechaelis-Menten type, and Leslie-Gower type, see [5], [9]. The Beddington-DeAngelis type as another type of functional response is used as a control to stabilize the interaction of the prey and the predator, see [10]. The use of these types of functional response is dependent on the characteristic of the prey and the predator. The Crowley-Martin response function is influenced by the predator density, catch rate, handling time, and the magnitude of disturbance among predators, see [11]. The prey-predator model with the Crowley-Martin response function has been applied for many purposes, see [12].

The Crowley-Martin response function was also applied to predict the dynamics of a phytoplankton-zooplankton system [13]. There are prey-predator models which consider two identical areas and populations can migrate to these areas. Some of the population models are useful, for example, a fish population model in fisheries management, when the populations are harvested in various ways and policies. There is a policy in the fisheries management where the population in an area is prohibited from being harvested while in the other area it is permitted. Several policies in harvesting include selective harvesting, harvesting with constant quotas, harvesting with constant effort. Harvesting activities in population dynamics have economic consequences. The populations are not only managed to be sustainable but also strived to provide the maximum benefit. In some prey-predator models, only the prey populations are harvested or only the predator populations are harvested, see [14]. There are also other researchers who considered both prey and predator populations to be harvested, see [15].

In this paper, we consider a prey-predator model with the Crowley-Martin response function in an ecosystem which is divided into two areas, namely, an area where fishing is permitted and other area where fishing is prohibited. The prey population can migrate into both areas. The modeled populations are the population of butini fish (*Glossogobius matanensis*) as the prey and the population of nila fish (*Oreochromis nilotichus*) as the predator. The butini is an endemic and native fish found in several lakes of East Luwu district, South Sulawesi province, Indonesia, see [16]. In this model, the nila fish as the predator is divided into two compartments according to where the fish is located. Both populations are allowed to be harvested in the permitted area. The model formed is a system of nonlinear differential equations and the constant harvesting efforts are used for both populations. The local stability is analyzed using the linearization method. Maximum profit is evaluated over a certain range of effort values. The surface of the profit function is given to visualize the maximum profit.

2 Material and Method

This research involves a prey-predator population model following the Crowley-Martin response function. The populations considered in this study are butini fish and nila fish that live in several lakes in East Luwu district. Both sets of fish can only be harvested in the permitted area. The location of the two sets of fish is divided into the permitted and prohibited areas for fishing, where the nila fish population can migrate into the two areas. The populations are divided into three compartments, namely, butini fish, nila fish that live in the prohibited area to be harvested, and the nila fish that live in the permitted area to be harvested. The growth models of the three compartments are expressed in the form of an autonomous system of nonlinear differential equations.

The interior equilibrium point of the model is confirmed and then stability analysis is carried out using the linearization method and checking the eigenvalues of the Jacobian matrix resulting from the linearized model around the interior equilibrium point. The butini fish and nila fish are harvested in the permitted area with constant harvesting efforts. In order to get the profit function which is the consequence of fishing activity, the cost function and revenue function should be defined. The profit function (π) is given by $\pi = TR - TC$, based on the total revenue function (TR), $TR = p_1E_1B^* + p_2E_2M^*$ and the total cost function (TC), $TC = c_1E_1 + c_2E_2$. The parameter E_i represents the harvesting efforts, p_i represents the price of fish catch per unit, and c_i represents the cost of fishing activities, where $i = 1, 2$.

The prey-predator population model is a nonlinear system and the interior equilibrium point cannot be stated explicitly. In order to perform the analysis, the parameter values of the model were used being partially obtained from data collection for the fish populations. Some of the relevant parameter values are obtained from various references and some other are assumed. The various ordered pairs of the harvesting efforts are taken within a range of values to get the interior equilibrium points. Therefore, stability of the equilibrium points and profit value are determined. From the simulation, we determine the ordered pair of efforts that gives the stable interior equilibrium point and maximize the profit.

3 Results and Discussion

3.1 Predator-prey population model

The dynamics of the predator and prey population with the Crowley-Martin response function is expressed in the form of a system of nonlinear differential equations. The environment in which the population lives is divided into two areas, the permitted and prohibited areas for harvesting. The predator population is divided into two compartments, depending on where the predator live. The predator can migrate between the two areas. The prey population is assumed to follow logistic growth. The predator and prey populations are harvested in the permitted area with constant harvesting efforts. The interaction between the prey and predator populations is shown in the following Figure 1. The growth rates of the prey-predator population with their interaction are stated in the system of nonlinear differential equations.

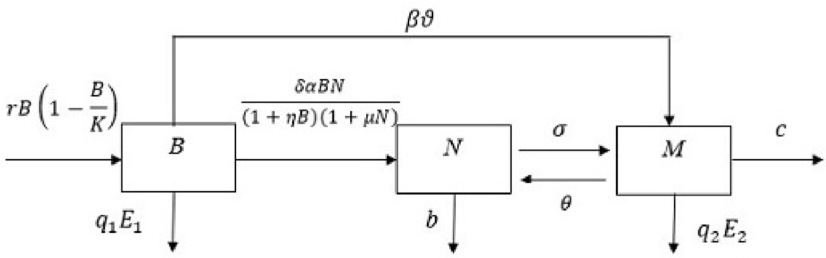

Figure 1: Interaction Diagram for the Prey and Predator Populations.

$$\frac{dB}{dt} = rB(1-\frac{B}{K}) - \frac{\alpha NB}{(1+\eta B)(1+\mu N)} - \beta MB - q_1E_1B, \tag{1}$$

$$\frac{dN}{dt} = \frac{\delta\alpha NB}{(1+\eta B)(1+\mu N)} - bN - \sigma N + \theta M, \tag{2}$$

$$\frac{dM}{dt} = \vartheta MB - cM + \sigma N - \theta M - q_2E_2M. \tag{3}$$

The symbol B is the size of the butini fish population as the prey, N and M state the size of the predator population in the permitted and prohibited area at time t, respectively. All parameters of the model are assumed to be positive. The description, meaning, and units of the parameters can be found in the related references, see [5]. For simplicity, we take $q_1 = q_2 = 1, r_1 = r - E_1, r_2 = b + \sigma$, and $r_3 = c + \theta + E_2$. Thus, the model (1, 2, 3) is rewritten as

$$\frac{dB}{dt} = r_1B(1-\frac{B}{K}) - \frac{\alpha NB}{(1+\eta B)(1+\mu N)} - \beta MB, \tag{4}$$

$$\frac{dN}{dt} = \frac{\delta\alpha NB}{(1+\eta B)(1+\mu N)} - r_2N + \theta M, \tag{5}$$

$$\frac{dM}{dt} = \vartheta MB - r_3M + \sigma N. \tag{6}$$

3.2 Equilibrium points and stability analysis

The possible non negative equilibrium points for the model (4, 5, 6) are $T_1 = (0,0,0)$, $T_2 = (K,0,0)$, $T_3 = (\omega, \frac{\omega\alpha_1+\alpha_2}{\mu\beta_1\alpha_2}, \frac{\sigma\omega\alpha_1-\alpha_2}{\mu\beta_1\beta_2\alpha_2})$, where ω are the roots of the equation $\delta\eta\mu r_1r_2\vartheta Z^5 + (-K\delta\eta\mu r_1r_2\vartheta^2 + \sigma\delta\eta\mu r_1\theta\vartheta - 2\delta\eta\mu r_1r_2r_3\vartheta + \delta\mu r_1r_2\vartheta^2)Z^4 + (-K\sigma\delta\eta\mu r_1\theta\vartheta + 2K\delta\eta\mu r_1r_2r_3\vartheta - K\alpha\beta\sigma\delta^2\vartheta + K\beta\sigma\delta\eta r_2 - K\delta\mu r_1r_2\vartheta^2 - \sigma\delta\eta\mu r_1r_3\vartheta + \delta\eta\mu r_1r_2r_3^2 - 2\delta\mu r_1r_2r_3\vartheta)Z^3 + (K\sigma\delta\eta\mu r_1r_3\theta - K\delta\eta\mu r_1r_2r_3^2 + K\alpha\beta\delta^2r_3 - K\beta\sigma^2\delta\eta\theta - K\beta\sigma\delta\eta r_1r_2 - K\delta\mu r_1r_2r_3^2 + \delta\mu r_1r_2r_3^2 - Kr_3^2\vartheta^2)Z^2 + (K\sigma\delta\mu r_1r_3\theta - K\delta\mu r_1r_2r_3^2 - K\alpha\sigma\delta r_3\theta + K\alpha\delta r_3^2r_2 + K\beta\sigma^2\delta\theta - K\beta\sigma\delta r_2r_3 - K\sigma^2\eta\theta^2 + 2K\sigma\eta\mu r_2r_3\theta - K\eta r_2^2r_3^2 - 2K\delta r_2\theta\vartheta + 2Kr_2^2r_3\vartheta)Z - K\delta^2\theta^2 + 2K\delta r_2r_3\theta - Kr_2^2r_3^2$, $\alpha_1 = \omega\eta r_2\vartheta - \omega\alpha\delta\vartheta - \alpha\delta r_3 + \sigma\eta\theta - \eta r_2r_3$, $\alpha_2 = \omega r_2\vartheta + \sigma\theta - r_2r_3$, $\beta_1 = \eta\omega + 1$, and $\beta_2 = \omega\vartheta - r_3$.

We focus to analyze the equilibrium point T_3 which is located in the first octant when $\omega > 0, \alpha_1 + \alpha_2 > 0$, and $\sigma\omega\alpha_1 > \alpha_2 > 0$. Because of the complexity of the system, we just consider the local stability of the interior equilibrium point.

3.3 Bionomic equilibrium and maximum profit

The bionomic equilibrium point is a condition where $\frac{dB}{dt} = \frac{dM}{dt} = \frac{dN}{dt} = 0$ and $\pi = 0$. The only interior equilibrium point satisfies the condition $T_3 = (\omega, \frac{\omega\alpha_1+\alpha_2}{\mu\beta_1\alpha_2}, \frac{\sigma\omega\alpha_1-\alpha_2}{\mu\beta_1\beta_2\alpha_2})$ which can be written in terms of E_1 and E_2 so that it becomes $T_3 = (B^*, N^*, M^*) = (\omega(r - E_1), \frac{\omega\alpha_1+\alpha_2(b+\sigma)}{\mu\beta_1\alpha_2}, \frac{\sigma\omega\alpha_1-\alpha_2(c+\theta+E_2)}{\mu\beta_1\beta_2\alpha_2})$. The total revenue function (TR) obtained from harvesting of the populations B and M evaluated at the equilibrium point T_3 is given by $TR(B^*, N^*, M^*) = TR(B^*)+TR(M^*) = p_1E_1B^*+p_2E_2M^*$. After substituting the values of B^* and M^* in the state of equilibrium, we get

$$TR = p_1\omega rE_1 - p_1\omega E_1^2 + \frac{p_2(\sigma\omega\alpha_1-\alpha_2)(c+\theta)E_2}{\mu\beta_1\beta_2\alpha_2} + \frac{p_2(\sigma\omega\alpha_1-\alpha_2)E_2^2}{\mu\beta_1\beta_2\alpha_2}.$$

The total cost function (TC) can be expressed as $C = c_1E_1 + c_2E_2$. Furthermore, the profit function (π) is given as

$$\pi = (p_1\omega r-c_1)E_1-p_1\omega E_1^2+\frac{(p_2(\sigma\omega\alpha_1 - \alpha_2)(c + \theta) - \mu\beta_1\beta_2\alpha_2c_3)E_2}{\mu\beta_1\beta_2\alpha_2}+\frac{p_2(\sigma\omega\alpha_1 - \alpha_2)E_2^2}{\mu\beta_1\beta_2\alpha_2}. \tag{7}$$

The profit function (7) now depends on the efforts E_1, E_2, and the parameter ω which is a positive root of the polynomial of degree five and cannot be written explicitly. The value of ω also depends on the efforts E_1 and E_2. As a standard procedure to get the maximum value of profit, we need to get the stationary points via the first partial derivative with respect to E_1 and E_2. Since ω cannot be stated in terms of E_1 and E_2, we evaluate the value of profit by taking various values of ordered pairs (E_1, E_2) and determine the interior equilibrium point T_3 and its stability by showing the eigenvalues. The eigenvalues are related to the Jacobian matrix evaluated at the equilibrium point T_3. Furthermore, the profit function at each value of ordered pairs (E_1, E_2) can be determined. In this study, we restrict the value of efforts as $0 \leq E_1, E_2 \leq E_{max}$, and $E_{max} = 1$. The ordered pair (E_1, E_2) to be considered is the ordered pair that gives the interior equilibrium point T_3 and is stable.

3.4 Simulation

In order to simulate the profil function, we set the values of parameters for the model equations (4), (5, (6) as follows: $K = 100, r = 0.7, \alpha = 0.3, \eta = 0.01, \mu = 0.01, \beta = 0.1, \vartheta = 0.01, \delta = 0.03, \sigma = 0.25, \theta = 0.25, b = 0.2$, and $c = 0.1$, see [5,17]. The values of these parameters are partly based on data collection. The interior equilibrium point T_3 with various ordered pairs of (E_1, E_2) are given in Table 1.

The various ordered pairs of the efforts $(E1, E2)$ give the interior equilibrium point T_3 and the stability is determined by inspection of the real part of eigenvalues. The equilibrium point is asymptoticaly stable when the real parts of eigenvalues are negative. The ordered pairs of efforts and eigenvalues of the interior equilibrium point T_3 are given in Table 2.

In order to simulate and determine the profit, we set the values of parameters related to the total revenue and total cost, namely, $p_1 = 3.5, p_2 = 1.3, c_1 = 0.5$ and $c_2 = 0.3$ in appropriate units. Together with the various values of ordered pairs of efforts, we determine the profit evaluated at the equilibrium point T_3 following the formula $\pi(E_1, E_2) = p_1B^*E_1 + p_2M^*E_2 - (c_1E_1 + c_2E_2)$. The ordered pairs of efforts and profit are given in Table 3.

Table 1: Ordered Pairs of Efforts and Interior Equilibrium Point T_3.

$E1/E2$	0	0.1	0.2	0.3
0	15.94, 1.52, 1.99	23.01, 1.52, 1.73	29.69, 1.51, 1.48	35.97, 1.47, 1.27
0.1	15.93, 1.26, 1.65	22.98, 1.23, 1.40	29.65, 1.19, 1.18	35.91, 1.14, 0.98
0.2	15.92, 1.00, 1.31	22.95, 0.95, 1.08	29.61, 0.89, 0.87	35.85, 0.81, 0.70
0.23	15.91, 0.92, 1.21	22.95, 0.87, 0.98	29.60, 0.80, 0.78	35.83, 0.72, 0.61
0.235	15.91, 0.91, 1.19	22.94, 0.85, 0.97	29.59, 0.78, 0.77	35.82, 0.70, 0.60
0.3	15.91, 0.74, 0.97	22.93, 0.67, 0.76	29.57, 0.58, 0.57	35.78, 0.48, 0.42
0.4	15.89, 0.48, 0.63	22.91, 0.39, 0.44	29.53, 0.28, 0.27	35.72, 0.16, 0.14
0.5	15.88, 0.22, 0.29	22.88, 0.11, 0.12	-	-

$E1/E2$	0.4	0.5	0.6	0.7
0	41.81, 1.43, 1.08	47.21, 1.38, 0.91	52.11, 1.32, 0.77	56.54, 1.26, 0.65
0.1	41.72, 1.08, 0.81	47.07, 1.01, 0.66	51.94, 0.93, 0.54	56.33, 0.85, 0.43
0.2	41.63, 0.73, 0.54	46.95, 0.63, 0.41	51.78, 0.54, 0.31	56.12, 0.44, 0.23
0.23	41.61, 0.62, 0.46	46.91, 0.52, 0.34	51.73, 0.42, 0.24	56.06, 0.32, 0.16
0.235	41.60, 0.61, 0.45	46.90, 0.51, 0.33	51.72, 0.40, 0.23	56.05, 0.29, 0.15
0.3	41.54, 0.38, 0.28	46.83, 0.26, 0.17	51.61, 0.15, 0.08	55.92, 0.03, 0.01
0.4	41.45, 0.03, 0.02	-	-	-

$E1/E2$	0.8	0.9	1
0	60.51, 1.19, 0.55	64.05, 1.13, 0.46	67.18, 1.07, 0.39
0.1	60.25, 0.76, 0.35	63.74, 0.69, 0.28	66.82, 0.61, 0.22
0.2	60.01, 0.34, 0.15	63.44, 0.24, 0.10	66.47, 0.16, 0.05
0.23	59.92, 0.21, 0.09	63.34, 0.12, 0.04	66.36, 0.02, 0.009
0.235	59.91, 0.19, 0.08	63.33, 0.09, 0.03	66.34, 0.002, 0.0009

Table 2: Ordered Pairs of Efforts and Eigenvalues of Interior Equilibrium Point T_3.

E_1/E_2	0	0.1	0.2	0.3
0	-0.02 ± 0.28I, -0.53	-0.03 ± 0.31I, -0.52	-0.05 ± 0.32I, -0.52	-0.07 ± 0.31I, -0.52
0.1	-0.02 ± 0.25I, -0.53	-0.04 ± 0.27I, -0.52	-0.06 ± 0.28I, -0.52	-0.08 ± 0.27I, -0.52
0.2	-0.03 ± 0.22I, -0.53	-0.05 ± 0.24I, -0.51	-0.07 ± 0.24I, -0.51	-0.09 ± 0.22I, -0.52
0.23	-0.03 ± 0.21I, -0.51	-0.05 ± 0.23I, -0.51	-0.07 ± 0.22I, -0.51	-0.09 ± 0.21I, -0.52
0.235	-0.03±0.21I, -0.53	-0.05±0.22I, -0.51	-0.07±0.22I, -0.51	-0.09±0.21I, -0.52
0.3	-0.03±0.19I, -0.52	-0.05± 0.21I, -0.52	-0.08± 0.18I, -0.51	-0.08±0.18I, -0.51
0.4	-0.04±0.15I, -0.52	-0.06±0.14I, -0.51	-0.09±0.11I, -0.51	-0.15, -0.08, -0.51
0.5	-0.05±0.09I, -0.52	-0.07±0.04I, -0.51	-	-

Table 3 shows that maximum profit is reached when the efforts $(E_1, E_2) = (0.235, 1)$ with $\pi_{max} = 217.05$. The profit becomes maximum when the predator population in the permitted area is harvested at the maximum level of efforts and the prey population is harvested at the level 0.235. The maximum profit occurs at the top of the surface of the profit function, as shown in Figure 2.

For the model without harvesting, we get an interior equilibrium point at the level

E_1/E_2	0.4	0.5	0.6	0.7
0	-0.09±0.31I, -0.54	-0.11±0.29I, -0.56	-0.13±0.27I, -0.58	-0.15±0.24I, -0.61
0.1	-0.11±0.26I, -0.53	-0.12±0.23I, -0.55	-0.14±0.21I, -0.58	-0.16±0.17I, -0.61
0.2	-0.07±0.24I, -0.51	-0.14±0.16I, -0.55	-0.16±0.11I, -0.58	-0.23, -0.12, -0.61
0.23	-0.12±0.17I, -0.53	-0.14±0.13I, -0.55	-0.01±0.05I, -0.58	-0.29, -0.07, -0.61
0.235	-0.12±0.17I, -0.53	-0.14±0.12I, -0.55	-0.16±0.03I, -0.58	-0.31, -0.06, -0.61
0.3	-0.13±0.11I, -0.52	-0.21, -0.09, -0.54	-0.31, -0.03, -0.57	-0.38, -0.006, -0.6
0.4	-0.27, -0.009, -0.52	-	-	-

E_1/E_2	0.8	0.9	1
0	-0.17±0.21I, -0.65,	-0.19±0.18I, -0.71,	-0.21±0.14I, -0.75
0.1	-0.18±0.12I, -0.65,	-0.20±0.04I, -0.70,	-0.32, -0.11, -0.76
0.2	-0.33, -0.06, -0.66	-0.39, -0.04, -0.71	-0.43, -0.02, -0.77
0.23	-0.36, -0.04, -0.66	-0.42, -0.02, -0.71	-0.45, -0.003, -0.77
0.235	-0.37, -0.03, -0.66	-0.42, -0.01, -0.72	-0.46, -0.0003, -0.77

Table 3: Ordered Pairs of Efforts and Profit.

$E1/E2$	0	0.2	0.4	0.6	0.8	1
0	0	54.62	103.91	142.36	171.36	193.88
0.1	3.29	60.67	112.26	152.66	183.20	206.97
0.2	5.472	64.64	117.70	159.27	190.74	215.27
0.23	5.907	65.43	118.79	160.54	191.96	216.84
0.235	5.971	65.55	118.94	160.72	192.01	217.05
0.3	6.532	66.53	120.21	162.23	-	-
0.4	6.476	66.362	119.82	-	-	-
0.5	5.308	-	-	-	-	-

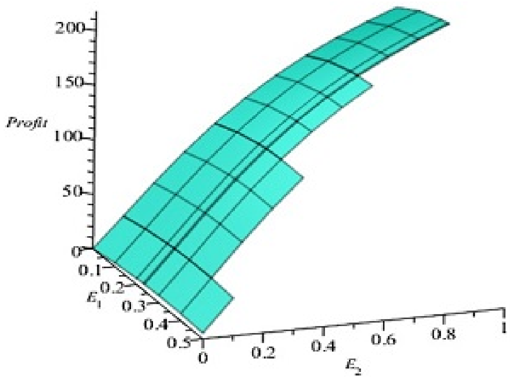

Figure 2: Surface of the Profit Function.

(15.94, 1.52, 1.99) and the related eigenvalues -0.02±0.28I, -0.53. This means that the prey (B) and the predators (M and N) will live sustainably. From Tables 1, 2 and 3, as the efforts of harvesting increase, the equilibrium points are still in the first octant and also remain stable but there are changes in the type of stability of the equilibrium point which are indicated by all the eigenvalues having negative real values. In addition, the value of the profit function also increases. When the prey and the predator are harvested at the level $(E_1, E_2) = (0.23, 1)$, the ordered pair of efforts gives an interior equilibrium point at the level (66.36, 0.02, 0.009), the eigenvalues -0.46, -0.0003, -0.77, and the profit at the level 216.84. The dynamics of the solution curve of the prey (B) and the predators (M and N) with the initial population $B(0) = 66.36, N(0) = 0.02$, and $M(0) = 0.009$ are shown in Figures 3, 4, 5.

Figures 3, 4, 5 show that with a given initial value of the prey and the predator

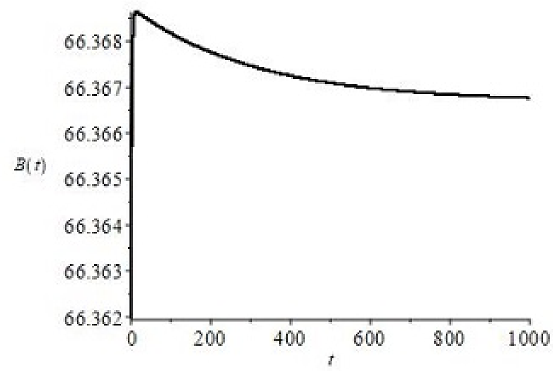

Figure 3: Solution Curve of Prey Population (B) with $t \in [0, 1000]$.

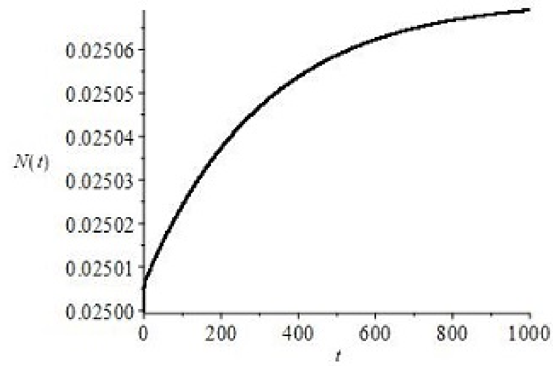

Figure 4: Solution Curve of Predator Population (N) with $t \in [0, 1000]$.

populations, there is initially little oscillatory motion. This is caused by the nonlinear term in the model and then the trajectories of the populations move monotonously toward the equilibrium point. It takes a long time to reach the equilibrium state. The ordered pair of efforts provides a stable interior equilibrium point and also almost maximizes the profit, the maximum profit is at the level 217.05.

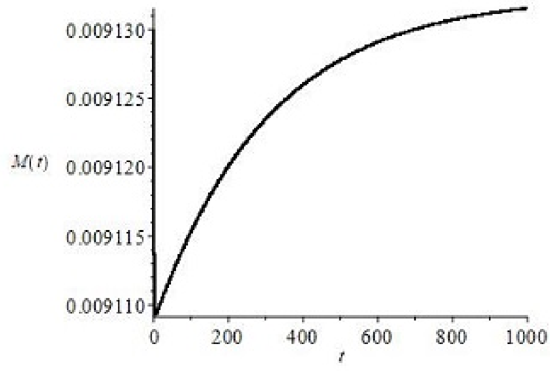

Figure 5: Solution Curve of Predator Population (M) with $t \in [0, 1000]$.

4 Conclusion

The model for growth of butini fish as the prey and nila fish as the predator in the permitted and prohibited areas for harvesting with the Crowley-Martin type response function and migration possibly has an interior equilibrium point. The prey and predator populations in the permitted area are harvested with constant efforts. The interior equilibrium point cannot be stated explicitly because of complexity of the nonlinear model. In order to get the maximum value of the profit function, several ordered pairs of harvesting efforts are evaluated to obtain a stable interior equilibrium point. Using the suitable parameter values and harvesting efforts, we get an ordered pairs of efforts that give a stable interior equilibrium point and maximize the profit.

The analysis and simulation show that if the level of harvesting effort for the prey population is increased, the equilibrium state for the prey and predator populations will decrease. This is because of the more prey populations are harvested, the lower number of prey populations exists. This has a consequence for predators having difficulty to get food, which results in the number of predator also decreasing. On the other hand, if the level of harvesting effort in the predator population is increased, this condition will result in a decrease in the number of predator population both in the permitted and prohibited areas for harvesting. This causes the prey population tending to increase because the number of predator population decreases. Harvesting with constant efforts for the prey and predator populations in the permitted area can obviously increase the number of the prey population, give maximum profit, and the populations also remain sustainable.

Acknowledgment

This research is funded by Institute for Research and Community Services Hasanuddin University with contract number: 1474/UN4.22/PT.01.03/2022.

References

[1] M. Z. Ndii and A. K. Supriatna. Stochastic Dengue Mathematical Model in the Presence of Wolbachia: Exploring the Disease Extinction. *Nonlinear Dynamics and System Theory* **20** (2) (2020) 214–227.

[2] A. Zaghdani. Mathematical Study of a Modified SEIR Model for the Novel SARS-Cov-2 Coronavirus. *Nonlinear Dynamics and System Theory* **21** (3) (2021) 326-336.

[3] Firman, S. Toaha and Kasbawati. Modification of the Trajectory Following Method for Asymptotic Stability in a System Nonlinear Control. *Nonlinear Dynamics and System Theory* **22** (2) (2022) 169–177.

[4] T. Herlambang, A. Y. P. Asih, D. Rahmalia, D. Adzkiya and N. Aini. The Effects of Pesticide as Optimal Control of Agriculture Pest Growth Dynamical Model. *Nonlinear Dynamics and System Theory* **22** (3) (2022) 281–290.

[5] B. Dubey, S. Agarwal and A. Kumar. Optimal Harvesting Policy of a Prey–Predator Model with Crowley–Martin Type Functional Response and Stage Structure in the Predator. *Nonlinear Analysis Modelling Control* **23** (4) (2018) 493–514.

[6] S. Toaha and M. I. Azis. Stability and Optimal Harvesting of Modified Leslie-Gower Predator-Prey Model. *IOP Conferences Series: Journal of Physics* (2018) 1-8.

[7] R. Sivasamy, K. Sathiyanathan and K. Balachandran. Dynamics of a Modified Leslie-Gower Model with Crowley-Martin Functional Response and Prey Harvesting. *Journal Applied Nonlinear Dynamics* **8** (4) (2019) 621–636.

[8] E. Y. Frisman, O. L. Zhdanova, M. P. Kulakov, G. P. Neverova and O. L. Revutskaya. Mathematical Modeling of Population Dynamics Based on Recurrent Equations: Results and Prospects. *Biology Bulletin* **48** (1) (2021) 1–15.

[9] P. K. Santra, G. S. Mahapatra and G. R. Phaijoo. Bifurcation and Chaos of a Discrete Predator-Prey Model with Crowley–Martin Functional Response Incorporating Proportional Prey Refuge. *Mathematical Problems in Engineering* **202** (1) (2020) 5309814.

[10] S. Chen, J. Wei and J.Yu. Stationary Patterns of a Diffusive Predator-Prey Model with Crowley-Martin Functional Response. *Nonlinear Analysis: Real World Applications* **39** (1) (2018) 33–57.

[11] S. Hossain, M. M. Haque, M. H. Kabir, M. O. Gani and S. Sarwardi. Complex Spatiotemporal Dynamics of a Harvested Prey–Predator Model with Crowley–Martin Response Function. *Results in Control and Optimization* **5** (1) (2021) 1–17.

[12] D. Didiharyono, S. Toaha, J. Kusuma and Kasbawati. Global Stability of Prey-Predator Model with Crowley-Martin Type Functional Response and Stage Structure for Predator. In: *Proceedings of the International Conference on Industrial Engineering and Operations Management Harbin* (2021) 714–724.

[13] T. Liao, H. Yu and M. Zhao. Dynamics of a Delayed Phytoplankton-Zooplankton System with Crowley-Martin Functional Response. *Advances in Difference Equations* **17** (5) (2017) 1–30.

[14] K. Chakraborty, S. Jana and T. K. Kar. Global Dynamics and Bifurcation in a Stage Structured Prey-Predator Fishery Model with Harvesting. *Applied Mathematics and Computation* **218** (5) (2012) 9271–9290.

[15] S. Toaha, J. Kusuma, Khaeruddin and Mawardi. Stability Analysis and Optimal Harvesting Policy of Prey-Predator Model with Stage Structure for Predator. *Applied Mathematical Sciences* **8** (159) (2014) 7923–7934.

[16] S. H. Nasution and R. Dina. Population Structure and Gonadal Maturity Stage of Endemic and Alien Fish Dominant Species in Lake Matano, South Sulawesi. *IOP Conference Series: Earth and Environmental Science* (2019) 1–14.

[17] S. Toaha, Firman and A. Ribal. Global Stability and Optimal Harvesting of Predator-Prey Model with Holling Response Function of Type II and Harvesting in Free Area of Capture. *Nonlinear Dynamics and Systems Theory* **22** (1) (2022) 105–116.