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Estimation of Forefinger Motion with Multi-DOF Using Advanced Kalman Filter

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Abstract: Data from the World Health Organization (WHO) of the year 2016 recorded that stroke cases were ranked second as a non-communicable disease that causes death, and the third leading cause of disability worldwide. Stroke can cause disability or weakness on one side of the body, including the upper limbs such as the fingers being difficult to move, so rehabilitation is required to restore the function of the hand. A finger arm robot is one solution to help accelerate the rehabilitation process specifically for finger movements. One of the efforts to develop a finger robot is finger motion estimation. It is started with the inverse kinematic modeling of the finger arm robot with 3 joints matching the structure of a human finger. One reliable estimation method frequently used is the Advanced Kalman Filter method. In this paper, the Advanced Kalman Filter is divided into two methods, that is, the Ensemble Kalman Filter (EnKF) and the Ensemble Kalman Filter Square Root (EnKF-SR). The focus of this paper is to estimate the fingers, especially the index finger of the left hand, using the EnKF and Square Root EnKF (SR-EnKF) methods. And, the simulation results show that both methods reached an accuracy of 99% when 400 ensembles were generated on a semicircular path by the EnKf-SR with lower error.

Keywords: *finger arm robot; EnKF; SR-EnKF; finger motion estimation.*

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1 Introduction

Data from the World Health Organization (WHO) of 2016 recorded that stroke cases were ranked second as a non-communicable disease that causes death and ranked third as a leading cause of disability worldwide. A stroke can cause weakness in one part or side of the body (hemiparesis), due to which a stroke sufferer finds it difficult to move and use parts of the respective side of the body [1]. A stroke can cause disability or weakness on one side of the body, including the upper limbs such as the fingers that are difficult to move, so rehabilitation is needed to restore the function of the hand. Hands and fingers are the most important and complex body parts that humans have. Their muscles can carry out any movement as the human brain commands, without having to control it one by one.

Robotics technology is currently developing rapidly along with advances in science and technology to assist medical rehabilitation, one of which is post-stroke rehabilitation, especially the rehabilitation of finger movements [2]. This is also due to the human desire to help each other in accelerating the recovery of post-stroke patients. The manufacture of technology in the form of robots can be inspired by the phenomena of living things, among others, by referring to the basic principles of movement of the human body. For instance, the way humans walk, talk, hold objects and others. The aim of medical rehabilitation is to maximize functional independence and ability of a patient to continue his or her pre-illness way of living or roles and to improve quality of his or her life.

A finger is one part of the human body, having an important role in human body movement to do various activities [2]. A human has a total of ten fingers functioning to hold objects. The working principles of the human finger are then used as the basis of developing a finger robot designed to hold objects.

A finger robot is one solution to help accelerate the rehabilitation process specifically for finger movements. One of the efforts to develop the finger robot is finger motion estimation [3]. One reliable estimation method frequently used is the Advanced Kalman Filter method. In this paper, the Advanced Kalman Filter method is divided into two methods, that is, the Ensemble Kalman Filter (EnKF) and the Ensemble Kalman Filter Square Root (EnKF-SR) ones. These EnKF and EnkF-SR methods are very reliable for both linear and nonlinear models [4], [5], [6]. The EnKF method was frequently used for estimating the motion and position of AUV [7], ASV [8]- [9] and missiles [10]. And in this paper, the finger estimation is carried out, particularly for the index finger of the left hand by using the EnKF and Square Root EnKF (SR-EnKF) methods on a semi-circular path, and the simulation results produce comparison of the accuracy of one motion estimation method and that of the other [11].

2 Inverse Kinematic Modeling of Finger Motion with 3 Joints

The following is a modeling analysis of the 3-joint finger arm robot [12]. Figure 1 shows the 3-joint arm robot using x and y coordinates in its working area. Just like the 2-joint arm robot, the 3-joint arm robot uses forward kinematics as an equation analysis [5].

The angle Ψ is the angle of the direction of the third part toward the X-axis, as in equation

$$\Psi = (\theta_1 + \theta_2 + \theta_3). \tag{1}$$

Figure 2 shows that the equations for the projection of link 1, link 2 and link 3 toward the x- and y-axis can be obtained by the analysis and combination of equations to locate

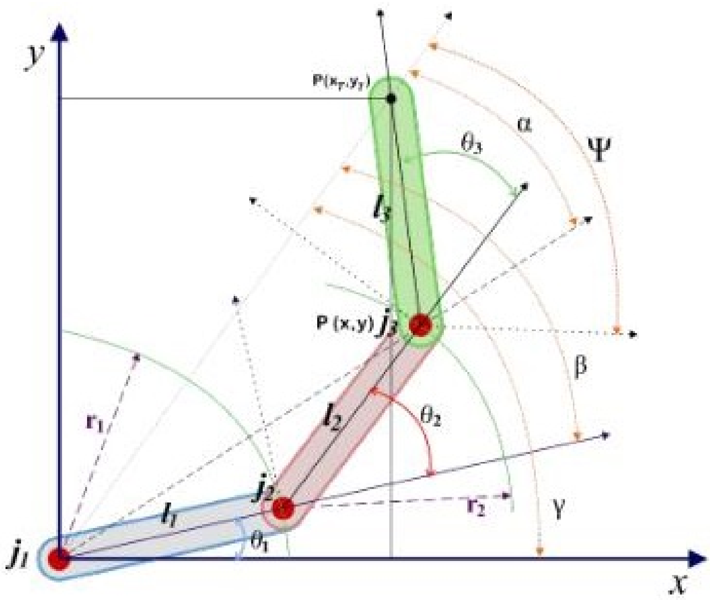

Figure 1: Configuration of a Finger Arm Robot with 3 Joints.

the coordinate points of X_T and Y_T as follows:

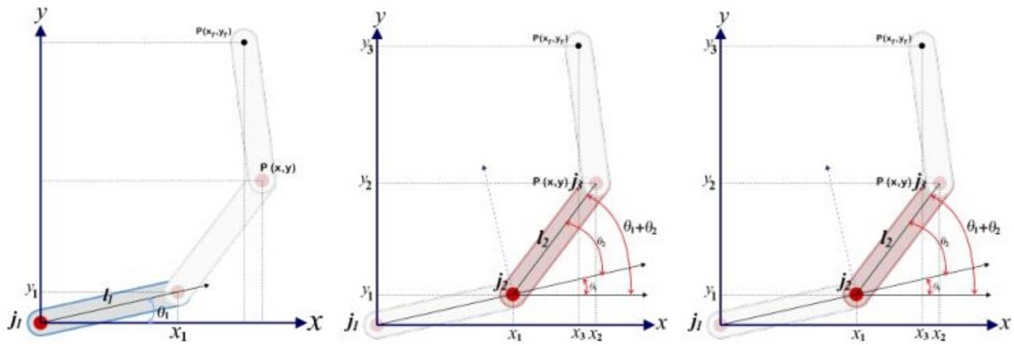

Figure 2: The First part, Second part and Third part of a Finger Arm Robot with 3 DOF.

$$
\begin{aligned}
x &= x_1 + x_2 + x_3, \\
x &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3),
\end{aligned} \tag{2}
$$

and

$$
\begin{aligned}
y &= y_1 + y_2 + y_3, \\
y &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3),
\end{aligned} \tag{3}
$$

so that

$$\begin{aligned} x_T &= l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3), \\ y_T &= l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3). \end{aligned} \tag{4}$$

Simplify by using the trigonometric identity formula [5]:

$$\begin{aligned} x_T &= x - l_3 \cos(\theta_1+\theta_2+\theta_3), \\ y_T &= y - l_3 \sin(\theta_1+\theta_2+\theta_3). \end{aligned} \tag{5}$$

The formula for the forward kinematic equation of 3 joint is

$$\begin{aligned} x &= l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + l_3 \cos(\theta_1+\theta_2+\theta_3), \\ y &= l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + l_3 \sin(\theta_1+\theta_2+\theta_3). \end{aligned} \tag{6}$$

For the inverse kinematics, if the coordinates of $P(x_T, y_T)$ and $P(x, y)$ are known, then θ_1 and θ_2 can be obtained by using the same equation as that applied to the two joint arm robot:

$$\begin{aligned} \theta_2 &= \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right), \\ \theta_1 &= \tan^{-1}\left(\frac{y(l_1+l_2\cos\theta_2) - xl_2\sin\theta_2}{x(l_1+l_2\cos\theta_2)+yl_2\sin\theta_2}\right). \end{aligned} \tag{7}$$

The angle $\Psi = (\theta_1+\theta_2+\theta_3)$ can be obtained by using $P(x_T, y_T)$ and $P(x, y)$ inserted into equations (5) and (6) so that θ_3 can be found.

By substituting $P(x_T, y_T)$ and $P(x, y)$ into equation (5), we get

$$l_3 \cos\Psi = 0, \tag{8}$$

whereas, by substituting $P(x_T, y_T)$ and $P(x, y)$ into equation (6), it becomes

$$\begin{aligned} y_T &= x - l_3 \sin\Psi, \\ l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_3\sin\Psi &= l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cos\Psi - l_3\sin\Psi, \\ 2l_3\sin\Psi - l_3\cos\Psi &= l_1(\cos\theta_1 - \sin\theta_1) + l_2(\cos(\theta_1+\theta_2) - \\ &\quad \sin(\theta_1+\theta_2)). \end{aligned} \tag{9}$$

Since in equation (8), $l_3 \cos\Psi = 0$, we obtain

$$\begin{aligned} \Psi &= \theta_1+\theta_2+\theta_3 \\ &= \sin^{-1}\left(\frac{l_1(\cos\theta_1-\sin\theta_1) + l_2(\cos(\theta_1+\theta_2) - \sin(\theta_1+\theta_2))}{2l_3}\right). \end{aligned} \tag{10}$$

Below is the picture of a finger arm robot and its finger parts.

3 Ensemble Kalman Filter and EnKF Square Root Algorithm

The Ensemble Kalman Filter and the Square Root Ensemble Kalman Filter (SR-EnKF) algorithms are summarized in Table 1.

Figure 3: The Arm Robot Image with a Focus on the Finger Arm Robot.

4 Simulation Results

This study started with the inverse kinematic modeling of a finger arm robot with 3 joints that matches the structure of the human fingers. In this section, two choices of the number of ensembles, that is, 300 ensembles and 400 ensembles were tested on a semi-circle track. Such a choice of track was because in a semi-circle, all the joints of a finger can move optimally.

In this trajectory, the diameter used is about 7.5 cm. This is due to the fact that the finger length of most people in Indonesia ranges from 7.5 to 8.2 cm. So, with a semi-circular movement having a diameter of about 7.5 cm, physical exercises for the index finger can be carried out thoroughly. And, the simulation results can be seen in Figures 4-7.

Figure 4 shows the simulation results by the EnKF and EnKF-SR methods using 300 ensembles and a time of 400 seconds. Figure 4 a) shows the forefinger movement, moving up to 8 cm of the X-axis, by both estimation methods with a small error of 0.1 for the EnKF method and 0.09 for the EnkF-SR method. Figure 4 b) shows the forefinger movement on the Y-axis is of only 2.5 cm, and the EnKF and EnKF-SR methods have sufficient accuracy.

Figure 5 shows the results of the simulation by the EnKF and EnKF-SR methods, producing a movement resembling a semi-circle with a diameter of $\sqrt{(8^2 + 2.5^2)} = \sqrt{70.25} =$ 8.3 cm, so overall if in terms of the diameter of about 7.5 cm, when using 300 ensembles, it has an error of about 10%.

Figure 6 shows the simulation results by the EnKF and EnKF-SR methods using 400 ensembles and a time of 400 seconds. Figure 6a) shows the forefinger movement, moving up to 6.8 cm of the X-axis, and by both estimation methods, having a small error of 0.09% for the EnKF method and 0.08% for the EnkF-SR method. Figure 6b) shows that the forefinger movement on the Y-axis is of only 3 cm, and the EnKF and EnKF-SR

EnKF	**EnKF-SR**
System Model and Measurement Model	
$x_{k+1} = f(u_k, x_k) + w_k, w_k \sim N(0, Q_k)$ $z_k = Hx_k + v_k, v_k \sim N(0, R_k)$	$x_{k+1} = f(u_k, x_k) + w_k, w_k \sim N(0, Q_k)$ $z_k = Hx_k + v_k, v_k \sim N(0, R_k)$
Initialization	
Generate N *ensemble* in accordance with initial estimate $\overline{x}_0$ $x_{0,i} = [x_{0,1}\ x_{0,2}\ x_{0,3}\ ...\ x_{0,Ne}]$ Determine initial value : $\widehat{x}_0 = \frac{1}{N_e}\sum\limits_{i=1}^{N} X_{0,i}$	Generate N *ensemble* in accordance with initial estimate $\overline{x}_0$ $x_{0,i} = [x_{0,1}\ x_{0,2}\ x_{0,3}\ ...\ x_{0,Ne}]$ Initial Mean *Ensemble* : $\overline{x}_{0,i} = x_{0,i}1_N$ *Ensemble* initial error : $\widetilde{x}_{0,i} = x_{0,i} - \overline{x}_{0,i} = x_{0,i}(I - 1_N)$
Prediction Stage	
$\widehat{x}^-_{k,i} = f(\widehat{x}_{k-1,i}, u_{k-1,i}) + w_{k,i}$ with $w_{k,i} \sim N(0, Q_k)$ Estimate : $\widehat{x}^-_k = \frac{1}{N_e}\sum\limits_{i=1}^{N} \widehat{x}^-_{k,i}$ Covariance error : $P^-_k = \frac{1}{N_e-1}\sum\limits_{i=1}^{N}(\widehat{x}^-_{k,i} - \widehat{x}^-_k)(\widehat{x}^-_{k,i} - \widehat{x}^-_k)^T$	$\widehat{x}^-_{k,i} = f(\widehat{x}_{k-1,i}, u_{k-1,i}) + w_{k,i}$ of which $w_{k,i} \sim N(0, Q_k)$ *Ensemble Mean* : $\overline{x}^-_{k,i} = \widehat{x}^-_{k,i}1_N$ *Ensemble Error* : $\widetilde{x}^-_{k,i} = \widehat{x}^-_{k,i} - \overline{x}^-_{k,i} = \widehat{x}^-_{k,i}(I - 1_N)$
Correction Stage	
$z_{k,i} = z_k + v_{k,i}$ with $v_{k,i} \sim N(0, R_k)$ Kalman gain : $K_k = P^-_k H^T(HP^-_k H^T + R_k)^{-1}$ Estimate : $\widehat{x}_{k,i} = \widehat{x}^-_{k,i} + K_k(z_{k,i} - H\widehat{x}^-_{k,i})$ $\widehat{x}_k = \frac{1}{N_e}\sum\limits_{i=1}^{N} \widehat{x}_{k,i}$	$z_{k,i} = z_k + v_{k,i}$ of which $v_{k,i} \sim N(0, R_k)$ $S_k = H\widetilde{x}^-_{k,i}, E_k = (v_1, v_2, ..., v_N)$, and $C_k = S_k S_k^T + E_k E_k^T$ *Ensemble Mean* : $\bar{x}_{k,i} = \bar{x}^-_{k,i} + \widetilde{x}^-_{k,i} S_k^T C_k^{-1}(\bar{z}_{k,i} - H\bar{x}^-_{k,i})$ Square root schema: - decompose eigenvalue of $C_k = U_k \Lambda_k U_k^T$ - compute matrices $M_k = \Lambda_k^{-1/2} U_k^T S_k^-$ - determine SVD from $M_k = Y_k L_k V_k^T$ *Ensemble Error* : $\widetilde{x}_{k,i} = \widetilde{x}^-_{k,i} V_k (I - L_k^T L_k)^{1/2}$ *Ensemble Estimate* : $\widehat{x}_{k,i} = \widetilde{x}_{k,i} + \bar{x}_{k,i}$

Table 1: EnKF and EnKF-SR Algorithms [13, 14].

methods have high accuracy.

Figure 7 shows the results of the simulation using the EnKF and EnKF-SR methods resulting in a movement resembling a semi-circle with a diameter of $\sqrt{(6.8^2 + 3^2)} = \sqrt{46.24 + 9} = \sqrt{55.24} = 7.43$ cm, so overall if viewed in terms of a diameter of about 7.5 cm, when using 400 ensembles, it has an error of about 0.09. In Table 2, it can be seen that the EnKF-SR method is more accurate than EnKF because there is a factor square root in the correction stage. Viewed in comparison of the numbers of ensembles, the generating of 400 ensembles is more accurate than that of 300 ensembles. When the

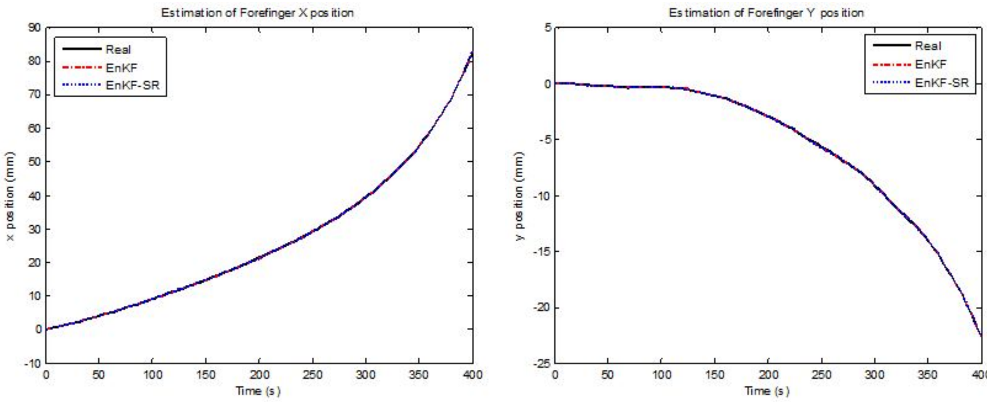

Figure 4: Estimation of Forefinger Motion using 300 Ensembles: a) X position, b) Y Position.

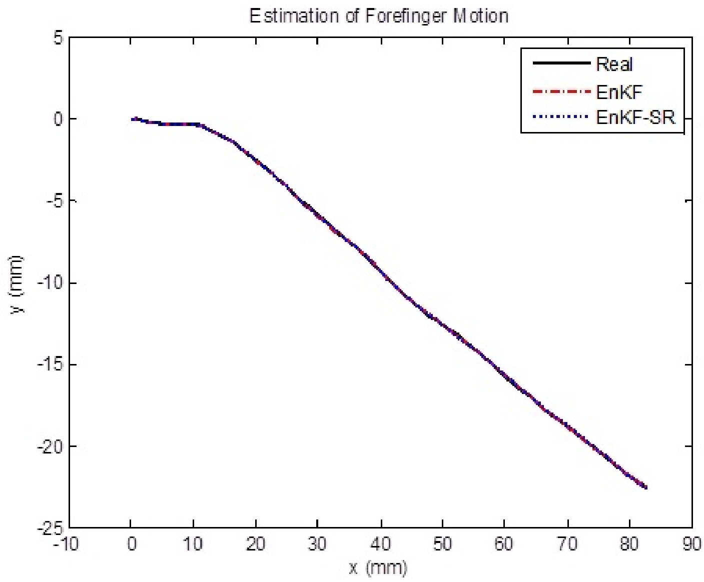

Figure 5: Estimation of Forefinger Motion in XY-Plane using 300 Ensembles.

index finger size of Indonesian people is generally around 7.5 cm (as the diameter) as a reference for the half-track trajectory, then the error for XY motion using 300 ensembles is about 10%, but that for XY motion using 400 ensembles is about 0.09%.

5 Conclusion

Based on the simulation results and the analysis above, it can be concluded that the EnKF and EnKF-SR methods were effective to estimate the movement of the index finger, especially for the finger size of the Indonesian people, with an accuracy of about

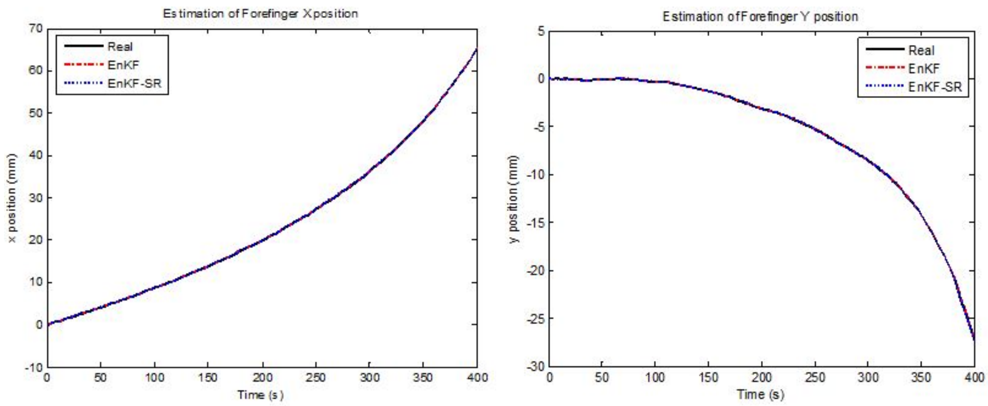

Figure 6: Estimation of Forefinger Motion using 400 Ensembles: a) X position, b) Y Position.

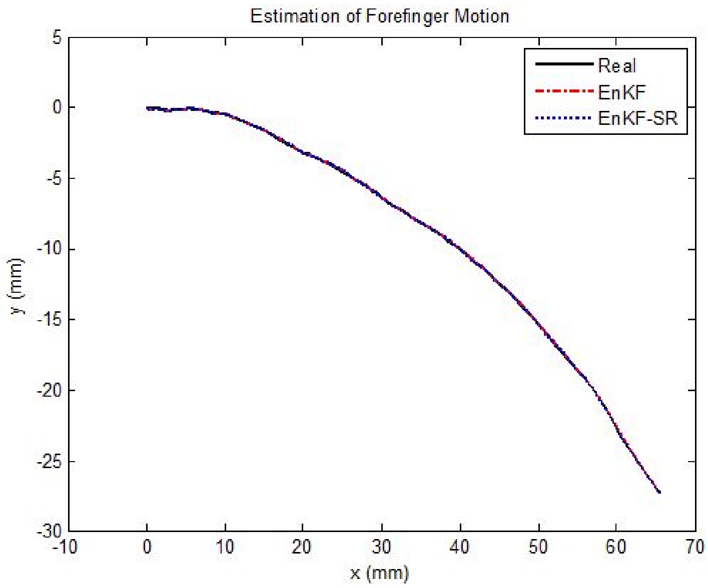

Figure 7: Estimation of Forefinger Motion in XY-Plane using 400 Ensembles.

99% and an error of 0.09%, and an error of 10% if using 300 ensembles. The error was obtained after comparing to the average finger size of the Indonesian people, about 7.5 as the diameter of the movement forming a semicircular path.

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	300 Ensembles		**400 Ensembles**	
	EnKF	**EnKF-SR**	**EnKF**	**EnKF-SR**
X Motion	0.1%	0.09%	0.094%	**0.091%**
Y Motion	0.12%	0.095%	0.09%	**0.087**
XY Motion (compare with real simulation)	0.13%	0.1%	0.093%	**0.09%**
XY Motion (compare with the real (average) finger size of Indonesian people)	10.2%	10%	0.092%	**0.09%**

Table 2: The Value of Motion Error by the EnKF and EnKF-SR Based on 400 Iterations.

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