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Tsunami Wave Simulation in the Presense of a Barrier

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Abstract: A tsunami is a series of waves that are generally caused by a vertical change in the seabed due to an earthquake beneath or on the seabed. Tsunamis usually strike coastal areas and result in damage to the shoreline, it can destroy buildings and roads and even take the lives of those who are in the area. One way to reduce the impact of a tsunami is to know the dangers of a tsunami, including natural signs. So, in this paper, it is shown by numerical simulation using the finite difference method, namely, by adding a barrier to the shallow water wave equation. The simulation results obtained in the presence of a barrier, show that the Tsunami waves are split due to hitting the barrier and experience a reduction in wave strength.

Keywords: *tsunami; shallow water equation; finite difference method.*

Mathematics Subject Classification (2010): 65L12, 76M20.

1 Introduction

Indonesia as a country that is located between three world plates, namely, the Eurasian, Indoaustralian and Pacific plates, has a high potential for natural disasters [1, 2]. These plates have a high seismic activity, which causes the emergence of many natural disasters [3,4], one of which is the occurrence of earthquakes as a primary impact of seismic activity and tsunamis as a secondary impact [2].

Tsunamis are one of the most dangerous natural disasters and damage the area around the coast [5]. Tsunamis arise as a result of displacement of large volumes of water due to earthquakes, volcanic eruptions, landslides or other phenomena that occur above or below the seabed [6]. The sea waves are not dangerous if their height does not exceed

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1 meter. However, it becomes disastrous when the wave energy is concentrated and the wavelength is greater than the depth of the sea. When the wave enters the shallow water zone, the wave velocity at the foreshore decreases sharply and the wave height increases tenfold [5].

The impact of the tsunami waves causes huge losses to humans, both in terms of the large number of casualties and the large losses in the economic field. Coastal areas affected by tsunami waves require a long time and expensive economic resources to restore [7]. Recorded in the last two decades, there have been 12 tsunamis out of 252 earthquakes with a total loss of 79.5 trillion rupiah [8]. Based on this, to minimize losses that will occur, disaster mitigation is needed.

Based on Law of the Republic of Indonesia Number 24 of 2007 concerning disaster management, mitigation is defined as a series of efforts to reduce disaster risk, both through physical development and awareness and increased capacity to face disaster threats. The embodiment of examples of mitigation activities is spatial planning, development arrangements, arrangements for infrastructure development and building layout, as well as the provision of education, counseling and training, both conventional and modern. In Indonesia, orientation in minimizing disaster risk is more towards emergency or curative handling and has not yet led to preventive aspects [9]. Therefore, it is necessary to improve the understanding of physical, engineering and social factors related to disaster mitigation implementation.

According to the initial joint rapid assessment report by the Central BMKG, BNPB, National Media, regional PUSDALOPS and community responses in the evaluation of the tsunami early warning system in the event of the Aceh earthquake and tsunami on April 11, 2012, no one really knew when the tsunami hit an area and how big was the strength of the tsunami until the time was so critical, it quickly passed and the earthquake was felt until the impact of the first wave [11]. The danger from tsunami waves is so unpredictable, sudden and extraordinary, it is almost impossible to avoid it [5]. However, other countries experiencing similar disasters have carried out several related studies in minimizing disaster risk by constructing tsunami barriers using analytical research methods and numerical modeling approaches [5].

The shallow water equation is commonly used in describing fluid problems that are based on physical conservation. With increased computation capabilities and refinement of the numerical aspects related to boundary conditions, it is possible to overcome the inherent limitations of the classical depth mean model. Shallow water equation models have been widely applied in atmospheric flows, storms, water flows around the pier, tsunami prediction, and so on.

Research on fluid computing has been widely studied, including the Airway Pressure Valve [12], Solitary Wave [13], Navier-Stokes Equation [14], Shallow Water Equation [15], [16]. The shallow water equations can be solved using the finite difference method. The method used to solve partial differential equations is the finite difference method [17,18].

In this paper, the concept of a tsunami barrier as an obstacle is presented as an attempt to deal with a complex breakwater configuration. The breakwater is studied to identify the hydrodynamic induced tsunami.

2 Research Method

2.1 Finite difference method

The Finite Difference Method is a method used in solving Partial Differential Equations (PDP) which can be used to approach the Taylor series [19]. The differential equation is an estimate of the value δ at the calculation points $U_{1,1}, U_{1,2}, \ldots, U_{i,j}, \ldots$ for the estimation it can be done by substituting the derivative of the partial differential equation using the difference estimate up to [20].

The first derivative can be calculated using the forward difference approach with $\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial t}$ of a differential equation based on the Taylor series which can be written as below:

$$\frac{\partial u(x,y,t)}{\partial x} = \frac{1}{\triangle x}\left(u(x+\triangle x,y,t)\right) - u(x,y,t), \quad (1)$$

$$\frac{\partial u(x,y,t)}{\partial y} = \frac{1}{\triangle y}\left(u(x,y+\triangle y,t)\right) - u(x,y,t), \quad (2)$$

$$\frac{\partial u(x,y,t)}{\partial t} = \frac{1}{\triangle t}\left(u(x,y,t+\triangle t)\right) - u(x,y,t). \quad (3)$$

If the approach is backwards, the first derivatives of $\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial t}$ can be written as below:

$$\frac{\partial u(x,y,t)}{\partial x} = \frac{1}{\triangle x}\left(u(x+\triangle x,y,t)\right) - u(x-\triangle x,y,t), \quad (4)$$

$$\frac{\partial u(x,y,t)}{\partial y} = \frac{1}{\triangle y}\left(u(x+\triangle x,y,t)\right) - u(x,y-\triangle y,t), \quad (5)$$

$$\frac{\partial u(x,y,t)}{\partial t} = \frac{1}{\triangle t}\left(u(x+\triangle x,y,t)\right) - u(x,y,t-\triangle t). \quad (6)$$

Second order center order is obtained:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\triangle x^2}\left(u(x+\triangle x,y,t) - 2u(x,y,t) - u(x-\triangle x,y,t)\right), \quad (7)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\triangle y^2}\left(u(x,y+\triangle y,t) - 2u(x,y,t) - u(x,y-\triangle y,t)\right), \quad (8)$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{\triangle t^2}\left(u(x,y,t+\triangle t) - 2u(x,y,t) - u(x,y,t-\triangle t)\right). \quad (9)$$

When using the subscript index i which is used to express the discrete point x, as well as the subscript index j which is used in expressing the discrete point y, and also the subscript index n used in expressing the discrete point t, it can be written as the equation below:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\cup^n_{i+1,j} - 2\cup^n_{i,j} + \cup^n_{i-1,j}}{\partial x^2}, \quad (10)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\cup^n_{i,j+1} - 2\cup^n_{i,j} + \cup^n_{i-1,j}}{\partial y^2}, \quad (11)$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\cup^{n+1}_{i,j} - 2\cup^n_{i,j} + \cup^{n-1}_{i-1,j}}{\partial t^2}. \quad (12)$$

2.2 Shallow water equation

The shallow water equation is generally used to model a surface wave of water that is influenced by gravity, for example, a wave flow on the surface of the seashore, lake, river, or on a smaller domain such as the water surface in a bathtub [16].

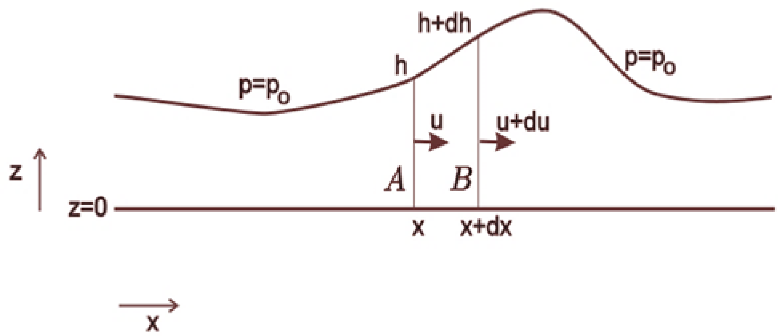

Figure 1: Shallow Water System Illustration [20].

The shallow water equation will take effect when the wavelength is greater than the depth. For a one-dimensional problem, the shallow water equation is stated as follows:

$$h_t + (uh)_x = 0, \tag{13}$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = 0. \tag{14}$$

Meanwhile, for a two-dimensional problem, the shallow water equation is stated as follows:

$$h_t + (hu)_x + (hv)_y = 0, \tag{15}$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y = 0, \tag{16}$$

$$(hu)_t + (huv)_x + \left(hu^2 + \frac{1}{2}gh^2\right)_y = 0, \tag{17}$$

where g is the Earth's gravitational constant, h is the height of the sea surface, (u, v) is the vector of the velocity of the water flow, and hu and hv are the momentum in two directions [16].

3 Result and Discussion

3.1 Tsunami wave analysis

In this study, the model used is a model of the shallow water equation to simulate the movement of a tsunami wave that has resistance by using the finite difference method. The shallow water equation comes from the mass conservation equation and the conservation of linear momentum (Navier-Stokes equation).

The mass equation in volume is defined as follows:

$$\begin{aligned}\frac{\partial m}{\partial t} &= \rho u(x)h(x) - \rho u(x+dx)h(x+dx)\\ &= -\rho\frac{\partial(uh)}{\partial x}dx,\end{aligned}$$

assuming $m = \rho h\,dx$, we obtain

$$\begin{aligned}\frac{\partial m}{\partial t} &= -\rho\frac{\partial(uh)}{\partial x}dx,\\ \frac{\partial \rho h}{\partial t}dx &= -\rho\frac{\partial(uh)}{\partial x}dx,\\ \frac{\partial h}{\partial t} &= -\frac{\partial(uh)}{\partial x},\\ \frac{\partial h}{\partial t}+\frac{\partial(uh)}{\partial x} &= 0,\\ h_t+(uh)_x &= 0. \qquad (18)\end{aligned}$$

Based on Newton's laws of motion applied to volume, we define

$$F = m\frac{du}{dt} = -\rho g h\frac{\partial h}{\partial x}dx$$

by applying $m = \rho h dx$, we then obtain

$$\begin{aligned}m\frac{du}{dt} &= -\rho g h\frac{\partial h}{\partial x}dx,\\ \rho h dx\frac{du}{dt} &= -\rho g h\frac{\partial h}{\partial x}dx,\\ \frac{du}{dt} &= -g\frac{\partial h}{\partial x}.\end{aligned}$$

Then the chain rule is applied to $\frac{d}{dt}$, we obtain

$$\begin{aligned}\frac{du}{dt} &= \frac{\partial u}{\partial t}+\frac{dx}{dt}\frac{\partial u}{\partial x},\\ &= \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}.\end{aligned}$$

So the equation is expressed by

$$\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x} &= -g\frac{\partial h}{\partial x},\\ \left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)dx &= \left(-g\frac{\partial h}{\partial x}\right)dx,\\ h\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right) &= -\frac{1}{2}gh\frac{\partial h}{\partial x},\\ h\frac{\partial u}{\partial t}+hu\frac{\partial u}{\partial x} &= -\frac{1}{2}gh\frac{\partial h}{\partial x},\\ h\frac{\partial u}{\partial t}+hu\frac{\partial u}{\partial x}+\frac{1}{2}gh\frac{\partial h}{\partial x} &= 0,\\ (hu)_t+\left(hu^2+\frac{1}{2}gh^2\right)_x &= 0. \qquad (19)\end{aligned}$$

For the problems adopted in this study, the shallow water equation does not have a Coriolis force. Thus, equation (15) is based on equation (18), while equation (16) and equation (17) are based on equation (19).

3.2 Discretization of tsunami wave time

Based on the shallow water equation which is approached by the finite difference method, equations 4, 5 and 6 are substituted in equations 16 and 17 in order to obtain

$$\begin{aligned}
u(i,j,n+1) &= \frac{(u(i+1,j,n)+u(i-1,j,n)+u(i,j+1,n)+u(i,j-1,n))}{4} \\
&-\frac{1}{2}\frac{dt}{dx}\left(\frac{u(i+1,j,n)^2}{2}-\frac{u(i-1,j,n)2}{2}\right) \\
&-\frac{1}{2}\frac{dt}{dy}v(i,j,n)\left(u(i,j+1,n)-u(i,j-1,n)\right) \\
&-\frac{1}{2}g\frac{dt}{dx}\left(h(i+1,j,n)-h(i-1,j,n)\right), \\
v(i,j,n+1) &= \frac{(v(i+1,j,n)+v(i-1,j,n)+v(i,j+1,n)+v(i,j-1,n))}{4} \\
&-\frac{1}{2}\frac{dt}{dy}\left(\frac{v(i,j+1,n)^2}{2}-\frac{v(i,j+1,n)2}{2}\right) \\
&-\frac{1}{2}\frac{dt}{dx}u(i,j,n)\left(v(i+1,j,n)-v(i-1,j,n)\right) \\
&-\frac{1}{2}g\frac{dt}{dy}\left(h(i,j+1,n)-h(i,j-1,n)\right),
\end{aligned}$$

$$\begin{aligned}
h(i,j,n+1) &= \frac{(h(i+1,j,n)+h(i-1,j,n)+h(i,j+1,n)+h(i,j-1,n))}{4} \\
&-\frac{1}{2}\frac{dt}{dx}u(i,j,n)\left(h(i+1,j,n)-b(i+1,j)\right) \\
&-\left(h(i-1,j,n)-b(i-1,j)\right) \\
&-\frac{1}{2}\frac{dt}{dy}v(i,j,n)\left(h(i,j+1,n)-b(i,j+1)\right) \\
&-\left(h(i,j-1,n)-b(i,j-1)\right) \\
&-\frac{1}{2}\frac{dt}{dx}\left(h(i,j,n)-b(i,j)\right)\left(u(i+1,j,n)-u(i-1,j,n)\right) \\
&-\frac{1}{2}\frac{dt}{dy}\left(h(i,j,n)-b(i,j)\right)\left(v(i,j+1,n)-v(i,j-1,n)\right)
\end{aligned}$$

with the following boundary conditions:

$$\begin{aligned}
u(1,i,n+1) &= \frac{10}{4}u(2,i,n+1)-2u(3,i,n+1)+\frac{1}{2}u(4,i,n+1), \\
v(1,i,n+1) &= \frac{10}{4}v(2,i,n+1)-2v(3,i,n+1)+\frac{1}{2}v(4,i,n+1), \\
h(1,i,n+1) &= \frac{10}{4}h(2,i,n+1)-2h(3,i,n+1)+\frac{1}{2}h(4,i,n+1).
\end{aligned}$$

The initial simulation of the formation of a wave ($t = 1$) is shown in Figure 2. At $t = 1$, it is assumed to be the time when the tsunami first appearance on the surface after a shift in the earth's plate under the sea. So its first appears on the surface is very large at the starting point of its appearance, then it travels towards the mainland.

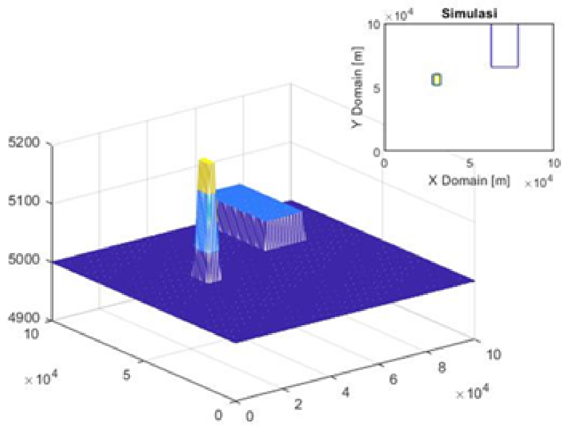

Figure 2: Tsunami Wave Simulation at t = 1.

Figure 2 shows simulation of a tsunami wave that appears to have a height that exceeds the height of the barrier by a height of 8 meters above sea level. Then, at the next time, the tsunami waves will move around, one of which being towards the shallow water where a barrier has been built. Based on the simulation, the tsunami wave will experience a collision with the barrier in 35 seconds since the initial appearance of the tsunami.

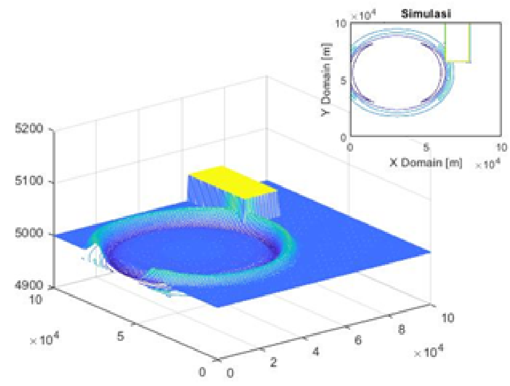

Figure 3: Tsunami Wave Simulation at t = 40.

The collision of the wave with the barrier is shown in Figure 3. As a result of the collision with the tsunami wave barrier, it breaks and can be suppressed. The damping by the barrier means that the area behind the barrier is not directly affected by the tsunami. The tsunami waves hit the area behind the barrier due to the impact of the tsunami from the area that is not protected by the barrier. The impact is not as high as that of a tsunami wave without a barrier.

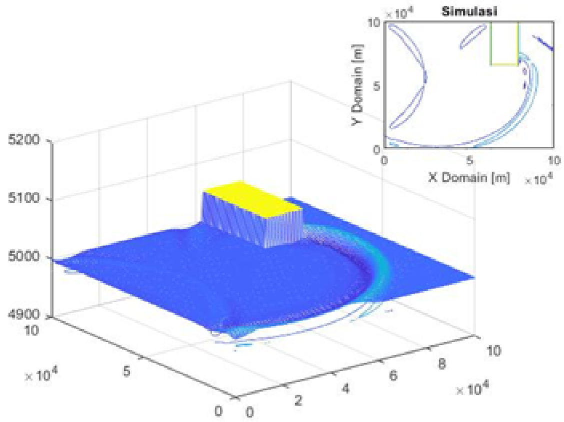

Figure 4: Tsunami Wave Simulation at t = 65.

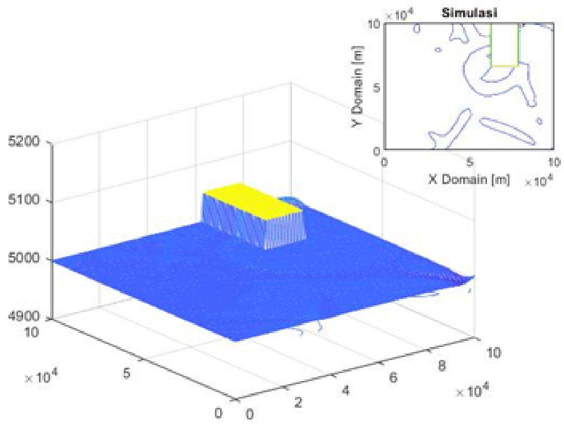

Figure 5: Tsunami Wave Simulation at t = 100.

Figure 4 shows the condition of the tsunami waves behind the damping barrier. The damping process is carried out continuously by the barrier that has been formed, until the waves calm down again. This condition is shown in Figure 5.

4 Conclusion

Numerical solutions and simulations have been carried out for the shallow water equation to represent a tsunami wave with the construction of a barrier. It is known that the construction of an obstacle can break up tsunami waves and can reduce the strength of the waves. Thus, if it is realized, it can really be an effort to reduce the impact of the tsunami disaster.

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