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Implementation of Infeasible Interior-Point Methods Based on a New Search Direction

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Abstract: In this paper, we present the implementation of infeasible interior-point methods (IIPMs) for linear and nonlinear optimization with the full-Newton step based on an algebraic equivalent transformation (AET). The algorithm was implemented in Matlab language, thus supporting the effectiveness of the method. Numerical tests demonstrate the behavior of the algorithms for different results of parameters.

Keywords: *infeasible interior-point methods; nonlinear systems; primal-dual methods; new search direction; nonlinear resonances.*

Mathematics Subject Classification (2010): 90C05, 90C51,93C10, 70K30.

1 Introduction

Linear optimization (LO) has numerous applications in different fields such as economics, logistics, engineering, nonlinear dynamics and systems (see, *e.g.*, [6], [7]). The classical method for solving LO problems is the simplex algorithm proposed by Dantzig [2] in 1947. The appearance of interior-point algorithms (IPAs) in LO is the result of a longer process. From the literature, we know that the first result is due to Frisch, who proposed the use of logarithmic barrier functions in LO [8]. Later on Fiacco and McCormick [5] developed the sequential unconstrained minimization technique (SUMT). Since then, the barrier functions have been extensively studied.

The result of Karmarkar obtained in 1984 [9] had a great impact on mathematical optimization from both theoretical and practical point of view. He derived projective scaling IPAs with better complexity than the ellipsoid algorithm and he claimed that his algorithm had better practical performance. Moreover, it turned out that the IPA

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approach to LO has a natural generalisation to the related field of convex nonlinear optimisation, which resulted in a new stream of research and an exellent monograph of Nesterov and Nemirovski [13]. This study opened the way into other new subfields of optimization such as semidefinite optimization (SDO), convex quadratic optimization (CQO), second-order cone optimization (SOCO), symmetric optimization (SO) and the complementarity problem (CP), with important applications in system theory, discrete optimization, and many other areas. The most important results related to IPAs for LO were summarized in the monographs written by Roos, Terlaky and Vial [16], Wright [19] and Ye [20]. Based on the starting point, two types of IPAs exist; feasible and infeasible algorithm. Feasible IPAs start from a feasible interior point and maintain feasibility during the whole process of the algorithms. Infeasible IPAs start from an infeasible interior point and they use two kinds of steps, feasibility and centering steps in each iteration. The first infeasible algorithms were introduced by Lustig [12] and Tanabe [18]. Kojima et al. [11] analyzed the complexity of these algorithms. In 2005, Roos [14] proposed a new algorithm, which uses only the full-Newton steps and starts from infeasible points. Takács [17] gave an application of infeasible interior-point algorithms. Several implementations of IPAs can be found in state-of-the-art solvers nowadays. The paper presents an implementation of original Roos's infeasible algorithm [2006, 2016], and a short updating algorithm [10], where the AET technique is used with the new function $\psi(t) = t^2$ to transform the central path equation. Numerical results show that the algorithm with the practical step size is more efficient than that with the fixed (theoretical) step size.

The outline of the paper is as follows. In Section 2, we briefly recall the new search direction based on the type of AET using the new function $\psi(t) = t^2$. In Section 3, we report some preliminary numerical results to demonstrate the computational performance of the proposed methods. Finally, some conclusions and remarks are made in Section 4.

We use the following notations throughout the paper: $\mathbb{R}^n$ is the n-dimensional Euclidean space with the inner product $\langle ., . \rangle$ and $\|.\|$ is the Euclidean norm which denotes the $2-$norm, $\mathbb{R}^{m\times n}$ is the space of matrices of order $m \times n$, x_i is the i-th component of x, xs is the component-wise product of vector x and s, respectively. The vector of ones is denoted by e.

2 Formulation of the Problem

We consider the LO problem in standard form

$$\min \left\{ c^T x : Ax = b, \quad x \geq 0 \right\}, \tag{P}$$

and its dual form

$$\max \left\{ b^T y : A^T y + s = c, \quad s \geq 0 \right\}, \tag{D}$$

where $A \in \mathbb{R}^{m\times n}$ with $rank(A) = m$, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$ with $y \in \mathbb{R}^m$ and $s \in \mathbb{R}^n$.

As usual, for IIPMs, we consider the starting point $(x^0, y^0, s^0) = \xi(e, 0, e)$ such that

$$\| (x^*, s^*) \|_\infty \leq \xi \Leftrightarrow 0 \leq x^* \leq \xi e \text{ and } 0 \leq s^* \leq \xi e. \tag{1}$$

For some primal-dual optimal solution (x^*, y^*, s^*), e is the all-one vector and ξ is a positive scalar. The triple (x, y, s) is the ϵ-solution of (P) and (D) if the norms of the residual vectors $b - Ax$ and $c - A^T y - s$ do not exceed ϵ, and also $x^T s$.

For any λ with $0 < \lambda \leq 1$, we consider the perturbed problem (P_λ) defined by

$$\left\{\min(c-\lambda r_c^0)^T x : b - Ax = \lambda r_b^0, \quad x \geq 0\right\}, \tag{P_λ}$$

and its dual form

$$\left\{\max(b-\lambda r_b^0)^T y : c - A^T y - s = \lambda r_c^0, \quad s \geq 0\right\}, \tag{D_λ}$$

where r_b^0 and r_c^0 denote the primal and dual initial residual vectors, respectively,

$$r_b^0 = b - A\xi e \text{ and } r_c^0 = c - \xi e.$$

Note that if $\lambda = 1$, then $x = x^0$ yields a strictly feasible solution of (P_λ), and $(y, s) = (y^0, s^0)$ is a strictly feasible solution of (D_λ). We conclude that if $\lambda = 1$, then (P_λ) and (D_λ) satisfy the interior point condition (IPC), we recall and develop some new results on the scaled search directions.

Lemma 2.1 *(cf. Theorem* 5.13 *in [20]) The original problems,* (P) *and* (D)*, are feasible if and only if for each* λ *satisfying* $0 < \lambda \leq 1$*, the perturbed problems* (P_λ) *and* (D_λ) *satisfy the IPC.*

Let (P) and (D) be feasible and $0 < \lambda \leq 1$. Then Lemma 2.1 implies that the problems (P_λ) and (D_λ) satisfy the IPC, and hence their central paths exist. This means that the system

$$\begin{cases} b - Ax = \lambda r_b^0, & x \geq 0, \\ c - A^T y - s = \lambda r_c^0, & y \in \mathbb{R}^m,\ s \geq 0, \\ xs = \mu e \end{cases} \tag{2}$$

has a unique solution for every $\mu > 0$. This solution consists of the μ-centers of the perturbed problems (P_λ) and (D_λ). The perturbed central path can be equivalently stated as follows:

$$\begin{cases} b - Ax = \lambda r_b^0, & x \geq 0, \\ c - A^T y - s = \lambda r_c^0, & y \in \mathbb{R}^m,\ s \geq 0, \\ \dfrac{xs}{\mu} = \sqrt{\dfrac{xs}{\mu}}. \end{cases} \tag{3}$$

From [10], we replace the third equation of system (3) by the equivalent equation $\psi(\frac{xs}{\mu}) = \psi(\sqrt{\frac{xs}{\mu}})$, where ψ is a real valued function differentiable on $\left(k^2 \to +\infty\right)$, where $0 \leq k < 1$, such that $2t\psi'(t^2) - \psi'(t)$, we get

$$\begin{cases} b - Ax = \lambda r_b^0, & x \geq 0, \\ c - A^T y - s = \lambda r_c^0, & y \in \mathbb{R}^m,\ s \geq 0, \\ \psi(\dfrac{xs}{\mu}) = \psi(\sqrt{\dfrac{xs}{\mu}}). \end{cases} \tag{4}$$

Let (x, y, s) be a feasible solution of (P_λ) and (D_λ). We consider the notation

$$f(x,y,s) = \begin{bmatrix} \lambda^+ r_b^0 - b + Ax \\ \lambda^+ r_c^0 - c + A^T y + s \\ \psi(\frac{xs}{\mu}) - \psi(\sqrt{\frac{xs}{\mu}}) \end{bmatrix} = 0,$$

where $\lambda^+ = (1-\theta)\lambda$ and $\theta \in (0,1)$, a new triple

$$(x_+, y_+, s_+) = (x + \Delta x, y + \Delta y, s + \Delta s)$$

is obtained thanks to the Newton method for solving the following system:

$$\begin{cases} A\Delta x = \theta\lambda^+ r_b^0, \\ A^T\Delta y + \Delta s = \theta\lambda^+ r_c^0 \\ \\ \dfrac{1}{\mu}\left(s\Delta x + x\Delta s\right) = \dfrac{-\psi\left(\dfrac{xs}{\mu}\right) + \psi\left(\sqrt{\dfrac{xs}{\mu}}\right)}{\psi'\left(\dfrac{xs}{\mu}\right) - \dfrac{1}{2\sqrt{\dfrac{xs}{\mu}}}\psi'\left(\sqrt{\dfrac{xs}{\mu}}\right)}. \end{cases} \tag{5}$$

Define the scaled search directions d_x and d_s as follows:

$$v = \sqrt{\frac{xs}{\mu}}, \quad d_x := \frac{v\Delta x}{x}, \quad d_s := \frac{v\Delta s}{s}, \tag{6}$$

then the system reduces to the system

$$\begin{cases} \bar{A}d_x = \theta\lambda^+ r_b^0, \\ \bar{A}^T\dfrac{\Delta y}{\mu} + d_s = \theta v\lambda^+ s^{-1} r_c^0, \\ d_x + d_s = p_v, \end{cases} \tag{7}$$

where

$$p_v = \frac{2\psi(v) - 2\psi(v^2)}{2v\psi'(v^2) - 2\psi'(v)} \text{ and } \bar{A} = A\ diag(\frac{x}{v}). \tag{8}$$

We consider the proximity measure defined by

$$\delta(v) = \delta(x, s; \mu) = \frac{\|p_v\|}{2} = \frac{1}{2}\|\frac{v\ -\ v^3}{2v^2\ -\ e}\|, \tag{9}$$

suppose that for some $\mu \in (0, \mu^0)$, we have a feasible solution (x, y, s) of the problem (P_λ) and (D_λ) with $\lambda = \dfrac{\mu}{\mu^0}$, such that $\delta(x, s; \mu) \leq \tau$, $\tau \in (0,1)$. Then the algorithm finds the feasible solution (x_+, y_+, s_+) of (P_{λ^+}) and (D_{λ^+}), where $\lambda^+ = (1-\theta)\lambda$ and $\theta \in (0,1)$. In this case, μ is reduced to $\mu^+ = (1-\theta)\mu$ and so $\delta(x_+, y_+; \mu^+) = \delta(v^+) \leq \tau$. If necessary, we repeat the procedure until an ϵ-solution is found.

Now we can define the generic infeasible interior-point algorithm for LO.

Algorithm 2.1 *The generic infeasible interior-point algorithm for LO.*

Input :
-Accuracy parameter $\epsilon > 0$,
-barrier update parameter θ *barrier update parameter* θ, $0 < \theta < 1$,
-threshold parameter $\tau > 0$,
Begin
$x = \xi e; y = 0; s = \xi e; \lambda = 1; \mu = \lambda\xi^2$.
while : $\max(x^T s, \|r_b\|, \|r_c\|) > \epsilon$ *do*
Begin
solve the systeme (7) *and use* (6) *to obtain* $(\Delta x, \Delta y, \Delta s)$*;*
$x = x + \Delta x;$ $y = y + \Delta y;$ $s = s + \Delta s;$
update of λ *and* μ
$\lambda = (1 - \theta)\lambda;$ $\mu = (1 - \theta)\mu;$
end
end

Lemma 2.2 *([10]) If* $\tau = \dfrac{1}{12}$ *and* $\theta = \dfrac{1}{22n}$, $n \geq 4$, *then* $\delta(v) \leq \tau$ *implies* $\delta(v^+) \leq \tau$.

Theorem 2.1 *(**Theorem** 1 [10]) If* (P) *and* (D) *are feasible and* $\xi > 0$ *such that* $\|(x^*; s^*)\|_\infty \leq \xi$ *for some optimal solutions* x^* *of* (P) *and* (y^*, s^*) *of* (D)*, then after at most*

$$22n \log \frac{\max(n\xi^2, \|r_b^0\|, \|r_c^0\|}{\epsilon}$$

iterations, the algorithm finds an ϵ*-optimal solution of* (P) *and* (D).

3 Numerical Results

In this section, we present an implementation of the IIPMs for LO, which demonstrates the influence of the update parameters θ and the dimension of the problem on the number of the iterations. The algorithm is coded in MATLAB (R 2014 a) and our experiments are performed on PC with Processor Genuine Intel (R) CPR T2080 @ 1, 73 GHZ installed memory (RAM) 2, 00GO. In all test problems, the starting point is designated by $(x^0, y^0, s^0) = \xi(e, 0, e)$ such that $e = (1, 1, \ldots, 1)^T$, we use (m, n) as the size of the problem, $\epsilon = 10^{-6}$and $\tau = \dfrac{1}{12}$ as our default accuracy parameter. The barrier update parameter θ is a given constant between 0 and 1, while in the theoretical version of the algorithm, $\theta = \dfrac{1}{22n}$. We denote by iter the number of iterations and by CPU the computing time in seconds. The primal and dual optimal solutions are denoted by x^*, (y^*, s^*), we tested the above mentioned algorithms in two different cases of the test: the full Newton step ($\alpha = 1$) and the practice step size $\alpha_{\max}$, which guarantees that the new iterates

$$\begin{cases} x_+ = x + \alpha_{\max}\Delta x, \\ s_+ = s + \alpha_{\max}\Delta s \end{cases}$$

remain nonnegative. To ensure the strict feasibility of the new iterates, we used a factor $\rho = 0.95$ to shorten the step length, thus the used step length is $\alpha_p = \rho\alpha_{\max}$ with $\alpha_{\max} = \min(\alpha_x, \alpha_s)$, or α_x and α_s are given by

$$\alpha_x = \begin{cases} \min\left(-\dfrac{x_i}{\Delta x_i}\right) & \text{if } \Delta x_i < 0, \\ 1 & \text{if } \Delta x_i \geq 0, \end{cases}$$

$$\alpha_s = \begin{cases} \min\left(-\dfrac{s_i}{\Delta s_i}\right) & \text{if } \Delta s_i < 0, \\ 1 & \text{if } \Delta s_i \geq 0. \end{cases}$$

3.1 Cases of full Newton step ($\alpha = 1$) with $\theta = \dfrac{1}{22n}$

3.1.1 Examples with fixed size

Example 3.1 $m = 2, n = 4$,

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & -3 \end{pmatrix}, b = \begin{pmatrix} 1 & 0.5 \end{pmatrix}^T, c = \begin{pmatrix} 1 & 2 & 3 & 2 & 4 \end{pmatrix}^T.$$

The optimal solution is

$$x^* = (0.875, 0, 0, 0.125)^T, y^* = (1.75, -0.75)^T, s^* = (0, 1, 1.25, 0)^T.$$

iter	*CPU*
1332	0.3279

Table 1: Number of iterations and computation time.

Example 3.2 $m = 3, n = 6$,

$$A = \begin{pmatrix} 2 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}, b = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T, c = \begin{pmatrix} 3 & -1 & 1 & 0 & 0 & 0 \end{pmatrix}^T.$$

The optimal solution is

$$x^* = (0.0000, 0.5000, 0.0000, 0.5000, 0.0000, 0.0000)^T,$$

$$y^* = (-0.5000, -0.0383, -0.5000)^T,$$

$$s^* = (4.5000, 0.0000, 1.4617, 0.0000, 0.4617, 0.5383)^T.$$

iter	*CPU*
2054	0.5914

Table 2: Number of iterations and computation time.

Example 3.3 $m = 5, n = 9$,

$$A = \begin{pmatrix} 0 & 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -2 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & -1 & -2 & 0 & 0 & 0 & 1 \\ 1 & 3 & 4 & 2 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$b = \begin{pmatrix} 1 & 2 & 3 & 2 & 1 \end{pmatrix}^T, \quad c = \begin{pmatrix} 1 & 0 & -2 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}^T.$$

The optimal solution is

$$x^* = (\ 0 \quad 0 \quad 0.25 \quad 0 \quad 0 \quad 0.5 \quad 1.25 \quad 3.5 \quad 2\ )^T, \quad y^* = \begin{pmatrix} 0 & 0 & 0 & 0 & -0.5 \end{pmatrix}^T,$$

$$s^* = \begin{pmatrix} 1.5 & 1.5 & 0 & 2 & 1.5 & 0 & 0 & 0 & 0 \end{pmatrix}^T.$$

iter	*CPU*
3241	1.5285

Table 3: Number of iterations and computation time.

3.1.2 Examples with variable size

Example 3.4 We consider the following example: $n = 2m$,
$A(i,j) = \begin{cases} 0 \text{ if } i \neq j \text{ and } i \neq j+m \\ 1 \text{ if } i = j \text{ and } i = j+m \end{cases}$, $c(i) = -1$, $c(i+m) = 0$ and $b(i) = 2$ for $i = 1, ..., m$.

The optimal solutions is obtained as follows:
$x^* = \begin{cases} 2 & \text{for } i = 1, ..., m, \\ 0 & \text{for } i = m+1, ..., n, \end{cases}$, $y^* = -1$ for $i = 1, ..., n$,
and $s^* = \begin{cases} 0 & \text{for } i = 1, ..., m, \\ 1 & \text{for } i = m+1, ..., n. \end{cases}$

We have the following results:

(m,n)	*iter*	*CPU*
$(10, 20)$	7390	6.2958
$(15, 30)$	11356	13.8918
$(25, 50)$	19493	45.4140
$(50, 100)$	40518	226.3179
$(100, 200)$	84093	1856.1149
$(200, 400)$	174293	61443.8710

Table 4: Number of iterations and computation time.

3.2 Cases of practice step size ($\alpha_{\max}$) with $\theta \in (0, 1)$

In this part, to improve the numerical results, we take $\theta \in (0, 1)$. Then we obtain the numerical results in the following tables.

3.2.1 Examples with fixed size

θ	$iter$	CPU
0.01	1514	0.3941
0.02	754	0.2678
0.05	298	0.1895
0.1	146	0.1646
0.2	70	0.1575
0.8	11	0.1868

Table 5: Number of iterations and computation time in Example 1.

θ	$iter$	CPU
0.01	1554	0.438790
0.02	774	0.283944
0.05	306	0.190252
0.1	150	0.167571
0.2	72	0.180073
0.4	32	0.184932

Table 6: Number of iterations and computation time in Example 2.

θ	$iter$	CPU
0.01	1633	0.7102
0.05	320	0.2167
0.02	813	0.4256
0.1	156	0.1827
0.7	15	0.2377
0.8	11	0.2349

Table 7: Number of iterations and computation time in Example 3.

3.2.2 Examples with variable size

θ	0.01		0.05		0.1	
(m, n)	$iter$	CPU	$iter$	CPU	$iter$	CPU
$(10, 20)$	1664	1.0255	329	0.3340	161	0.2524
$(15, 30)$	1715	1.7057	337	0.4459	165	0.2990
$(25, 50)$	1765	2.7304	347	0.6487	170	0.4376
$(50, 100)$	1834	8.4164	361	1.5884	176	0.86035
$(100, 200)$	1903	38.0307	374	7.0592	176	4.5225
$(200, 400)$	1972	182.1165	388	36.1221	183	29.9616

θ	0.2		0.3	
(m,n)	$iter$	CPU	$iter$	CPU
$(10,20)$	74	0.2840	47	0.6049
$(15,30)$	76	1.2352	48	0.6665
$(25,50)$	78	2.1898	49	1.3937
$(50,100)$	81	4.3522	–	–
$(100,200)$	84	16.9204	–	–
$(200,400)$	–	–	–	–

Table 8: Number of iterations and computation time in Example 4.

4 Concluding Remarks

In this paper, we have proposed an implementation of the IIPMs for linear and nonlinear optimization based on the AET proposed in [10]. Some preliminary numerical results are provided to reveal the influence of the update parameters θ and the dimension of the problem on the number of iterations. Through these results, we notice that the number of iterations and the computation time to reach the optimal solution are a bit large. To improve these results, we proposed other choices of the parameter θ and the step size α different from the theorecal values. It was found that these values decreased the number of iterations and the computation time. For further research, it is necessary to think of a simple strategy to determine the appropriate values of the parameter θ which keeps the iteration in the interior of the feasible domain. Furthermore, this algorithm may be possible to extend to the semidefinite linear optimization, quadratic programming and linear complementarity problem with these choices of the step size.

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