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Weights Optimization Using Firefly Algorithm for Dengue Fever Optimal Control Model by Vaccination, Treatment, and Abateseae

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Abstract: Indonesia is one of tropical countries where dengue fever disease can spread through *Aedes aegypti* mosquitoes and sometimes cause deaths. There are many control strategies to bound the spread of dengue fever: vaccination for controlling susceptible humans, treatment for controlling infected humans, and abateseae (larvacides for killing the mosquito larvae). Optimal control is used for minimizing the number of infected humans, larvae, infected mosquitoes, the cost of vaccination, the cost of treatment, and the cost of abateseae. Due to the cost of the objective function depending on weights, in this research, we will apply the Firefly Algorithm (FA) to optimize the weights minimizing the cost of the objective function. The FA is based on the behavior of flashing characteristics of fireflies. Simulations have been applied and we can obtain the comparison of the number of humans and mosquitoes with and without control. In addition, we also obtain the optimal weight related to the number of infected humans, the number of larvae, the number of infected mosquitoes, the cost of vaccination, the cost of treatment, and the cost of abateseae, respectively.

Keywords: *dengue fever; optimal control; firefly algorithm; vaccination; treatment; abateseae.*

Mathematics Subject Classification (2010): 49L12, 65L06.

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1 Introduction

Indonesia is one of tropical countries where dengue fever disease can spread through *Aedes aegypti* mosquitoes and sometimes cause deaths. Based on the data of the Directorate of Animal Disease Control Source, Ministry of Health Department of the Republic of Indonesia, in 2011, there were 126,908 cases of dengue with 1,125 deaths [14].

Dengue fever disease is caused by *Aedes aegypti* mosquitoes. The mosquitoes have 4 life stages: egg, larva, pupa and adult (mosquito). Mosquitoes can live and reproduce inside and outside the home. The mosquitoes are most frequently found in tropical and subtropical areas of the world. *Aedes aegypti* historically is considered to be a primary vector of viral diseases such as dengue fever, chikungunya and yellow fever. Generally, the habitats of *Aedes aegypti* are the areas lacking piped water systems and depend on water storage containers to lay their eggs. Male and female mosquitoes feed on nectar of plants. However, female mosquitoes need blood in order to produce eggs, and they are active in the daytime. *Aedes aegypti* prefers biting people but it also bites dogs and other domestic animals, mostly mammals. Only female mosquitoes bite to obtain blood for laying eggs.

The purpose of modelling epidemics is to provide policies designed to control the spread of the disease [7]. There are many control strategies to bound the spread of dengue fever: vaccination for controlling susceptible humans, treatment for controlling infected humans, and abateseae (larvacides for killing the mosquito larvae). Optimal control is used for minimizing the number of infected humans, larvae, infected mosquitoes, the cost of vaccination, the cost of treatment, and the cost of abateseae [6].

From the previous researches, a mathematical model to look for stability of the disease or for controlling the disease has been contructed [13]. In [16], the dengue fever control has been applied by vaccination to control the number of susceptible humans to be recovered humans. However, in Indonesia, dengue fever controls have been applied by vaccination to control the number of susceptible humans to be recovered humans, and fogging for devastating the mosquitoes [17]. In this paper, we construct a mathematical model for controlling dengue fever by vaccination for controlling susceptible humans, treatment for controlling infected humans, and abateseae (larvacides for killing the mosquito larvae) for controlling larvae.

In the earlier research from Michalewicz, by heuristic optimization like the Genetic Algorithm (GA), we can determine an optimal control minimizing the objective function based on the natural selection of chromosomes [5]. In this research, the Firefly Algorithm (FA) will be used. The FA was discovered by Xin-She Yang in 2008. It is based on the behavior of flashing characteristics of fireflies. These insects communicate, search for a prey, and find mates using bioluminescence with varying flaying patterns. One of the characteristics of fireflies is the less bright one will move toward the brighter one. A brighter firefly indicates a better objective function as a fitness function [4].

In the optimal control problem, weight selection is applied by trial and error [2]. Due to the cost of the objective function depending on weights [8], [11], in this research, we will apply the Firefly Algorithm to optimize the weights minimizing the cost of the objective function. In the previous research, the Ant Colony Optimization (ACO) has been applied for SEIR contagious disease [6], [9], [10]. The artificial Bee Colony (ABC) has been applied for influenza disease [7].

Simulations have been applied and we can obtain the comparison of the number of humans and mosquitoes with and without control. In addition, we also obtain optimal weights related to the number of infected humans, the number of larvae, the number of infected mosquitoes, the cost of vaccination, the cost of treatment, and the cost of abateseae, respectively.

2 Optimal Control Dengue Fever Model

Generally, the disease can be modeled as a SIR (Susceptible, Infected, Recovered) epidemic model [6]. In the SIR epidemic model, there are three compartments of individuals: susceptible, infected, and recovered. A susceptible individual can be an infected individual after making contact with an infected individual based on disease transmission rate. An infected individual can be a recovered individual when the symptoms of the disease have gone based on recovery rate [1], [12].

2.1 Mathematical model of dengue fever

The dengue fever model is the development of a standard SIR epidemic model. In the dengue fever model, there are two different populations such as mosquito as a vector and human as a host. The compartments of the dengue fever model can be seen in Figure 1 where in the mosquito as a vector one, there are larvae (mosquitoes in aquatic phase) A_m, susceptible mosquitoes S_m, and infected mosquitoes I_m, while in the human as a host one, there are susceptible humans S_h, infected humans I_h, and recovered humans R_h. The mathematical model of dengue fever can be constructed in equations (1) - (8):

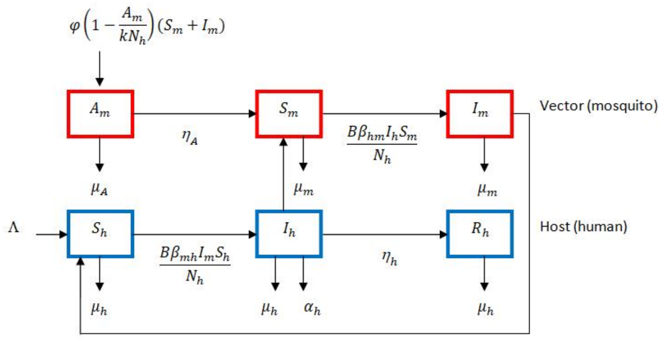

Figure 1: Compartments of the Dengue Fever Model.

$$
\begin{aligned}
\frac{dS_h}{dt} &= \Lambda - B\beta_{mh}\frac{I_m}{N_h}S_h - \mu_h S_h - u_1 S_h, && (1)\\
\frac{dI_h}{dt} &= B\beta_{mh}\frac{I_m}{N_h}S_h - \eta_h I_h - \mu_h I_h - \alpha_h I_h - u_2 I_h, && (2)\\
\frac{dR_h}{dt} &= \eta_h I_h + u_1 S_h + u_2 I_h - \mu_h R_h, && (3)\\
\frac{dA_m}{dt} &= \varphi\left(1 - \frac{A_m}{kN_h}\right)(S_m + I_m) - \eta_A A_m - \mu_A A_m - u_3 A_m, && (4)\\
\frac{dS_m}{dt} &= \eta_A A_m - B\beta_{hm}\frac{I_h}{N_h}S_m - \mu_m S_m, && (5)
\end{aligned}
$$

$$\frac{dI_m}{dt} = B\beta_{hm}\frac{I_h}{N_h}S_m - \mu_m I_m, \tag{6}$$

$$N_h(t) = S_h(t) + I_h(t) + R_h(t), \tag{7}$$

$$N_m(t) = S_m(t) + I_m(t), \tag{8}$$

with the positive solutions

$$S_h(t) \geq 0, I_h(t) \geq 0, R_h(t) \geq 0, A_m(t) \geq 0, S_m(t) \geq 0, I_m(t) \geq 0.$$

The parameters used in the model above are:
Λ : The recruitment rate (birth or immigration) of the human population.
μ_h : The natural death rate of humans.
μ_m : The natural death rate of mosquitoes (adult phase).
μ_A : The natural death rate of mosquitoes (aquatic phase).
B : The average daily biting (per day) of the mosquito.
β_{mh} : The transmission probability (per bite) from infected mosquitoes to humans.
β_{hm} : The transmission probability (per bite) from infected humans to mosquitoes.
φ : The number of eggs at each deposit per capita (per day).
η_h : The recovery rate of the human population.
η_A : The maturation rate from larvae to adult mosquitoes (per day).
α_h : The death by the disease rate of humans.

In the human population, the model can be explained as follows. At the susceptible compartment, the recruitment rate (birth or immigration) can increase the number of susceptible. However, the disease transmission rate due to the contact with infected mosquitoes through bitings and the natural death rate can decrease the number of susceptible. At the infected compartment, the disease transmission rate due to the contact with infected mosquitoes through bitings can increase the number of infected. However, the natural death rate, death by the disease rate, and recovery rate can decrease the number of infected. At the recovered compartment, the recovery rate can increase the number of recovered. However, the natural death rate can decrease the number of recovered.

In the mosquito population including larva in aqua phase, the model can be explained as follows. At the larvae compartment, the recruitment rate can increase the number of larvae. However, the maturation rate and natural death rate can decrease the number of larvae. At the susceptible compartment, the maturation rate of larvae can increase the number of susceptible. However, the disease transmission rate due to the contact with infected humans through bitings and the natural death rate can decrease the number of susceptible. At the infected compartment, the disease transmission rate due to the contact with infected humans through bitings can increase the number of infected. However, the natural death rate can decrease the number of infected.

In addition, there are the control function of susceptible humans vaccinated, u_1, the control function of infected humans treated, u_2, and the control function of larvae killed by abateseae, u_3. The effectiveness range of u_1, u_2 and u_3 is $[0, 1]$, where the value 0 means the control functions fail or are not applied, and the value 1 means the control functions are successful or applied entirely.

The objective function which will be minimized is

$$J(u,v)=\int_0^{t_f}\left(W_1 I_h(t)^2 + W_2 A_m(t)^2 + W_3 I_m(t)^2 + W_4 u_1(t)^2 + W_5 u_2(t)^2 + W_6 u_3(t)^2\right) dt \tag{9}$$

with the weights $W_1 > 0, W_2 > 0, W_3 > 0, W_4 > 0, W_5 > 0, W_6 > 0$. From the model, we want to minimize the number of infected humans, the number of larvae, the number of infected mosquitoes, the cost of vaccination, the cost of treatment, and the cost of abateseae.

The goal is finding u_1^*, u_2^*, u_3^* such that

$$J(u_1^*, u_2^*, u_3^*) = \min(J(u_1, u_2, u_3)). \tag{10}$$

2.2 Pontryagin's maximum principle

If u_1^*, u_2^*, u_3^* are the optimal control, there exist the adjoint variables

$$\begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \end{pmatrix}$$

which satisfy the following [3]:

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial S_h} = \lambda_1 B\beta_{mh}\frac{I_m}{N_h} + \lambda_1\mu_h + \lambda_1 u_1 - \lambda_2 B\beta_{mh}\frac{I_m}{N_h} - \lambda_3 u_1, \tag{11}$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial I_h} = \lambda_2\eta_h + \lambda_2\mu_h + \lambda_2\alpha_h + \lambda_2 u_2 - \lambda_3\eta_h - \lambda_3 u_2 + \lambda_5 B\beta_{hm}\frac{S_m}{N_h} - \lambda_6 B\beta_{hm}\frac{S_m}{N_h} - 2W_1 I_h, \tag{12}$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial R_h} = \lambda_3\mu_h, \tag{13}$$

$$\frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial A_m} = \lambda_4\varphi\frac{S_m + I_m}{kN_h} + \lambda_4\eta_A + \lambda_4\mu_A + \lambda_4 u_3 - \lambda_5\eta_A - 2W_2 A_m, \tag{14}$$

$$\frac{d\lambda_5}{dt} = -\frac{\partial H}{\partial S_m} = -\lambda_4\varphi + \lambda_4\varphi\frac{A_m}{kN_h} + \lambda_5 B\beta_{hm}\frac{I_h}{N_h} + \lambda_5\mu_m - \lambda_6 B\beta_{hm}\frac{I_h}{N_h}, \tag{15}$$

$$\frac{d\lambda_6}{dt} = -\frac{\partial H}{\partial I_m} = \lambda_1 B\beta_{mh}\frac{S_h}{N_h} - \lambda_2 B\beta_{mh}\frac{S_h}{N_h} - \lambda_4\varphi + \lambda_4\varphi\frac{A_m}{kN_h} + \lambda_6\mu_m - 2W_3 I_m, \tag{16}$$

with the final conditions $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = \lambda_4(T) = \lambda_5(T) = \lambda_6(T) = 0$, where the Hamiltonian is

$$\begin{aligned} H \;=\; & W_1 I_h(t)^2 + W_2 A_m(t)^2 + W_3 I_m(t)^2 + W_4 u_1(t)^2 + W_5 u_2(t)^2 + W_6 u_3(t)^2 \\ & \lambda_1\left(\Lambda - B\beta_{mh}\frac{I_m}{N_h}S_h - \mu_h S_h - u_1 S_h\right) + \\ & \lambda_2\left(B\beta_{mh}\frac{I_m}{N_h}S_h - \eta_h I_h - \mu_h I_h - \alpha_h I_h - u_2 I_h\right) + \\ & \lambda_3\left(\eta_h I_h + u_1 S_h + u_2 I_h - \mu_h R_h\right) + \end{aligned}$$

$$\lambda_4 \left(\varphi \left(1 - \frac{A_m}{kN_h} \right) (S_m + I_m) - \eta_A A_m - \mu_A A_m - u_3 A_m \right) +$$
$$\lambda_5 \left(\eta_A A_m - B\beta_{hm} \frac{I_h}{N_h} S_m - \mu_m S_m \right) + \lambda_6 \left(B\beta_{hm} \frac{I_h}{N_h} S_m - \mu_m I_m \right). \quad (17)$$

Furthermore, we can find the optimal control u_1^*, u_2^*, u_3^* :

$$\begin{aligned}
\frac{\partial H}{\partial u_1} &= 0, \\
2W_4 u_1 - \lambda_1 S_h + \lambda_3 S_h &= 0, \\
u_1 &= \min \left(1, \max \left(0, \frac{(\lambda_1 - \lambda_3) S_h}{2W_4} \right) \right), \\
\frac{\partial H}{\partial u_2} &= 0, \\
2W_5 u_2 - \lambda_2 I_h + \lambda_3 I_h &= 0, \\
u_2 &= \min \left(1, \max \left(0, \frac{(\lambda_2 - \lambda_3) I_h}{2W_5} \right) \right), \\
\frac{\partial H}{\partial u_3} &= 0, \\
2W_6 u_3 - \lambda_4 A_m &= 0, \\
u_3 &= \min \left(1, \max \left(0, \frac{\lambda_4 A_m}{2W_6} \right) \right). \qquad (18)
\end{aligned}$$

2.3 Forward-backward sweep method

The forward backward sweep method applied to the optimal control dengue fever model can be designed as follows [3]. Suppose the state variables and the adjoint variables are

$$f_1 = \frac{dS_h}{dt}, f_2 = \frac{dI_h}{dt}, f_3 = \frac{dR_h}{dt}, f_4 = \frac{dA_m}{dt}, f_5 = \frac{dS_m}{dt}, f_6 = \frac{dI_m}{dt},$$

$$g_1 = \frac{d\lambda_1}{dt}, g_2 = \frac{d\lambda_2}{dt}, g_3 = \frac{d\lambda_3}{dt}, g_4 = \frac{d\lambda_4}{dt}, g_5 = \frac{d\lambda_5}{dt}, g_6 = \frac{d\lambda_6}{dt}.$$

The algorithm to compute the objective function as the fitness function with the parameter weights $W_1 > 0, W_2 > 0, W_3 > 0, W_4 > 0, W_5 > 0, W_6 > 0$ is:
control_dengue$(W_1, W_2, W_3, W_4, W_5, W_6)$ while (process has not converged yet) $u_{old} = 0$.

1. Compute the solution of state variables forward with the initial condition $x(0) = (S_h(0), I_h(0), R_h(0), A_m(0), S_m(0), I_m(0))$ using the Runge-Kutta fourth-order method:

$$k_{1i} = f_i \left(t, x_i(t), u_1(t), u_2(t), u_3(t) \right), i = 1, 2, \ldots, 6,$$

$$\begin{aligned}
k_{2i} &= f_i \left(t + \frac{h}{2}, x_i(t) + \frac{h}{2} k_{1i}, \frac{u_1(t) + u_1(t+h)}{2}, \right. \\
&\qquad \left. \frac{u_2(t) + u_2(t+h)}{2}, \frac{u_3(t) + u_3(t+h)}{2} \right), i = 1, 2, \ldots, 6,
\end{aligned}$$

$$k_{3i} = f_i\left(t+\frac{h}{2}, x_i(t)+\frac{h}{2}k_{2i}, \frac{u_1(t)+u_1(t+h)}{2}, \frac{u_2(t)+u_2(t+h)}{2}, \frac{u_3(t)+u_3(t+h)}{2}\right), i=1,2,\ldots,6,$$

$$k_{4i} = f\left(t+h, x_i(t)+hk_{3i}, u_1(t+h), u_2(t+h), u_3(t+h)\right), i=1,2,\ldots,6,$$

$$x_i(t+h) = x_i(t)+\frac{h}{6}\left(k_{1i}+2k_{2i}+2k_{3i}+k_{4i}\right), i=1,2,\ldots,6.$$

2. Compute the solution of adjoint variables backward with the final condition $\lambda(0) = (\lambda_1(T), \lambda_2(T), \lambda_3(T), \lambda_4(T), \lambda_5(T), \lambda_6(T))$ using the Runge-Kutta fourth order method:

$$k_{1i} = g_i\left(t, x_i(t), \lambda_1(t), u_2(t), u_3(t)\right), i=1,2,\ldots,6,$$

$$k_{2i} = g_i\left(t-\frac{h}{2}, \frac{x_i(t)+x_i(t-h)}{2}, \lambda_i(t)-\frac{h}{2}k_{1i}, \frac{u_1(t)+u_1(t-h)}{2}, \frac{u_2(t)+u_2(t-h)}{2}, \frac{u_3(t)+u_3(t-h)}{2}\right), i=1,2,\ldots,6,$$

$$k_{3i} = g_i\left(t-\frac{h}{2}, \frac{x_i(t)+x_i(t-h)}{2}, \lambda_i(t)-\frac{h}{2}k_{2i}, \frac{u_1(t)+u_1(t-h)}{2}, \frac{u_2(t)+u_2(t-h)}{2}, \frac{u_3(t)+u_3(t-h)}{2}\right), i=1,2,\ldots,6,$$

$$k_{4i}=g_i\left(t-h, x_i(t-h), \lambda_i(t)-hk_{3i}, u_1(t-h), u_2(t-h), u_3(t-h)\right), i=1,2,\ldots,6.$$

$$\lambda_i(t-h) = \lambda_i(t)+\frac{h}{6}\left(k_{1i}+2k_{2i}+2k_{3i}+k_{4i}\right), i=1,2,\ldots,6.$$

3. Compute the optimal control u_1^*, u_2^*, u_3^* using equations (18).

4. Update the optimal control

$$u_1 \leftarrow \frac{u_1+u_{1,old}}{2}, u_2 \leftarrow \frac{u_2+u_{2,old}}{2}, u_3 \leftarrow \frac{u_3+u_{3,old}}{2} \tag{19}$$

End

5. Compute the objective function as the fitness function

$$J(u_1,u_2,u_3) = \sum_{k=0}^{T-1}\left(W_1I_h(k)^2+W_2A_m(k)^2+W_3I_m(k)^2+W_4u_1(k)^2+ W_5u_2(k)^2+W_6u_3(t)^2\right). \tag{20}$$

3 Firefly Algorithm

The Firefly Algorithm (FA) was discovered by Xin-She Yang in 2008. It is based on the behavior of flashing characteristics of fireflies. These insects communicate, search for a prey, and find mates using bioluminescence with varying flaying patterns. The FA is based on the rules [4]:

1. All fireflies are unisex so they attract one another.
2. Attractiveness is proportional to firefly brightness. For any couple of flashing fireflies, the less bright one will move toward the brighter one. Attractiveness is proportional to brightness and they both decrease as their distance increases. If there is no a brighter one than a particular firefly, it will move randomly.

The brightness of a firefly is affected or determined by the landscape of the objective function. In the FA, the attractiveness of a firefly is assumed to be determined by its brightness which is related to the objective function. The brightness of a firefly at a particular location x can be chosen as $f(x)$, where $f(x)$ is the objective function. However, if the attractiveness β is relative, it should be judged by the other fireflies. Thus, it will vary with the distance r_{ij} between the firefly i and the firefly j.

In this algorithm, the weights used are $W_1, W_2, W_3, W_4, W_5, W_6$ related to the number of infected humans, the number of larvae, the number of infected mosquitoes, the cost of vaccination, the cost of treatment, and the cost of abateseae, respectively.

The overall algorithm for optimizing the weights $W_1, W_2, W_3, W_4, W_5, W_6$ using the FA is as follows:

1. Generate the initial population position of fireflies $x^i = \left(W_1^i, W_2^i, W_3^i, W_4^i, W_5^i, W_6^i\right), i = 1, 2, \ldots, \max pop$, and compute the fitness value
$$f(x^i) = control\_dengue \left(W_1^i, W_2^i, W_3^i, W_4^i, W_5^i, W_6^i\right) i = 1, 2, \ldots, \max pop.$$
2. Determine the best firefly in the population with its position
$$i^{\min} \leftarrow arg \min_i \left(f(x^i), i = 1, 2, \ldots, \max pop\right), \tag{21}$$
$$x^{i^{\min}} \leftarrow arg \min_{x^i} \left(f(x^i), i = 1, 2, \ldots, \max pop\right). \tag{22}$$
3. Do the iteration as follows:
$for\ i = 1 : \max pop$
$for\ j = 1 : \max pop$
$if\ (f(x)^j < f(x^i))$.

   a. Compute the distance between the firefly i and the firefly j
$$r_{ij} = \left\|x^i - x^j\right\| = \sqrt{\sum_{t=1}^{T} \left(x_t^i - x_t^j\right)^2}.$$
   b. Compute the attractiveness function of a firefly $\beta \leftarrow \beta_0 e^{-\gamma r_{ij}}$.

   c. Generate $u_i = \alpha \left(rand - \frac{1}{2}\right)$, with $rand \sim U(0, 1)$.

d. Update the movement of the firefly i

$$x^i \leftarrow (1-\beta)\, x^i + \beta x^j + u_i$$

end
end
end
Generate $u_{i^{\min}} = \alpha\left(rand - \frac{1}{2}\right)$, with $rand \sim U(0,1)$.
Update the movement of the best firefly

$$x^{i^{\min}} \leftarrow x^{i^{\min}} + u_{i^{\min}}.$$

4. Repeat step 3 until stopping criteria is achieved.

4 Simulation Results

Parameters used in the FA simulations are $\beta_0 = 1, \gamma = 5, \alpha = 0.1$ with the number of fireflies being 10 and maximum iterations being 50. Parameters used in the dengue fever model are [15], [16]:

Parameters	Value
The recruitment rate (birth or immigration) of the human population Λ	3
The natural death rate of humans μ_h	$\frac{1}{(70\times 365)}$
The natural death rate of mosquitoes (adult phase) μ_m	0.0741
The natural death rate of mosquitoes (aquatic phase) μ_A	0.2
The average daily biting (per day) of the mosquito B	0.5
The transmission probability (per bite) from infected mosquitoes to humans β_{mh}	0.38
The transmission probability (per bite) from infected humans to mosquitoes β_{hm}	0.38
The number of eggs at each deposit per capita (per day) φ	3
The recovery rate of the human population η_h	0.17
The maturation rate from larvae to adult mosquitoes (per day) η_A	0.0541
The death by the disease rate of humans α_h	0.000457

Table 1: Parameters of the Dengue Fever Model.

The simulations of the optimal control dengue fever model can be seen in Figures 3-5, while Figure 2 is the FA simulation.

Figure 2 shows the optimization process of the FA. At the first iteration, the positions of fireflies are random. In the optimization process, we update the brightness of fireflies so that the fireflies move toward the brighter firefly with the minimum fitness function. Optimal weights obtained are $W_1 = 0.641, W_2 = 0.110, W_3 = 6.040, W_4 = 5.581, W_5 = 7.443, W_6 = 1.990$ with the minimum fitness being 4.529×10^{12}.

Figure 3 shows the numerical solution for larvae with and without control. The number of larvae with control is lower than that without control because of the abateseae effect which decreases the number of larvae. The decreasement of larvae will cause the decrease of the number of susceptible mosquitoes and infected mosquitoes.

Initial Value	Value
Susceptible humans $S_h(0)$	39850
Infected humans $I_h(0)$	50
Recovered humans $R_h(0)$	100
Larvae $A_m(0)$	50
Susceptible mosquitoes $S_m(0)$	1500
Infected mosquitoes $I_m(0)$	100

Table 2: Initial Value of Dengue Fever Model.

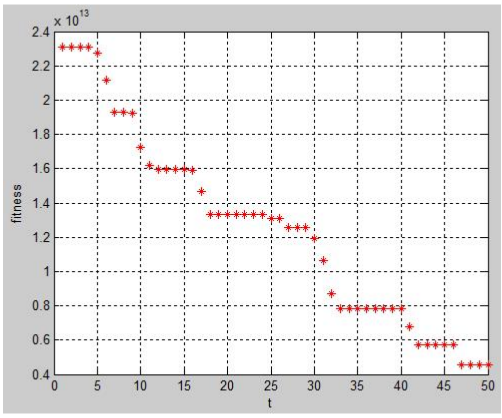

Figure 2: The FA Optimization Process.

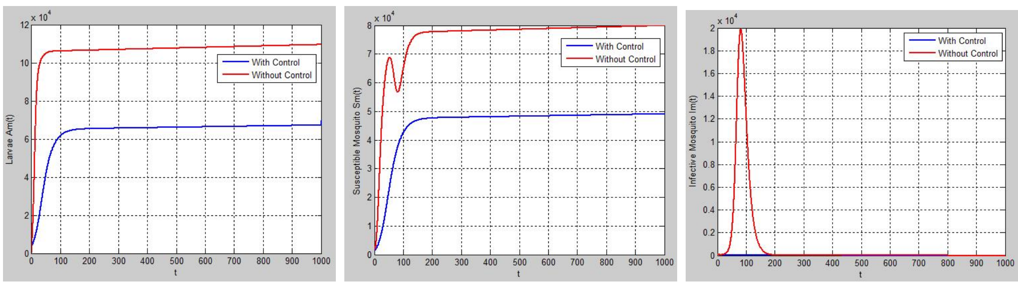

Figure 3: Numerical Solutions for Mosquitoes as a Vector. (a) Larvae. (b) Susceptible Mosquitoes. (c) Infected Mosquitoes.

Figure 4(a) shows the numerical solution for susceptible humans with and without control. The number of susceptible humans with control is lower than that without control because of the vaccination effect which decreases the number of susceptible humans. Figure 4(b) shows the numerical solution for infected humans with and without control. The number of infected humans with control is lower than that without control because of the treatment effect which decreases the number of infected humans. Figure 4(c) shows the numerical solution for recovered humans with and without control. The number of recovered humans with control is higher than that without control because of the vaccination and treatment effect which increases the number of recovered humans.

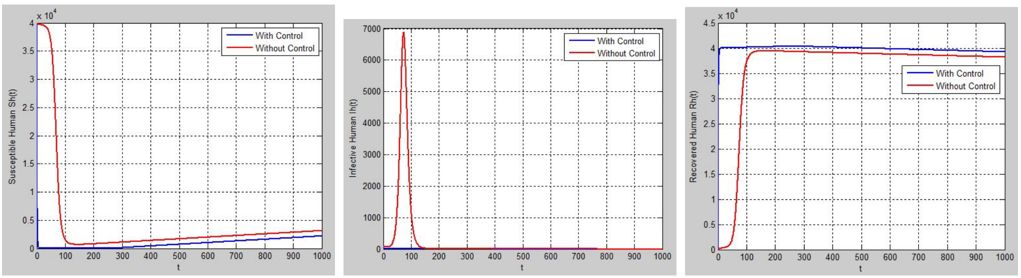

Figure 4: Numerical Solutions for Humans as a Host. (a) Susceptible Humans. (b) Infected Humans. (c) Recovered Humans.

Figure 5 shows the control function of vaccination, the control function of treatment and the control function of abateseae. Each of the control functions has the range of effectiveness between 0 to 1, where the value 0 means the control functions fail or are not applied and the value 1 means the control functions are successfull or applied entirely.

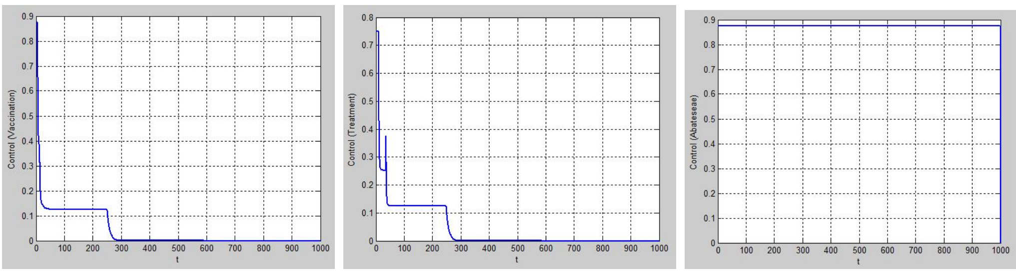

Figure 5: Control Function Solutions. (a) Vaccination. (b) Treatment. (c) Abateseae.

5 Conclusion

The FA can optimize the weights of the optimal control dengue fever model. From the simulations, the positions of fireflies are random. In the optimization process, we update the brightness of fireflies so that the fireflies move toward the brighter firefly with the minimum fitness function. When the FA has obtained optimal weights related to the number of infected humans, the number of larvae, the number of infected mosquitoes, the cost of vaccination, the cost of treatment, and the cost of abateseae, respectively, the optimal weights are applied in dengue fever simulation. Based on the parameters of the dengue fever model, we can compare the numerical solutions for larvae, susceptible mosquitoes, infected mosquitoes in the mosquito population and the susceptible humans, infected humans, and recovered human in the human population when the vaccination, treatment, and abateseae controls are applied.

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