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An Adaptive Step Size for Chaotic Local Search Algorithm

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Abstract: In this paper, a chaotic strategy based on a 2-D chaotic application is proposed. This method reduces the search space of optimized variables and improves the search precision, which has higher search efficiency. In order to solve the problem between fast convergence and low steady-state, a suitable step size control is proposed. The simulation results show that the new algorithm has faster convergence.

Keywords: *chaos; global optimization; evolutionary algorithms; step size control; chaos optimization algorithm.*

Mathematics Subject Classification (2010): 34D45, 70K55.

1 Introduction

Chaos is one of the few concepts in mathematics that cannot usually be defined in a word or statement. The study of chaos has been rapidly developed since Lorenz's influential book [7], and Li and York's pioneer paper [8]. R. L. Devaney has been provided one of the most popular and accepted definitions of chaos, in which chaotic systems exhibit a sensitive dependence on the initial conditions, topological transitivity, and dense periodic orbits [2]. Recently, there has been an increasing interest in controlling and utilizing chaos, particularly among the physicists, mathematicians, engineering and technological communities. The noun "chaos" and the adjective "chaotic" are used to describe the time behavior of a system when this behavior is a sensitive dependence on the initial conditions, aperiodic (it never exactly repeats), and apparently random or "noisy". The key word here is apparently. Underlying this apparent chaotic randomness is an order determined, in some sense, by the equation describing the system [7–9,11]. The

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combination of optimization methods and fundamentals of chaotic systems has attracted an increased interest in various fields in recent years. The chaos optimization algorithm is a new global optimization, which used chaotic variables directly in the search for the optimal solution. Referring to the properties of chaotic systems, it is clear that the ergodicity, self-similarity, regularity, and intrinsic stochastic property of chaos make it more possible to obtain the global optimal solution by the chaos optimization than by the method adopted before. It can more easily escape from local minima than other stochastic algorithms. The optimization algorithms based on the chaos theory are search methodologies that differ from any of the existing traditional stochastic optimization techniques [2–4, 13]. So, the chaos optimization algorithm (COA) is used to greatly reduce computational cost and select the optimal threshold value, and finally, to enhance segmentation performance [5, 6, 12]. The paper is organized as follows: in the next section, we introduce the proposed approach with a new strategy based on two phases of global/local chaotic search using the Gingerbreadman map. In Section 2, illustrative examples with the discussion of the results are presented and conclusions are offerred.

2 The Principal of Chaos Optimization

Non-linear systems with complex dynamics have lately been the subject of intense research and exploration, giving birth to *chaos theory.* Chaotic systems are deterministic systems that exhibit irregular behavior and sensitive dependence on the initial conditions. Chaos theory studies the behavior of systems that follow deterministic laws but appear random and unpredictable, i.e., dynamical systems. Chaotic variables can go through all states in certain ranges according to their own regularity without repetition [8]. A chaotic map is a map that exhibits some type of chaotic behavior. In this work, we applied a chaotic map that is common in the literature, namely, the Gingerbreadman map. The mathematical form of a chaotic two-dimensional map, which maps the unit square $I \times I$, where $I = [0, 1]$, onto itself in a one-to-one manner, is chosen.
Later on, we will use this map in the chaotic searches.

2.1 Chaos model

In most COA methods [3], chaos variables are generated by the logistic map [1, 2]. It is possible to change the form of this map to obtain other chaotic attractors, but in this paper, we assume a Gingerbreadman two-dimensional discrete map can generate chaos variables. The Gingerbreadman map is a discrete-time dynamical system [9–11]. It is one of the most studied examples of dynamical systems that exhibit chaotic behavior. The Gingerbreadman map takes a point (x_n, y_n) on the plane and maps it to a new point

$$\begin{cases} y_1(k) = 1 - a(y_1(k-1))^2 + by(k-1), \\ y(k) = y_1(k-1), \end{cases} \tag{1}$$

where k is the iteration number. In this work, the values of y are normalized in the range $[0, 1]$ to each decision variable in the uni-dimensional space of the optimization problem. This transformation is given by

$$z_i(k) = \frac{(x_i(k) - L_i)}{(U_i - L_i)}.$$

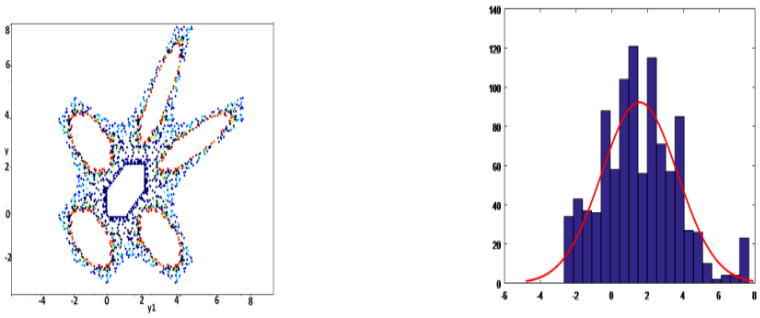

Figure 1: A chaotic Gingerbreadman attractor obtained for a = 1 and b = 1.

The parameters used in this work are $a = 1$ and $b = 1$, these values are suggested by (1). An example of the evolution of a new map is shown in Fig.1. The properties of stochastic sensitivity to the initial value and ergodicity of the two-dimensional map (1) are expressed in Fig.1 by iterating 1000 times.

3 Design of the Algorithm

Recently, the idea of using chaotic sequences instead of random sequences has been noticed in the research field such as chaos optimization. Li and Jiang [3] presented a chaos optimization algorithm (COA) that can solve complex optimization problems. The most important advantages of the COA are summarized as: easy implementation, short execution time, and speed-up of the search. Observations, however, reveal that the COA also has some problems including: (i) the COA is effective only for small decision spaces; (ii) the COA easily converges in the early stages of the search process [8]. Figure 2 shows the flowchart of the proposed algorithm.

Consider the following optimization problem on the minimum of functions. If the target function $f(x_i)$ is continuous and differentiable, the object problem to be optimized is find x_i to minimize $f(x_i); x_i \in [L_i, U_i]; i = 1, 2, ..., n$.

The main procedures of this algorithm are shown as follows:

Input :

M_g : maximum number of iterations of the global search.
M_l : maximum number of iterations of the local search.
$M_l + M_g$: stopping criterion of the chaotic optimization method in iterations.
λ : step size in the chaotic local search.

Output :

X^* : best solution from the current run of the chaotic search.
f^* : best objective function (minimization problem).
Then the basic steps of the chaos optimization algorithm based on the chaos variable from chaos map (1) are expressed as follows [2]:

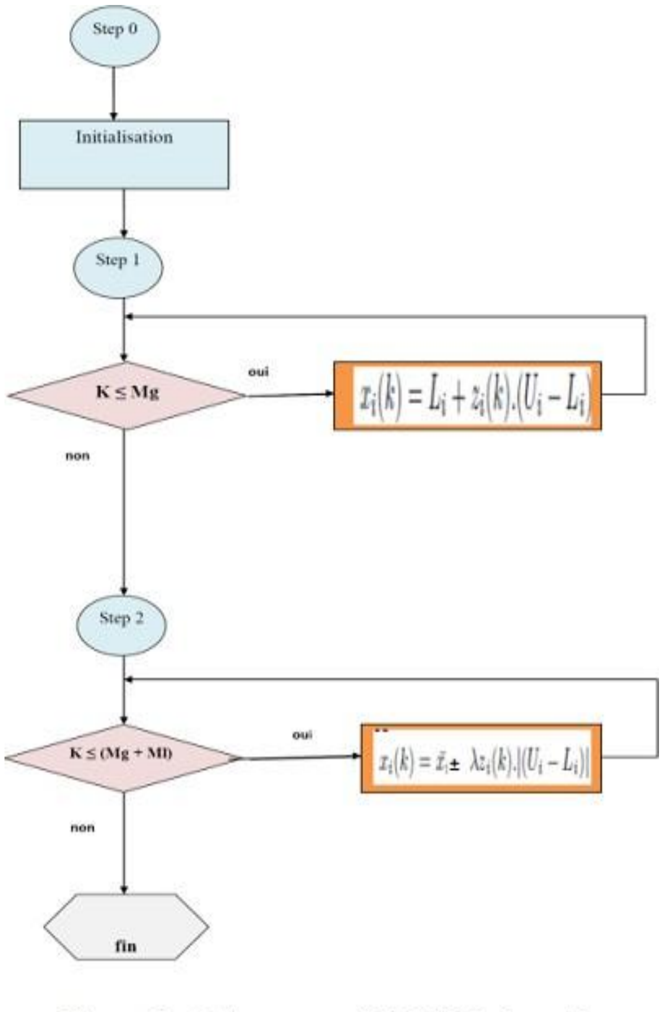

Figure 2: Diagram of the COA.

3.1 Step-size control

It is well-established that the convergence of a chaos optimization algorithm directly depends on how it controls the step size. Moreover, the step-size control influences to a large extent the rate at which a chaos optimization algorithm approaches the optimum. The step-size adaptation mechanisms are all based on the idea that the smaller the step size, the higher the probability of sampling good solutions.

4 Numerical Results

In order to verify the typical function of this paper to optimize the effectiveness of the algorithm, the 4-target function expression is as follows [14, 15]:

1. F_1 is the Rosenbrock function,

$$F_1 = 100(x_1^2 - x_2)^2 + (1 - x_1)^2. \tag{2}$$

- Search domain : $-2.048 \leq x_i \leq 2.048, i = 1, 2$.
- Number of local minima : no local minima except the global one.
- The global minima : $\bar{x} = (1, 1), f(\bar{x}) = 0$.

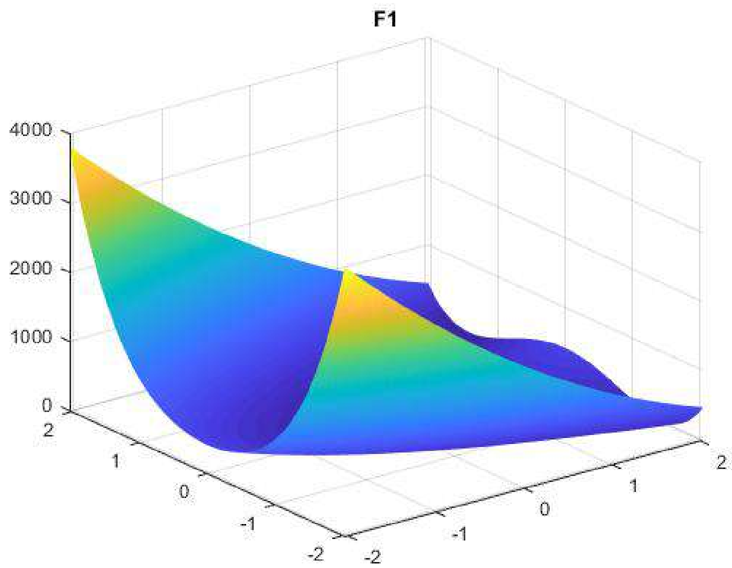

Figure 3: The Rosenbrock function.

2. F_2 is the Goldstein-Price function,

$$\begin{aligned} F_2 = (1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)) \\ (30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_12 + 48x_2 - 36x_1x_2 + 27x_2^2)). \end{aligned} \quad (3)$$

- Search domain : $-2 \leq x_i \leq 2, i = 1, 2$.
- Number of local minima: several local minima.
- The global minima : $\bar{x} = (0, 1), f(\bar{x}) = 3$.

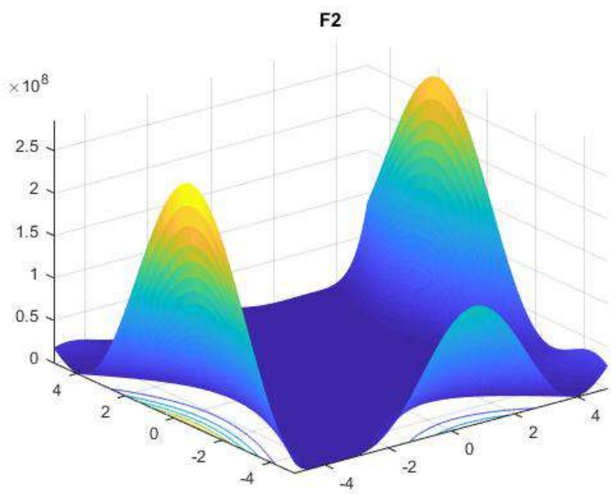

Figure 4: The Goldstein-Price function.

3. F_3 is the Easom function,

$$F_3 = -cos(x_1)cos(x_2)exp(-(x_1 - pi)^2 - (x_2 - pi)^2). \quad (4)$$

- Search domain : $-10 \leq x_i \leq 10, i = 1, 2$.

- Number of local minima : several local minima.
- The global minima : $\bar{x} = (\pi, \pi), f(\bar{x}) = -1$.

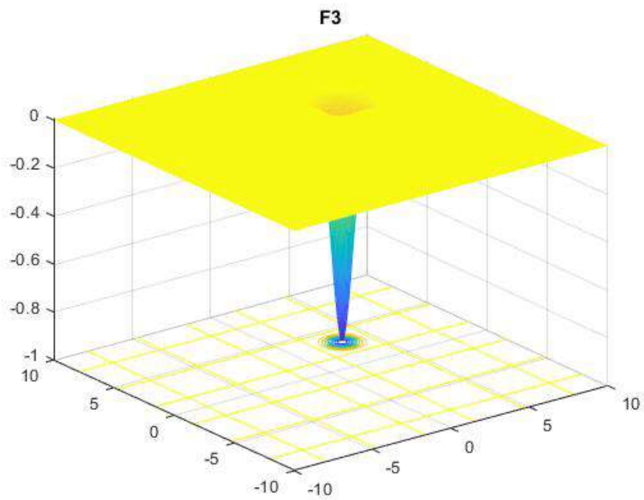

Figure 5: The Easom function

4. F_4 is the Schaffer function,

$$F_4 = -0.5 + (\frac{((sin\sqrt{(x_1^2 + x_2^2)})^2 - 0.5)}{(1 + .001(x_1^2 + x_2^2))^2}). \quad (5)$$

- Search domain : $-4 \leq x_i \leq 4, i = 1, 2$.
- Number of local maxima : infinite local maxima.
- The global maximum: $\bar{x} = (0, 0), f(\bar{x}) = -1$.

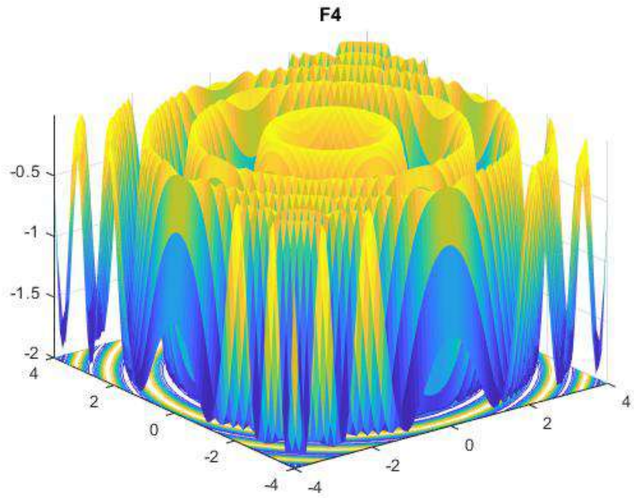

Figure 6: The Schaffer function.

Function F_1 is the Rosenbrock function, which has global minima $\bar{x} = (1, 1)$, and optimal objective function value $F_1(\bar{x}) = 0$. The function F_2 is the Goldstein-Price function, which has infinite local minima and one global minimum $\bar{x} = (0, 1); F_2(\bar{x}) = 3$. The function F_3 is the Easom function, which has many local minima and one global minimum $\bar{x} = (-\pi, \pi)$ and $F_3(\bar{x}) = -1$. The function F_4 is the Schaffer function, which has infinite local maxima and one global maximum $\bar{x} = (0, 0)$, and $F_4(\bar{x}) = -1$. These four nonlinear multimodal functions are often used to test the convergence, efficiency, and accuracy of the optimization algorithms [3].

During the chaotic local search, the step size λ is an important parameter in the convergence behavior of the optimization method, which adjusts small ergodic ranges around. The step size λ is employed to control the impact of the current best solution on the generating of a new trial solution. A small λ tends to perform exploitation to refine results by local search, while a large one tends to facilitate a global exploration of search space [2,13]. A suitable value for the step size λ usually provides a balance between global and local exploration abilities and consequently, a reduction of the number of iterations required to locate the optimum solution.

In this work, using the same number of function evaluations : $M_g + M_l$, we perform 50 runs with different initial conditions for the mapping of the tested values of step size in the chaotic optimization method based on the Gingerbreadman map which are described as follows:

- $\lambda = 0.1; M_g = 2200; M_l = 300.$

	Best Value	Mean Value	Std. Dev	$(\bar{x_1}, \bar{x_2})$	Time
F_1	0.0000	0.0003	0.0003	(0.9913, 0.9831)	33.9249s
F_2	3.0031	3.0031	0.0000	(0.0032, -0.9978)	34.4000s
F_3	-0.9961	-0.9961	0.0000	(3.1364 , 3.0906)	37.6054s
F_4	-0.9993	-0.9989	0.0001	(-0.0310, 0.0080)	35.6340s

Table 1: The COA based on the Gingerbreadman map.

- $\lambda = 0.001; M_g = 2200; M_l = 300.$

	Best Value	Mean Value	Std. Dev	$(\bar{x_1}, \bar{x_2})$	Time
F_1	0.0000	0.0000	0.0000	(1.0001, 1.0002)	33.8875s
F_2	3.0000	3.0000	0.0000	(-0.0001, -1.0001)	33.9260s
F_3	-1.0000	-0.9997	0.0001	(3.1438 , 3.1411)	37.1161s
F_4	-0.9999	-0.9998	0.0001	(-0.0006, -0.0017)	35.7883s

Table 2: The COA based on the Gingerbreadman map.

- $0.001 \leq \lambda \leq 0.1; M_g = 2200; M_l = 300.$

	Best Value	Mean Value	Std. Dev	$(\bar{x_1}, \bar{x_2})$	Time
F_1	0.0000	0.0001	0.0001	(1.0044, 1.0088)	35.2739s
F_2	3.0000	3.0027	0.0010	(0.0024, -0.9985)	35.4629s
F_3	-0.9994	-0.9963	0.0008	(3.1374 , 3.0952)	38.2361s
F_4	-0.9996	-0.9989	0.0001	(-0.0290 , 0.0063)	36.9733s

Table 3: The COA based on the Gingerbreadman map.

5 Conclution

The chaos optimization method based on the Gingerbreadman map (COGM methodologies) was successfully validated for testing four different cost functions. From the case studies and comparison of the results through three tested COGM approaches it has been shown that the parameter of step size λ is essential for the good convergence profile. In this context, the parameter λ regulates the trade-off between the global and local exploration abilities of the chaotic local search. However, the future works will include a detailed study of self-adaptive heuristics for the step size design.

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