Nonlinear Dynamics and Systems Theory, **23** (3) (2023) 283–294

On the Dynamics and FSHP Synchronization of a New Chaotic 3-D System with Three Nonlinearities

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Received: January 12, 2023; Revised: May 26, 2023

Abstract: This paper reports on a novel chaotic system with three nonlinearities. Firsltly, some properties of the system are studied including equilibrium points and their stability, the Lyapunov exponent and Kaplan-Yorke dimension. Also, the system dynamics are studied by numerical mathematical tools, namely, the Lyapunov exponent spectrum, bifurcation diagrams and 0-1 test. Also, we have studied a type of synchronization, a full-state hybrid projective synchronization (FSHPS), between master and slave chaotic systems. We design suitable controllers to achieve this type of synchronization by using the Lyapunov stability criteria of the integer-order linear system. Finally, the effectiveness of the proposed scheme for this type of synchronization is demonstrated by an illustrative example with numerical simulation in Matlab.

Keywords: *chaotic system; strange attractor; Lyapunov exponent; Lyapunov stability theory; adaptive control; synchronization.*

Mathematics Subject Classification (2010): 34C28, 34D08, 37B25, 37B55, 37D45, 93D05, 93D20.

1 Introduction

In the fields of nonlinear systems dynamics and Chaos theory, a chaotic system is a nonlinear deterministic system that displays a complex, unpredictable behavior and extreme sensitivity to initial conditions. Chaotic systems are applied in many disciplines such as biology, ecology, economics, science and engineering [1-4], etc. They have many different and common application areas such as neural networks, image and sound encryption, robotics, cryptography and secure communication [5-13]. In 1963, Lorenz discovered the

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first 3-d chaotic system [14]. After that, several chaotic systems have been designed by many researchers, they are: the Rossler, Chen, Zhou, Vaidyanathan, Yu-Wang, Hannachi systems [15-20], etc. After the work done by Pecora and Carroll [21], the chaos synchronization between chaotic systems has been extensively studied by different theoretical and experimental methods and due to the powerful and multiple applications of synchronization using chaotic systems in various fields such as secure communication, telecommunication, cryptography and encryption [22-26], the study of chaos and synchronization in dynamical systems has attracted a considerable attention, and an intense competition has begun among researchers for finding new chaotic systems and developing different types and methods of synchronization for those systems. The synchronization of chaotic systems has been presented in diverse works, where different techniques were employed to synchronize two chaotic systems. In recent years, we find that diverse types and methods of synchronization have been developed, among them there are the active control [27-28], sliding mode control [29-31], backstepping control [32], adaptive control [33-36], function projective synchronization [37], modified projective synchronization [38], hyprid projective synchronization [39], full state hybrid projective synchronization [40], inverse full state hybrid projective synchronization [41]. In this work, a new 3-D chaotic system with three nonlinearities is introduced. Basic dynamical properties of this new chaotic system are studied, namely, the equilibrium points and their stability, dissipativity and Lyapunov exponent, Lyapunov exponent spectrum, Kaplan-Yorke dimension, bifurcations. Also, we have studied a type of synchronization, a full-state hybrid projective synchronization (FSHPS), using the new systems. We design suitable controllers to achieve this type of synchronization by using the Lyapunov stability criteria of the integer-order linear system. Finally, the effectiveness of the proposed scheme for this type of synchronization is demonstrated by an illustrative example with numerical simulation in Matlab.

This paper is organized as follows. In Section 2, a description of the novel chaotic system is given. In Sections 3, the FSHP synchronization using the new chaotic system is investigated. The new system and another new system are used in Section 4 to demonstrate the effectiveness of the proposed method. Finally, the conclusion is given in Section 4.

1.1 Description of the novel chaotic system

A novel 3-D autonomous chaotic system is expressed as follows:

$$\begin{cases} \frac{dx}{dt} = a(y-x), \\ \frac{dy}{dt} = cx - y - xz, \\ \frac{dz}{dt} = e^{xy} - y^2 - bz, \end{cases} \tag{1}$$

where x, y, z are the state variables and a, b, c are the positive real parameters.

There are eight terms on the right-hand side but it mainly relies on three nonlinearities, namely, e^{xy}, y^2 and xz, respectively.

System (1) can generate a new double scroll strange attractor for the parameters $a = 10, b = 3, c = 35$ with the initial conditions $[1, 1, 1]$ as displayed in Figs.2-3. We note the new chaotic attractor is different from that of the Lorenz system or any existing systems.

1.2 Basic properties

In this section, some basic properties of the system (1) are given. We start with the equilibrium points of the system and check their stability at the initial values of the parameters a, b, c.

1.3 Equilibrium points

Put the equations of the system (1) equal to zero, i.e.,

$$\begin{cases} a(y-x)=0, \\ cx-y-xz=0, \\ e^{xy}-y^2-bz=0. \end{cases} \tag{2}$$

A simple calculation yields the unique equilibrium point

$$p_1=\left(0,0,\frac{1}{3}\right). \tag{3}$$

1.4 Stability

In order to check the stability of the equilibrium points, we derive the Jacobian matrix at a point $p\,(x,y,z)$ of the system (1):

$$J(p)=\begin{pmatrix} -a & a & 0 \\ c-z & -1 & -x \\ ye^{xy} & -2y+xe^{xy} & -b \end{pmatrix}. \tag{4}$$

For p_1, we obtain three eigenvalues:

$$\lambda_1=\frac{1}{6}\sqrt{3}\sqrt{4403}-\frac{11}{2},\lambda_2=-\frac{1}{6}\sqrt{3}\sqrt{4403}-\frac{11}{2},\lambda_3=-3. \tag{5}$$

Since all the eigenvalues are real, the Hartma-Grobman theorem implies that p_1 is a saddle point which is unstable according to the Lyapunov theorem on stability.

1.4.1 Lyapunov exponents and Kaplan-Yorke dimension

For the chosen parameter values of a, b, c, the Lyapunov exponents of the novel chaotic system (1) are obtained using Matlab with the initial conditions $(x\,(0)\,, y\,(0)\,, z\,(0))=(1,1,1)$ as

$$L_1=0.955333, L_2=-0.00158345, L_3=-14.9537. \tag{6}$$

Since the spectrum of Lyapunov exponents (6) has a maximal positive value L_1, it follows that the 3-D novel system (1) is a chaotic system. Moreover, the sum of all the Lyapunouv exponents is negative, which implies that the system is dissipative. The Kaplan-Yorke dimension of system (1) is calculated as

$$D_{KL}=2+\frac{L_1+L_2}{|L_3|}=2.0638. \tag{7}$$

The Lyapunov exponents spectrum and the chaotic attractor of system (1) in 2-D and 3-D are shown in Figs.1-3.

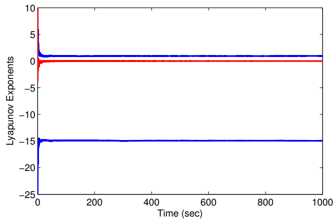

Figure 1: Lyapunov exponents spectrum.

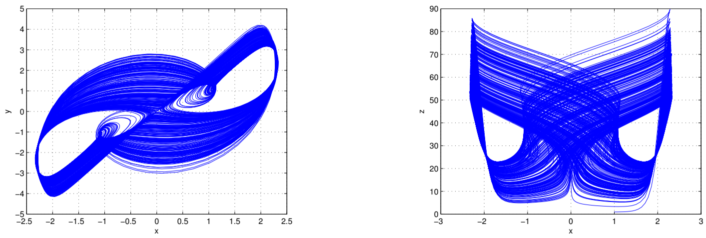

Figure 2: Chaotic attractor of system (1) in the x-y and x-z plane.

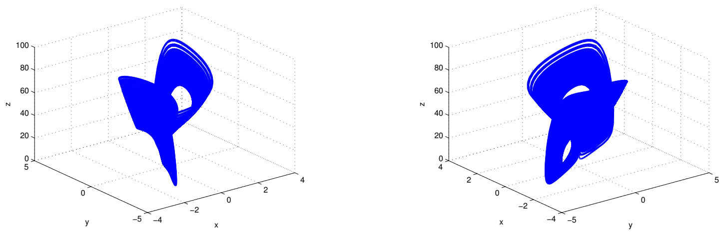

Figure 3: 3-D view of the chaotic attractor of system (1) in the x-y-z and y-x-z space.

2 Dynamics of the System

In this section, we investigate numerically the dynamical behavior of the system (1) using the largest Lyapunov exponents spectrum and bifurcation diagrams.

Figs.4-6 show the largest Lyapunov exponents spectrum and the bifurcation diagrams of system (1) with respect to the parameters a, b, c, respectively. Obviously, when $a \in [0, 20]$, $b \in [0, 10]$, $c \in [20, 40]$, the behavior of system (1) is either chaotic, periodic or converges to an equilibrium. When $a \in]1.07, 1.3] \cup]4, 8.7[\cup]9.3, 11.2[$, $b \in [2.6, 3.2] \cup [3.5, 10]$, $c \in [22.23, 22.6] \cup [23, 40]$, the maximum Lyapunov exponent is positive, implying that the new system (1) is chaotic in this range of parameters. For $a \in [11.47, 20]$, $b \in [1.25, 2.5]$, $c \in [20, 21.9]$, the maximum Lyapunov exponent almost always equals zero, implying that the new system (1) has a periodic orbit. The maximum Lyapunov exponent is negative when $a \in [0, 1.06] \cup [1.43, 4[\cup]11.2, 11.46[$, $b \in [2.3267697, 2.3886929] \cup$

$[3.2490194, 3.2743673] \cup [3.3644489, 3.437006] \cup [3.4626194, 3.4800358]$, $c \in]21.9, 22] \cup [22.8, 22.87]$, which means that the trajectories of the new system (1) is fall to converge to equilibria. Figs.7 shows the different behavior of system (1): for $a = 3.02$ converging to an equilbrium, for $c = 32$ chaotic, for $c = 21.9$ and $b = 1.75$ periodic.

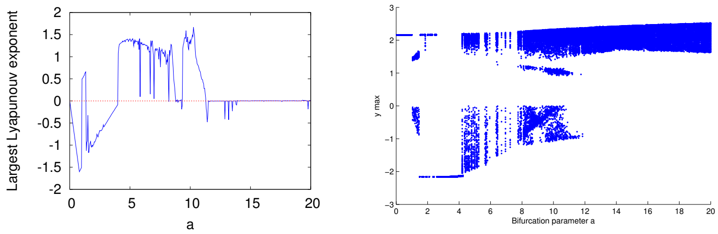

Figure 4: Lyapunov exponents spectrum and bifurcation diagram for $a \in [0, 20]$.

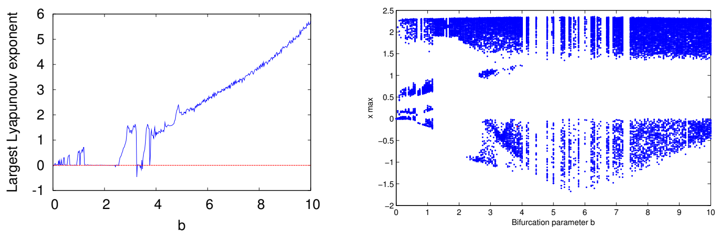

Figure 5: Lyapunov exponents spectrum and bifurcation diagram for $b \in [0, 10]$.

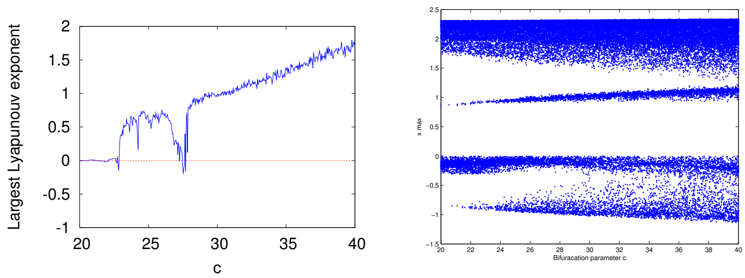

Figure 6: Lyapunov exponents spectrum and bifurcation diagram for $c \in [20, 40]$.

2.1 0-1 test for system (1)

The 0-1 test was proposed by Gottwald and Melbourne, it is a test approach for distinguishing regular and chaotic dynamics in deterministic dynamical systems [42]. This test depends on the rapport k_c, if it is close to one, then the system has a chaotic behavior and if it is close to zero, then the system has a regular behavior.

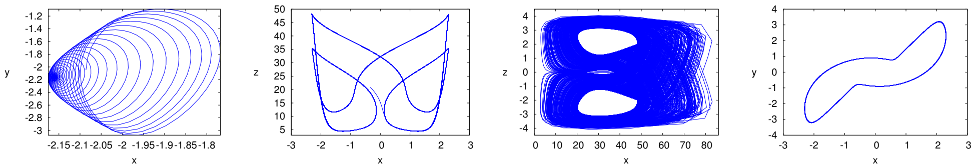

Figure 7: Different behavior of system (1): For a=3.02 converging to an equilbrium, for c=32 chaotic, for c=21.9 and b=1.75 periodic.

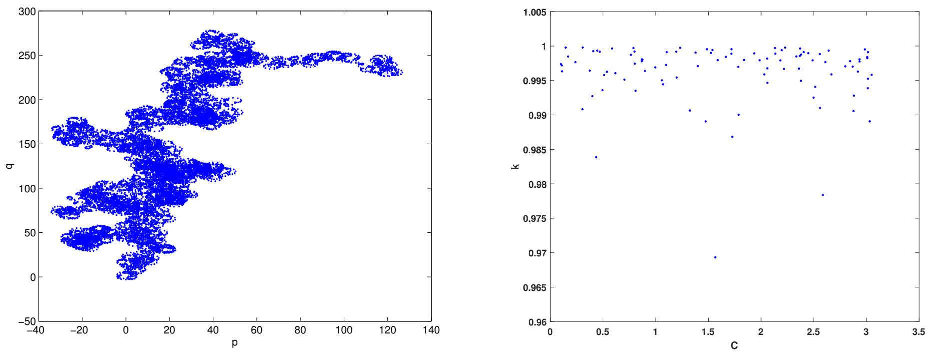

Figure 8: Brownian motion in the (p-q) plane and Kc Plot for the new system (1).

In Matlab, we choose the random constant ($C \in [0; \pi]$), as a result, we find the rapport $k_c = 0.9990$ which is close to one as shown in Fig.8, moreover, we obtain a Brownian motion in the (p-q) plane, which means that the novel system (1) has a chaotic behavior as shown in Fig.8.

3 Master-Slave Synchronization of Non-Identical 3-D Novel Chaotic Systems Using FSHP Method

We consider the drive system given by

$$\dot{x_i}(t) = f_i(X(t)), i = 1, .., n, \tag{8}$$

where $X(t) = (x_1, x_2, ..., x_n)^T$ is the state vector of the system (8), $f_i : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ for $i = 1, .., n$ are nonlinear functions, and the response system is the system given by

$$\dot{y_i}(t) = \sum_{j=1}^{n} b_{ij} y_j(t) + g_i(Y(t)) + u_i, \ i = 1, .., n, \tag{9}$$

where $Y(t) = (y_1, y_2, ..., y_n)^T$ is the state vector of the system (9), $g_i : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ for $i = 1, .., n$ are the nonlinear functions, u_i are the controllers to be designed so that the system (8) and the system (9) are synchronized.

Now, we introduce the definition of FSHPS [40] between master and slave systems.

Definition 3.1 FSHPS occurs between master and slave systems (8) and (9) when there are controllers u_i, i=1,2,...,n, and given real numbers $(\alpha_{ij})_{1\leq i,j\leq n}$ such that the synchronization errors

$$e_i\left(t\right) = y_i\left(t\right) - \sum_{j=1}^{n} \alpha_{ij} x_j\left(t\right),\ i = 1,..,n, \tag{10}$$

satisfy $\lim_{t\to+\infty} e_i\left(t\right) = 0$.

Full-state hybrid projective synchronization (FSHPS) is one of the most noticeable types. It has been widely used in the synchronization of chaotic systems. In this type of synchronization, each slave system state achieves synchronization with the linear combination of master and system states. The state errors for (8) and (9) are

$$e_i = y_i - \sum_{j=1}^{n} \alpha_{ij} x_j, i = 1,..,n. \tag{11}$$

Consequently, the error dynamic system is given by

$$\dot{e}_i = \left(\sum_{j=1}^{n} b_{ij} y_j\left(t\right) + g_i\left(Y\left(t\right)\right)\right) + U_i - \sum_{j=1}^{n} \alpha_{ij} f_j\left(X\left(t\right)\right),\ i = 1,..,n. \tag{12}$$

The error system can be described as

$$\dot{e}_i = \sum_{j=1}^{n} b_{ij} e_j\left(t\right) + \left(\sum_{j=1}^{n} b_{ij} y_j\left(t\right) - \sum_{j=1}^{n} b_{ij} e_j\left(t\right) + g_i\left(Y\left(t\right)\right)\right) + U_i - \sum_{j=1}^{n} \alpha_{ij} f_j\left(X\left(t\right)\right), \tag{13}$$

$i = 1,..,n$, i.e.,

$$\dot{e}_i = \sum_{j=1}^{n} b_{ij} e_j\left(t\right) + R_i + U_i, i = 1,..,n, \tag{14}$$

where

$$R_i = \left(\sum_{j=1}^{n} b_{ij} y_j\left(t\right) - \sum_{j=1}^{n} b_{ij} e_j\left(t\right) + g_i\left(Y\left(t\right)\right)\right) - \sum_{j=1}^{n} \alpha_{ij} f_j\left(X\left(t\right)\right), i = 1,..,n. \tag{15}$$

Rewrite error system (14) in the compact form

$$\dot{e} = Be + R + U, \tag{16}$$

where $B = (b_{ij})_{n\times n}$ and $e = (e_1, e_2, ..., e_n)^T, R = (R_i)_{1\leq i\leq n}, U = (U_i)_{1\leq i\leq n}$.

Theorem 3.1 *FSHPS between the master system (8) and the slave system (9) will occur under the following control law:*

$$U = -\left(R + Ce\right) \tag{17}$$

with C being a feedback gain matrix selected so that $B - C$ is a negative definite matrix.

Proof. By inserting (17) into (16), we get

$$\dot{e} = (B - C)\, e. \tag{18}$$

If we choose matrix C so that $B - C$ is negative, then all the eigenvalues $\lambda_i, i = 1, 2, 3,$ of $(B - C)$ stay in the left-half plane, i.e., $Re\,(\lambda_i) < 0$, which ensures, according to the Lyapunov stability theory, that error system (18) is asymptotically stable. Hence the synchronization between the sytem (8) and the sytem (9) is achieved.

4 Illustrative Example

In this section, the new system (1) and another new system (19) are used to demonstrate the effectiveness of the proposed method.

As a driving system, we considier the chaotic system [20] given by

$$\begin{cases} \frac{dx_1}{dt} = a(x_2 - x_1), \\ \frac{dx_2}{dt} = cx_1 - x_1x_3, \\ \frac{dx_3}{dt} = -x_1x_2 + b(x_1 - x_3), \end{cases} \tag{19}$$

where $a = 13$, $b = 2.5$, $c = 50$, and as a response system, we consider the controlled system of system (1) given by

$$\begin{cases} \frac{dy_1}{dt} = a(y_2 - y_1) + u_1, \\ \frac{dy_2}{dt} = cy_1 - y_2 - y_1y_3 + u_2, \\ \frac{dy_3}{dt} = e^{y_1y_2} - y_2^2 - by_3 + u_3, \end{cases} \tag{20}$$

where $a = 10$, $b = 3$, $c = 35$.

According to the above method, for FSHPS, we have

$$B = \begin{pmatrix} -10 & 10 & 0 \\ 35 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} \tag{21}$$

and the choice of $C = \begin{pmatrix} 0 & 10 & 0 \\ 35 & 4 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ and $(\alpha_{ij})_{1\leq i,j\leq 4} = \begin{pmatrix} 1 & 8 & 5 \\ 7 & 2 & 1 \\ 6 & 0 & -3 \end{pmatrix}$ yields

$$\begin{cases} R_1 = 10e_1 - 10e_2 - 399.5x_1 - 13x_2 + 12.5x_3 - 10y_1 + 10y_2 + 5x_1x_2 + 8x_1x_3, \\ R_2 = e_2 - 35e_1 - 11.5x_1 - 91x_2 + 2.5x_3 + 35y_1 - y_2 + x_1x_2 + 2x_1x_3 - y_1y_3, \\ R_3 = 3e_3 + 85.5x_1 - 78x_2 - 7.5x_3 - 3y_3 + e^{y_1y_2} - y_2^2 - 3x_1x_2 \end{cases} \tag{22}$$

and

$$(U_1, U_2, U_3)^T = -\left(R + C\,(e_1, e_2, e_3)^T\right) \tag{23}$$

$$= \begin{pmatrix} 399.5x_1 - 10e_1 + 13x_2 - 12.5x_3 + 10y_1 \\ -10y_2 - 5x_1x_2 - 8x_1x_3 \\ 11.5x_1 - 5e_2 + 91x_2 - 2.5x_3 - 35y_1 \\ +y_2 - x_1x_2 - 2x_1x_3 + y_1y_3 \\ 78x_2 - 85.5x_1 - e_3 + 7.5x_3 + 3y_3 - e^{y_1y_2} \\ +y_2^2 + 3x_1x_2 \end{pmatrix}. \tag{24}$$

The error system is given by

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} -10 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \tag{25}$$

then the eigenvalues of the matrix $(B - C)$ are given by $\lambda_1 = -10$, $\lambda_2 = -5$, $\lambda_3 = -1$, which all are negatives. Hence the error system is asymptotically stable [43] and the synchronization between the two systems (19) and (20) is achieved.

We used the classical fourth-order Runge-Kutta method with the step size $h = 10^{-6}$ to solve the system of differential equations (25). The initial conditions of the drive system (19) and the response system (20) are chosen as $(x_1(0), x_2(0), x_3(0)) = (20, 10, -10)$, $(y_1(0), y_2(0), y_3(0)) = (-20, -10, -50)$, respectivelly. $(e_1(0), e_2(0), e_3(0)) = (-70, -160, -200)$, $(z_1(0), z_2(0), z_3(0)) = (50, 150, 150)$ with $z_i = \sum_{j=1}^{3} \alpha_{ij} x_j, i = 1, 2, 3.$ In Fig.10, the time-history of the synchronization errors $e_1(t); e_2(t); e_3(t)$ is depicted.

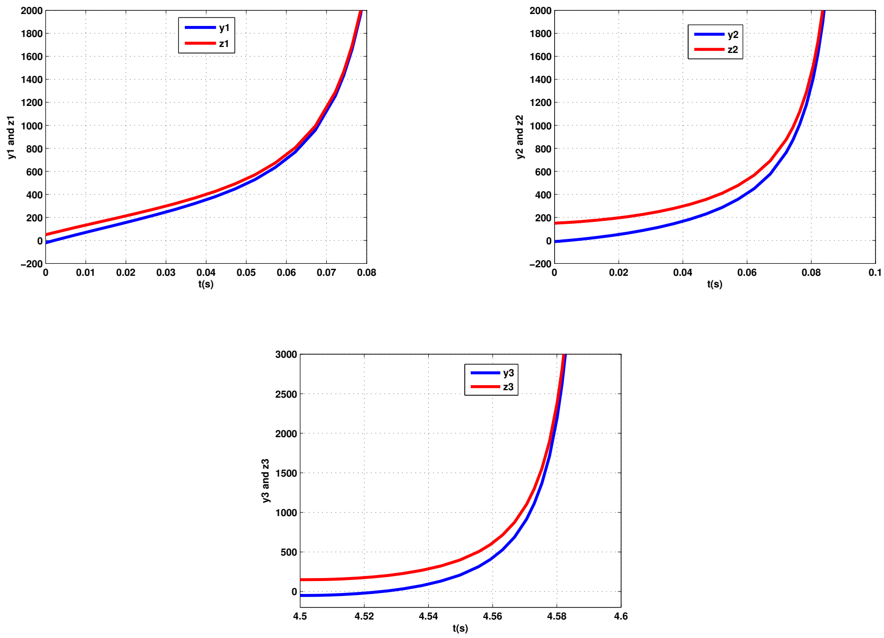

Figure 9: Synchronization between $z_i, y_i, i = 1, 2, 3$.

5 Conclusion

In this work, a new 3-D chaotic system with three nonlinearities is introduced. Basic dynamical properties of this new chaotic system are studied including equilibrium points and their stability, dissipativity, the Lyapunov exponent, Kaplan-Yorke dimension, Lyapunov exponent spectrum and bifurcation diagrams. Moreover, the synchronization problem for

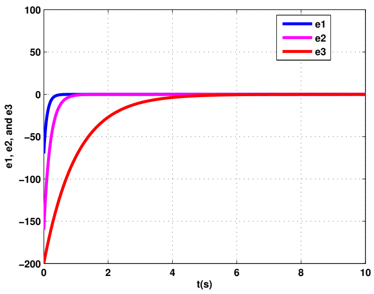

Figure 10: The time-history of the synchronization errors $e_1(t); e_2(t); e_3(t)$.

globally synchronizing the non-identical 3-D chaotic systems is solved using the FSHP method and Lyapunov stability criteria of the integer-order linear system. Numerical simulations using MATLAB have been shown to illustrate our results for the new chaotic system and the considered synchronization scheme.

6 Concluding remarks

The main important points in this work are:

- The new chaotic attractor is different from that of the Lorenz system or any existing systems.
- The dynamics of the novel system is investigated by means of the largest Lyapunov exponents spectrum and bifurcation diagrams of the system with respect to the system parameters.
- We achieved FSHP synchronyzation between non-identical 3-D chaotic systems using the new system and the Lyapunov stability theory.

The novel system and the obtained results of this work have many applications in many fields such as secure communication and signal encryption. Therefore, further research of the system is still important and will be taken into consideration in a future work.

Acknowledgment

The author would like to thank the editor-in-chief and the referees for their valuable suggestions and comments.

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