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Motion Estimation of Third Finger Using Ensemble and Unscented Kalman Filter for Inverse Kinematic of Assistive Finger-Arm Robot

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Abstract: The Islamic Hospital (RSI) Jemursari and RSI A. Yani always attempts to achieve an optimal community health status by health maintenance, health improvement (promotive), disease prevention (preventive), healing (curative) and recovery (rehabilitative) approaches in a comprehensive, integrated, and sustainable way. Specialists in prosthetics and orthotics, as the health professionals who are members of the medical rehabilitation team unit in Indonesia, are responsible for carrying out medical rehabilitation activities. The goal of the medical rehabilitation is to achieve its maximum functional competence and to prevent recurrent attacks. For this, a biomedical technology, that is, an assistive finger-arm robot, is required to help the recovery. The assistive finger-arm robot is one solution to assist the recovery process of paresis patients, specifically for finger movement. One of the research and development efforts on the assistive finger-arm robot is finger motion estimation. Several reliable motion estimation methods frequently used are the Unscented Kalman Filter (UKF) and Ensemble Kalman Filter (EnKF) methods, which are very reliable for either forward and inverse kinematic models or nonlinear models. Therefore, both methods were used in this study. Before the estimation was carried out, we started with modeling the inverse kinematics of the finger-arm robot as a platform for emulating the real movement of the fingers, to be specific, the third finger only. In this case, the third finger size was taken from the Surabaya citizens from Indonesia. The simulation results show that both methods had a fairly small error of about 2.5% -4.23%.

Keywords: *finger-arm robot; UKF; EnKF; inverse kinematic models.*

Mathematics Subject Classification (2010): 93E10, 62F10.

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1 Introduction

University of Nahdlatul Ulama Surabaya (UNUSA) is under the Surabaya Islamic Hospital (RSI) Foundation running the Jemursari Islamic Hospital and the Wonokromo Islamic Hospital. To support RSI Jemursari and RSI A. Yani in Wonokromo in an effort to achieve optimal community health status by applying health maintenance, health improvement (promotive), disease prevention (preventive), healing (curative) and recovery (rehabilitative) approaches in a comprehensive, integrated, and sustainable way, it attempts to develop any relevant and effective devices [1].

Specialists in prosthetics and orthotics as professionals who are members of the medical rehabilitation team unit in Indonesia are responsible for carrying out medical rehabilitation activities [1]. The rehabilitation program is a form of integrated health services with medical, psychosocial, educational vocational approach aiming to achieve its maximum function and to prevent repeated attacks. This rehabilitation program provides services using a multidisciplinary approach involving neurologists, medical rehabilitation doctors, nurses, physiotherapists, occupational therapists, medical social workers, psychologists as well as clients and families. One of the medical rehabilitation activities is the recovery of paresis sufferers. Paresis is a condition characterized by weakness of body movement, or partial loss of body movement or movement disorders. Among them is weakness in the fingers of post-stroke patients. For that, we need a biomedical technology to help recovery after stroke [2].

One of such technologies is robotic finger-arm assistance [3]. An assistive finger-arm robot is one solution to assist the recovery process of paresis patients, especially the movement of the fingers. One of such efforts to develop the assistive finger-arm robot is estimation of finger motion [2]. Several reliable estimation methods frequently used are the Ensemble Kalman Filter (UKF) and Ensemble Kalman Filter (EnKF) methods, which are very reliable for either forward and inverse kinematic models or nonlinear models. In its application, prior to estimation, we start with modeling the inverse kinematics of the finger-arm robot as a platform for mimicing the real movement of the fingers, to be more specific, the third finger. In this case, for this study, the third finger size is taken from the citizens of Surabaya, Indonesia. The UKF and EnKF methods are also frequently used to estimate the motion and position of submarines [4–6] and surface ships [7,8]. But the object of this study is finger motion estimation.

2 Finger Arm Motion Modelling

Specifically, the aim of the study in this paper was to estimate the motion of the fingers, especially the third finger of the left hand, using the EnKF and UKF methods by comparing the accuracy rates of these two motion estimation methods.

Here is the analysis of the finger-arm robot with 3 joints.

Figure 1 shows the 3-joint arm robot using the x and y coordinates in its working area. Just like the 2-joint arm robot, the 3-joint arm robot uses forward kinematics as an equation analysis [9].

The angle Ψ is the angle of the direction of the third part toward the X-axis, as in equation (1),

$$\Psi = (\theta_1 + \theta_2 + \theta_3). \tag{1}$$

Figure 2 is used to find the equation for the projections of link 1, link 2 and link 3 about the x-axis and y-axis, the projections can be analyzed and combined into an

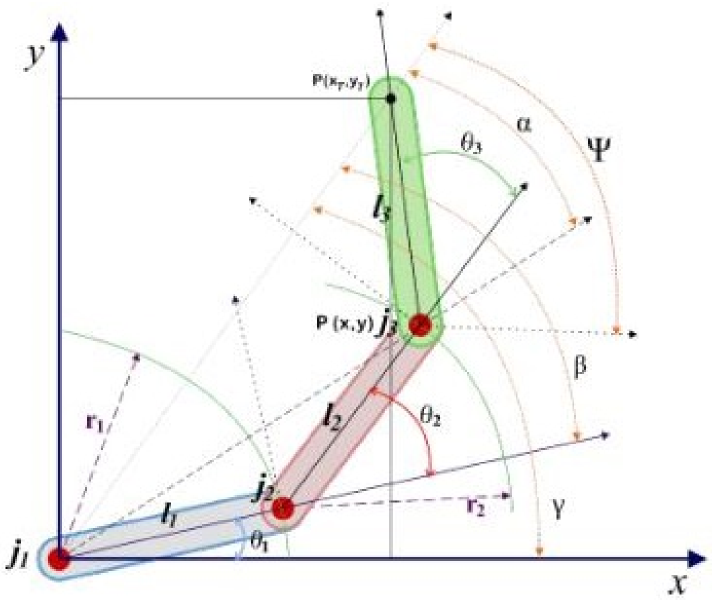

Figure 1: Configuration of the Finger-Arm Robot with 3 Joints.

equation.

For inverse kinematics, if the coordinates of $P(X_T, Y_T$ and $P(x,y)$ are known, then the angles θ_1, θ_2 and Ψ are as follows:

$$\begin{aligned} \theta_2 &= \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right), \\ \theta_1 &= \tan^{-1}\left(\frac{y(l_1+l_2\cos\theta_2)-xl_2\sin\theta_2}{x(l_1+l_2\cos\theta_2)+yl_2\sin\theta_2}\right), \end{aligned} \tag{2}$$

$$\begin{aligned} \Psi &= \theta_1+\theta_2+\theta_3 \\ &= \sin^{-1}\left(\frac{l_1(\cos\theta_1-\sin\theta_1)+l_2(\cos(\theta_1+\theta_2)-\sin(\theta_1+\theta_2))}{2l_3}\right). \end{aligned} \tag{3}$$

Below is the picture of three joints of the finger of a Surabaya citizen.

3 Ensemble and Unscented Kalman Filter Algorithms

The Ensemble and Uscented Kalman Filter algorithms are summarized in Table 1.

4 Simulation Results

This study started with forward kinematics modeling to obtain the motion on the X and Y axes and, after that, to obtain an inverse kinematic model of a robotic finger prosthetic arm with 3 joint parts that match the structure of the human finger.

Figure 2: Image of the arm robot with a focus on its finger prosthetic arms.

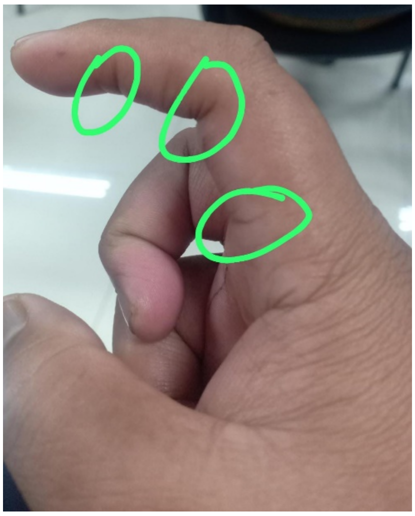

Figure 3: Picture of three joints of the finger of a Surabaya citizen.

In the numerical computation, the focus was on the movement of the third finger, the average length of which for Surabaya people, part of the Javanese tribe in Indonesia, is around 7.8 – 8.3 cm.

In this paper, the study used and compared the accuracy of the UKF and EnKF methods by comparing the four simulation results, that is, those by generating 500 and 600 ensembles and at 300 and 400 iterations. The third finger motion chosen in this simulation was the third finger motion in the form of a semicircular trajectory because

EnKF	**UKF**
System Model and Measurement Model	
$x_{k+1} = f(u_k, x_k) + w_k, w_k \sim N(0, Q_k)$ $z_k = Hx_k + v_k, v_k \sim N(0, R_k)$	$x_{k+1} = f(u_k, x_k) + w_k, w_k \sim N(0, Q_k)$ $z_k = Hx_k + v_k, v_k \sim N(0, R_k)$
Initialization	
Generate N *ensemble* in accordance with initial estimate $\overline{x}_0$ $x_{0,i} = [x_{0,1}\ x_{0,2}\ x_{0,3}\ ...\ x_{0,Ne}]$ Determine initial value : $\widehat{x}_0 = \frac{1}{N_e}\sum_{i=1}^{N} X_{0,i}$	$\widehat{x}_0 = E[x_0]$ $P_{x_0} = E[(x_0 - \widehat{x}_0)(x_0 - \widehat{x}_0)^T]$ $\widehat{x}_0^a = E[x^a] = E[\widehat{x}_0^T\ 0\ 0]^T$
Prediction Stage	
$\widehat{x}_{k,i}^- = f(\widehat{x}_{k-1,i}, u_{k-1,i}) + w_{k,i}$ with $w_{k,i} \sim N(0, Q_k)$ Estimate : $\widehat{x}_k^- = \frac{1}{N_e}\sum_{i=1}^{N}\widehat{x}_{k,i}^-$ Covariance error : $P_k^- = \frac{1}{N_e-1}\sum_{i=1}^{N}(\widehat{x}_{k,i}^- - \widehat{x}_k^-)(\widehat{x}_{k,i}^- - \widehat{x}_k^-)^T$	$X_{k\|k-1}^- = f(X_{k\|k-1}^x, X_{k\|k-1}^v)$ Estimate : $\widehat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} X_{i,k\|k-1}^x$ Covariance error : $P_{x_k}^- = \sum_{i=0}^{2L} W_i^{(c)}(X_{i,k\|k-1}^x - \widehat{x}_k^-)(X_{i,k\|k-1}^x - \widehat{x}_k^-)^T$
Correction Stage	
$z_{k,i} = z_k + v_{k,i}$ with $v_{k,i} \sim N(0, R_k)$ Kalman gain : $K_k = P_k^- H^T(HP_k^- H^T + R_k)^{-1}$ Estimate : $\widehat{x}_{k,i} = \widehat{x}_{k,i}^- + K_k(z_{k,i} - H\widehat{x}_{k,i}^-)$ $\widehat{x}_k = \frac{1}{N_e}\sum_{i=1}^{N}\widehat{x}_{k,i}$ Covariance error : $P_k = [I - K_k H]P_k^-$	$P_{\bar{z}_k,\bar{z}_k} = \sum_{i=0}^{2L} W_i^{(c)}(Z_{i,k\|k-1} - \widehat{z}_k^-)(Z_{i,k\|k-1} - \widehat{z}_k^-)^T$ Kalman gain : $P_{x_k,\bar{z}_k} P_{\bar{z}_k,\bar{z}_k}^{-1}$ Estimate : $\widehat{x}_k = \widehat{x}_k^- + K_k(Z_k - \widehat{Z}_k^-)$ Covariance error : $P_{x_k} = P_{x_k}^- - K_k P_{\bar{z}_k} K_k^T$

Table 1: EnKF and UKF Algorithms [10–13].

within that range, all finger joints could move optimally. With a semi-circular motion with a diameter of about 7.8 cm, the physical exercise on the third finger could be done to the maximum. The simulation results can be seen in Figures 4–7.

Figures 4 and 5 represent the simulation results of the estimated motion of the third finger resembling a semi-circle by generating 500 and 600 ensembles with 300 iterations, while Figures 6 and 7 used 400 iterations in the simulations.

Figure 4 shows the results of the simulations by the UKF and EnKF methods, using 300 iterations and generating 500 Ensembles for the EnKF method, resulted in a motion resembling a semi-circle with a diameter of $\sqrt{7.3^2 + 1.8^2} = \sqrt{53.29 + 3.24} = \sqrt{56.53} = 7.158$ cm, so overall, for the diameter of about $7.8 - 8.3$ cm, and using 500 ensembles, it had an error of around 3.6%, in other words, it gained an accu-

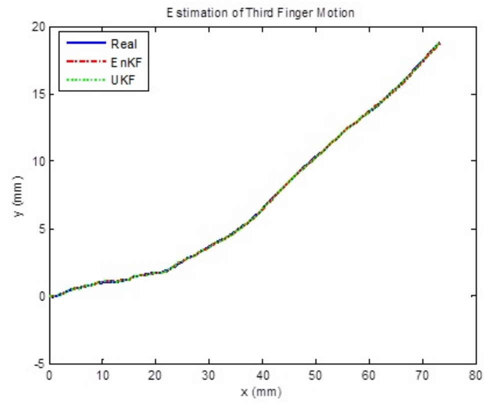

Figure 4: Estimation of Third Finger Motion in XY Plane using UKF and EnKF with 300 iterations and 500 Ensembles

racy of about 96.7%, while the UKF method produced a motion with a diameter of $\sqrt{7.25^2 + 1.82^2} = \sqrt{52.5625 + 3.3124} = \sqrt{55.8749} = 7.47$ resulting in an error of about 4.23%, in other words, it gained an accuracy of about 95.77%.

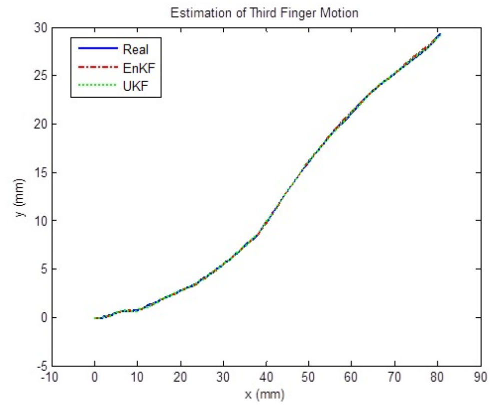

Figure 5: Estimation of Third Finger Motion in XY Plane using UKF and EnKF with 300 iterations and 600 Ensembles.

Figure 5 shows the results of the simulations by the UKF and EnKF methods, using 300 iterations and generating 500 ensembles, the EnKF method resulted in a motion resembling a semi-circle with a diameter of $\sqrt{8^2 + 2.9^2} = \sqrt{64 + 8,41} = \sqrt{72.41} = 8.509$ cm, so overall, for the diameter of about $7.8 - 8.3$ cm, and using 600 ensembles, it had an error of about 2.51%, in other words, it gained an accuracy of about 97.49%. Meanwhile, the UKF method produced a motion with a diameter of $\sqrt{8.1^2 + 2.92^2} = \sqrt{65.61 + 8.5264} = \sqrt{74.1364} = 8.61$, resulting in an error of about $3,73\%$, in other words, it gained an accuracy of about 96.27%.

According to Table 2, it is clear that the Ensemble Kalman Filter was more accurate than the UKF for both 500 and 600 ensembles. However, the UKF method had a faster

	EnKF with 500 Ensembles	UKF	EnKF with 600 Ensembles	UKF
XY Motion	96.7%	95.77 %	97.49%	96,27%
Simulation Time	10.85 s	10. 66 s	13.31 s	13,1 s

Table 2: Accuracy rate of motion estimation of third finger using UKF and EnKF methods with 300 iterations.

simulation time because it did not generate a number of ensembles. Overall, in terms of the level of accuracy above 95%, both methods were effective and can be used to effectively estimate the motion of the assistive finger-arm robot.

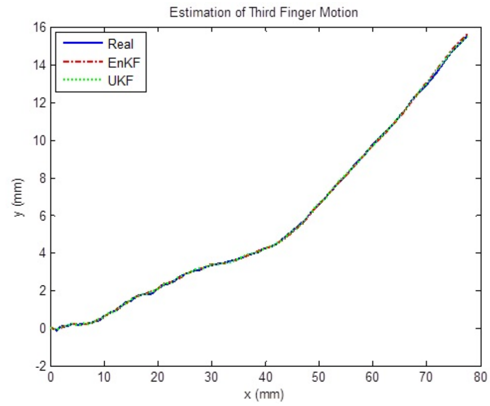

Figure 6: Estimation of Third Finger Motion in XY Plane using UKF and EnKF with 400 iterations and 500 Ensembles.

Figure 6 shows the results of the simulations by the UKF and EnKF methods, using 400 iterations and generating 500 ensembles, the EnKF method produced a motion resembling a semi-circle with a diameter of $\sqrt{7.8^2+1.5^2} = \sqrt{60.84+2.25} = \sqrt{63.09} = 7.94$ cm, so overall, for the diameter of about $7.8cm - 8.3$ cm, the range of 7.94 cm is included in the average length range of the third finger of Surabaya residents, in other words, there is no error or one has 100% accuracy. Meanwhile, the UKF method produced a motion with a diameter of $\sqrt{7.82^2+1.53^2} = \sqrt{61.1524+2.3409} = \sqrt{63.4993} = 7.968$, which is included within the average length range of the third finger of Surabaya people.

Figure 7 shows the results of the simulations by the UKF and EnKF methods, using 400 iterations and generating 600 ensembles, the EnKF method produced a motion resembling a semi-circle with a diameter of $\sqrt{7.8^2+2.4^2} = \sqrt{60.84+5.76} = \sqrt{66.6} = 8.16$ cm, so overall, for the diameter of about $7.8-8.3$ cm, the range of 8.16 cm is included within the range of the average length of the third finger of Surabaya people, in other words there is no error or one has 100% accuracy. Meanwhile, the UKF method produced a motion with a diameter of $\sqrt{7.77^2+2.36^2} = \sqrt{60.3729+5.5696} = \sqrt{65.9425} = 8.12$ cm, so overall, for the diameter of about $7.8-8.3$ cm, the range of 8.12 cm is included within the range of the average length of the third finger of Surabaya people, in other words, there is no error or one has 100% accuracy.

According to Table 3, it is clear that using the Ensemble Kalman Filter and Unscented

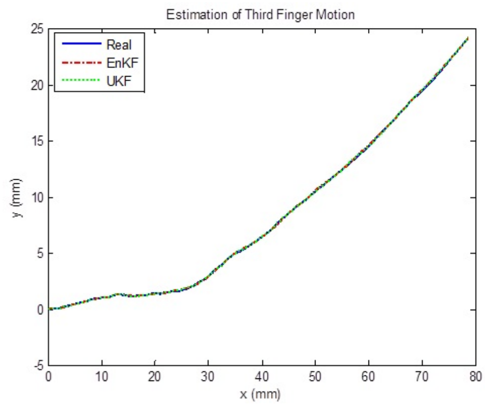

Figure 7: Estimation of Third Finger Motion in XY Plane using UKF and EnKF with 400 iterations and 600 Ensembles.

	EnKF with 500 Ensembles	**UKF**	**EnKF with 600 Ensembles**	**UKF**
XY Motion	100%	100 %	100%	100%
Simulation Time	15.45 s	15.18 s	17.62 s	17,33 s

Table 3: Accuracy rate of third finger motion estimation using UKF and EnKF with 400 iterations.

Kalman Filter produced the same accuracy of 100%. However, the UKF method had a faster simulation time because it did not generate a number of ensembles. Overall, in terms of the level of accuracy above 95%, both methods were effective and applicable to estimate the motion of the assistive finger-arm robot.

5 Conclusion

Based on the simulation results and the analysis above, it was found out that the EnKF and UKF methods were able to effectively estimate the third finger motion, especially for the finger size of the Javanese people in Indonesia, with an accuracy of around 95% – 100%. The error generated by the EnKF method in the simulation using 300 iterations was about 2.51% – 3.6%, while the UKF method produced an error of 3.7% – 4.2%. However, the simulation results using 400 iterations by the two methods, EnKF and UKF, had the same 100% accuracy rate. Overall, in terms of the level of accuracy above 95%, the two methods were effective and applicable to estimate the motion of the assistive finger-arm robot.

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