



Modelling and MultiSim Simulation of a New Hyperchaos System with No Equilibrium Point

A. Sambas^{1,2}, S. Vaidyanathan³, S. F. Al-Azzawi⁴, M. K. M. Nawawi^{5,6*},
M. A. Mohamed¹, Z. A. Zakaria¹, S. S. Abas¹ and M. Mamat¹

¹ Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Besut Campus,
22200, Terengganu, MALAYSIA.

² Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya,
Tasikmalaya, 46196, INDONESIA.

³ Center for Control Systems, Vel Tech University, Avaddi, 60062, Chennai, INDIA.

⁴ Department of Mathematics, College of Computer Science and Mathematics, University of
Mosul, Mosul 41002, IRAQ.

⁵ Institute of Strategic Industrial Decision Modeling, School of Quantitative Sciences,
Universiti Utara Malaysia, 06010, UUM Sintok, Kedah, MALAYSIA.

⁶ School of Quantitative Sciences, Universiti Utara Malaysia, 06010, UUM Sintok, Kedah,
MALAYSIA.

Received: April 16, 2023; Revised: October 2, 2023

Abstract: Crypto-devices and encryption applications make good use of nonlinear dynamical systems with hyperchaotic attractors due to their inherent complexity. Using four quadratic nonlinearities, a new 12-term hyperchaos system with hidden attractor is proposed in this research paper. It is established that the new hyperchaos system has no balance point and a hidden attractor exists for the system. Coexisting attractors and multistability are also proven to exist for the new system. The Kaplan-Yorke fractal dimension is determined for the new hyperchaos system with hidden attractor. MultiSim circuit design and simulation are carried out for the validation and real-world applications of the new hyperchaos system with hidden attractor. Finally, the chaos control results based on feedback control are also derived for the new 4D hyperchaotic system with no equilibrium point.

Keywords: *hidden attractor; hyperchaotic systems; MultiSim design; chaos control.*

Mathematics Subject Classification (2010): 34A34, 34D06, 34H10, 70Q05, 93B52.

* Corresponding author: <mailto:mdkamal@uum.edu.my>

1 Introduction

Chaos applications of dynamical models arise in various engineering domains such as nonlinear oscillatory systems [1, 2], biological models [3, 4], circuit devices [5, 6], ANN models [7, 8], chemical models [9, 10], finance models [11, 12], robotics [13, 14], mechanical systems [15, 16] etc.

Dynamical systems with positive Lyapunov index numbers are addressed as hyperchaotic systems [17] which possess diverse engineering applications such as secure devices [18, 19], crypto-devices [20, 21], memristor devices [22–24], neural networks [25, 26], etc. The hyperchaotic attractor extends in two or more directions concurrently as compared to a chaotic attractor with a singularly positive Lyapunov exponent. It implies that the hyperchaotic attractor performs significantly better in many real-world applications such as secure communication and encryption because it has a topological structure that is highly complex.

Hyperchaotic models are broadly divided as hyperchaos systems exhibiting self-excited or hidden attractors. The hyperchaos models with no balance point belong to the class of systems exhibiting hidden attractors [27, 28]. Li et al. [29] proposed an optical image encryption scheme based on the fractional Fourier transform and five-dimensional host-induced nonlinearity fractional-order laser hyperchaotic system. Erkan et al. [30] studied a novel two-dimensional (2D) chaotic system depending on the Schaffer function and the recursive 2D discrete model of the Schaffer function has superior chaotic behavior. Yu et al. [31] introduced two-dimensional logistic-adjusted-sine map (2D-LASM) and four-dimensional quadratic autonomous hyperchaotic system (4D-QAHS). Also, the experimental results demonstrate that the encryption approach is secure, with an average information entropy of 7.9972. Hosny et al. [32] proposed a novel utilization of fractional-order chaotic systems in color image encryption using the 4D hyperchaotic Chen system of fractional-order combined with the Fibonacci Q-matrix. Pavithran et al. [33] proposed a novel encryption process based on Deoxyribonucleic Acid (DNA) cryptography, a hyperchaotic system and a Moore machine. Alkhayyat et al. [34] studied a novel four-dimensional continuous-time dynamical system with the features of hyperchaotic phenomenon, dissipativity, rich dynamics, unstable equilibrium point, multistability and cryptographic S-box application.

In this work, we report a new 4-D nonlinear dynamical system with a hyperchaotic attractor. We show that the proposed nonlinear dynamical system has no balance point and it displays a hidden hyperchaotic attractor. We illustrate the qualitative properties of the proposed nonlinear dynamical system with a hyperchaotic attractor via MATLAB signal plots, balance points, multi-stability, coexisting hyperchaotic attractors, etc. The proposed nonlinear dynamical system with a hyperchaotic attractor has good applications in cryptosystems [35, 36] and secure communications [37].

For practical implementation of the hyperchaos systems, designs carried out via electronic circuits [38–40] or FPGA [41, 42] are immensely useful. In this research work, we exhibit MultiSim circuit design of the proposed new hyperchaos system in the four-dimensional space with no balance point.

This paper is organised as follows. Section 2 describes the dynamics and basic properties of the new 4D hyperchaotic system. Electronic circuit using MultiSim (Version 14.0) of the new 4D hyperchaotic system is given in Section 3. Section 4 presents control results using feedback control for the new 4D hyperchaotic system. Section 5 contains the conclusions of this work.

2 A Four-Dimensional Hyperchaos Systems with No Balance Point

A novel four-dimensional dynamical system is described in this work as follows:

$$\begin{cases} \dot{\xi} = \alpha(\eta - \xi) - \eta\zeta + \omega, \\ \dot{\eta} = -\gamma\xi\zeta + \beta\eta - \varepsilon\zeta^2 + \omega, \\ \dot{\zeta} = \xi\eta - \vartheta, \\ \dot{\omega} = -\xi - \eta. \end{cases} \quad (1)$$

The four-dimensional vector $K = (\xi, \eta, \zeta, \omega)$ designates the state of the system (1). It is remarked that there are four quadratic nonlinearities (in the first, second and third differential equations). It will be established using the Lyapunov Index (LI) values spectrum analysis in MATLAB that there is hyperchaos in the system (1) when the parameters assume the values

$$\alpha = 18, \beta = 6, \gamma = 14, \vartheta = 10, \varepsilon = 0.2. \quad (2)$$

For the time-series analysis of the state K of the system (1), we assume the initial state as

$$\xi(0) = 0.2, \eta(0), \zeta(0) = 0.4, \omega(0) = 0.2. \quad (3)$$

Using Wolf's procedure [43], the Lyapunov Index (LI) values of the 4-D model (1) are numerically found in seconds (see Figure 1) as follows:

$$\mu_1 = 2.8033, \mu_2 = 0.0535, \mu_3 = 0, \mu_4 = -14.8266. \quad (4)$$

The existence of two positive LI values (*viz.* μ_1, μ_2) summarily signifies that the model (1) has hyperchaos nature. As the total of all LI values in the LI spectrum is seen to be negative, the model (1) has also dissipative motion of all its trajectories converging to the hyperchaotic attractor. The Kaplan-Yorke dimension of the hyperchaotic system (1) is computed as follows:

$$D_{KY} = 3 + \frac{\mu_1 + \mu_2 + \mu_3}{\mu_4} = 3.1927. \quad (5)$$

Figures 1 and 2 show the LI values and signal plots of the model (1) simulated in MATLAB for the parameter set $(\alpha, \beta, \gamma, \vartheta, \varepsilon) = (18, 6, 14, 10, 0.2)$. The MATLAB plots show the high complexity of the hyperchaos system (1).

Multistability means two or more attractors coexist together with different initial conditions and has been found in many nonlinear systems. Let the parameters be fixed as $\alpha = 18, \beta = 6, \gamma = 14, \vartheta = 10, \varepsilon = 0.2$ and we suppose that the initial states of the system (1) are picked as $K_0 = (0.2, 0.4, 0.4, 0.2)$ and $\Lambda_0 = (-0.8, 0.8, -0.8, 0.8)$. Figure 3 shows the multistability of the hyperchaotic system (1) with two coexisting hyperchaotic attractors emanating from K_0 (blue color) and Λ_0 (red color), respectively.

3 Multisim Simulation of the Novel Hyperchaos System with Hidden Attractor

This study will consider the analog circuit implementation of the new double-scroll hyperchaos system described in (1). Figure 4 shows a four channels electronic circuit scheme

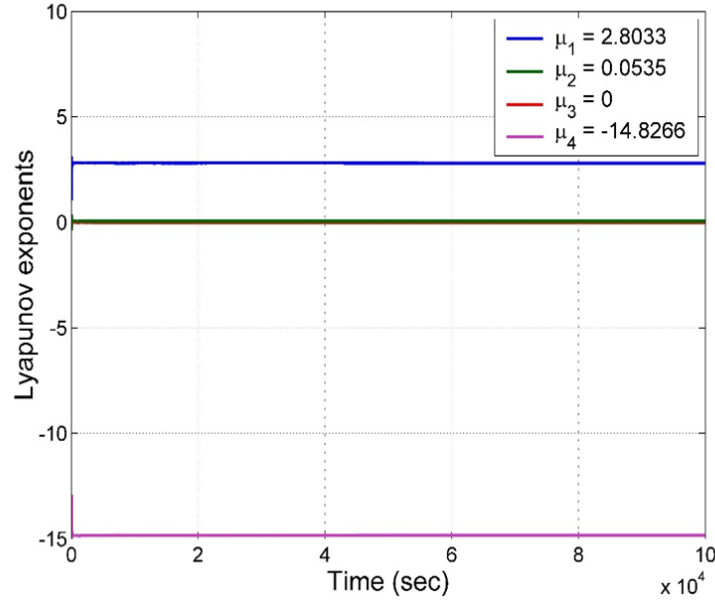


Figure 1: Lyapunov index values spectrum for the new hyperchaos system (1).

with variables ξ, η, ζ, ω from the system (1). For circuit implementation, we rescale the state variables of the new chaotic system (1) as follows: $\xi = \frac{1}{2}\xi, \eta = \frac{1}{2}\eta, \zeta = \frac{1}{2}\zeta$ and $\omega = \frac{1}{2}\omega$. In the new coordinates $(\xi, \eta, \zeta, \omega)$, the chaotic system (1) becomes

$$\begin{cases} \dot{\xi} = \alpha(\eta - \xi) - 2\eta\zeta + \omega, \\ \dot{\eta} = -2\gamma\xi\zeta + \beta\eta - 2\epsilon\zeta^2 + \omega, \\ \dot{\zeta} = 2\xi\eta - \frac{1}{2}\vartheta, \\ \dot{\omega} = -\xi - \eta. \end{cases} \quad (6)$$

By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations can be derived in the following form:

$$\begin{cases} \dot{\xi} = \frac{1}{C_1 R_1} \eta - \frac{1}{C_1 R_2} \xi - \frac{1}{10 C_1 R_3} \eta \zeta + \frac{1}{C_1 R_4} \omega, \\ \dot{\eta} = -\frac{1}{10 C_2 R_5} \xi \zeta + \frac{1}{C_2 R_6} \eta - \frac{1}{10 C_2 R_7} \zeta^2 + \frac{1}{C_2 R_8} \omega, \\ \dot{\zeta} = \frac{1}{10 C_3 R_9} \xi \eta - \frac{1}{C_3 R_{10}} V_1, \\ \dot{\omega} = -\frac{1}{C_4 R_{11}} \xi - \frac{1}{C_4 R_{12}} \eta. \end{cases}$$

Here, ξ, η, ζ, ω are the voltages across the capacitors C_1, C_2, C_3 and C_4 , respectively.

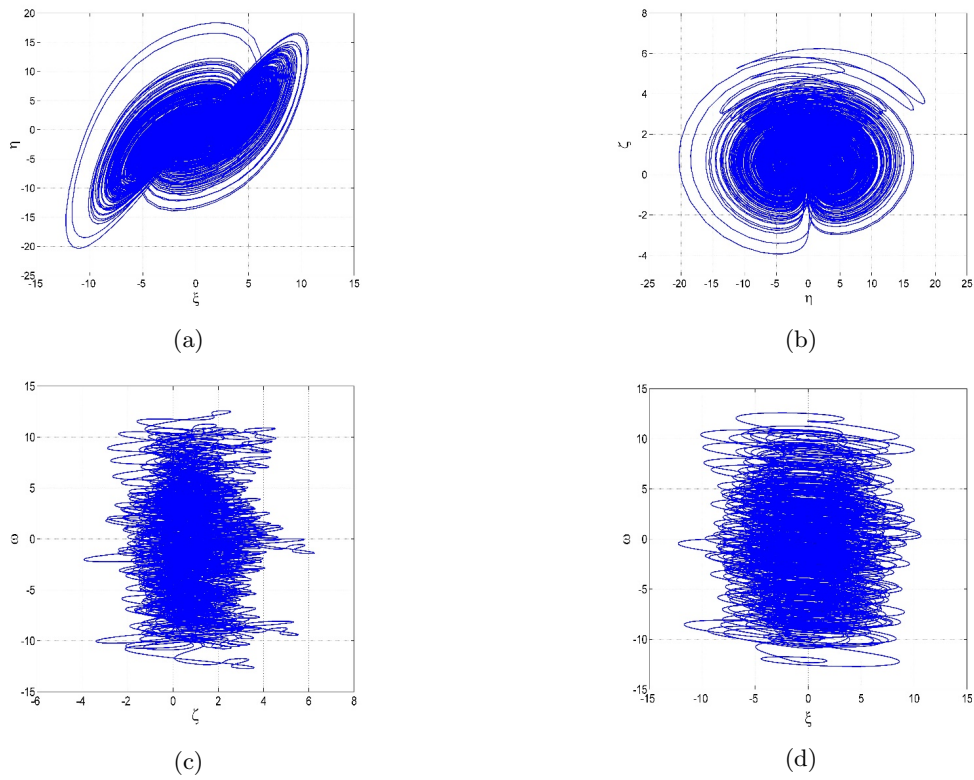


Figure 2: Signal plots of the new hyperchaos system (1) simulated in MATLAB.

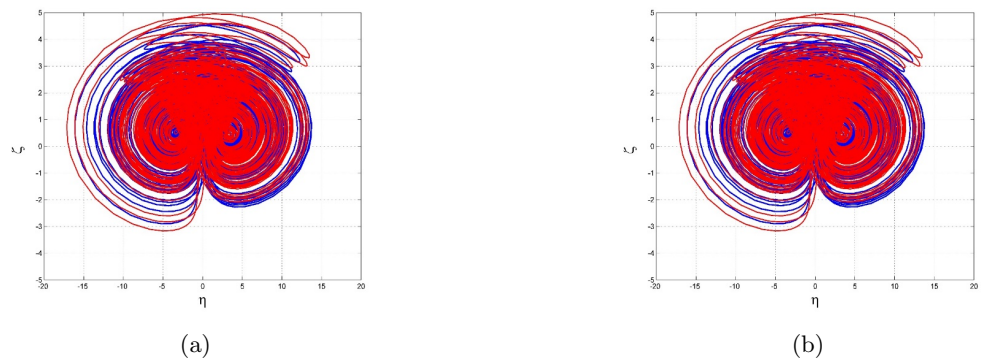


Figure 3: Phase portraits of the coexisting hyperchaotic attractors (1) for the initial states $K_0 = (0.2, 0.4, 0.4, 0.2)$ and $\Lambda_0 = (-0.8, 0.8, -0.8, 0.8)$.

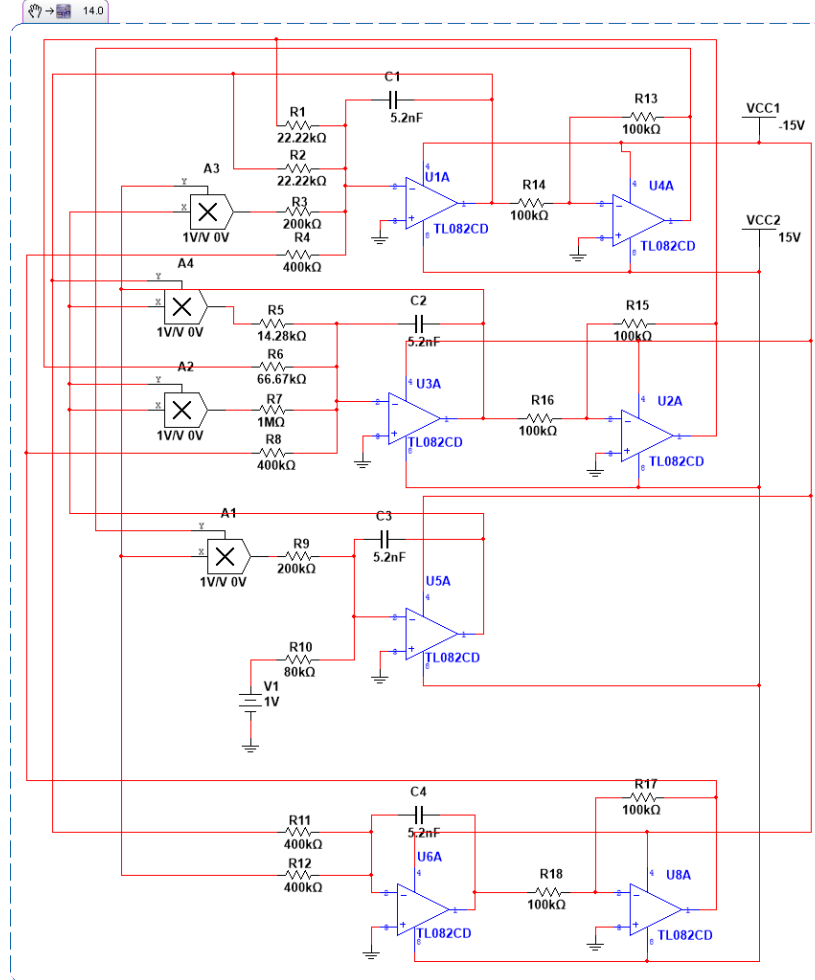


Figure 4: Circuit design for the new hyperchaotic two-scroll system.

We choose the values of the circuit elements as $R_1 = R_2 = 22.22 \text{ k}\Omega$, $R_3 = R_9 = 200 \text{ k}\Omega$, $R_4 = R_8 = R_{11} = R_{12} = 400 \text{ k}\Omega$, $R_5 = 14.28 \text{ k}\Omega$, $R_6 = 66.67 \text{ k}\Omega$, $R_7 = 1 \text{ M}\Omega$, $R_{10} = 80 \text{ k}\Omega$, $R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = 100 \text{ k}\Omega$, $C_1 = C_2 = C_3 = C_4 = 5.2 \text{ nF}$. The corresponding phase portraits on the oscilloscope are shown in Figure 5. The agreement between the Multisim results (Figure 5) and the MATLAB plots (Figure 2) shows the feasibility of the proposed hyperchaotic system.

4 Chaos Control

The system (1) is modified by introducing feedback controllers $U = [u_1, u_2, u_3, u_4]^T$ and is expressed as

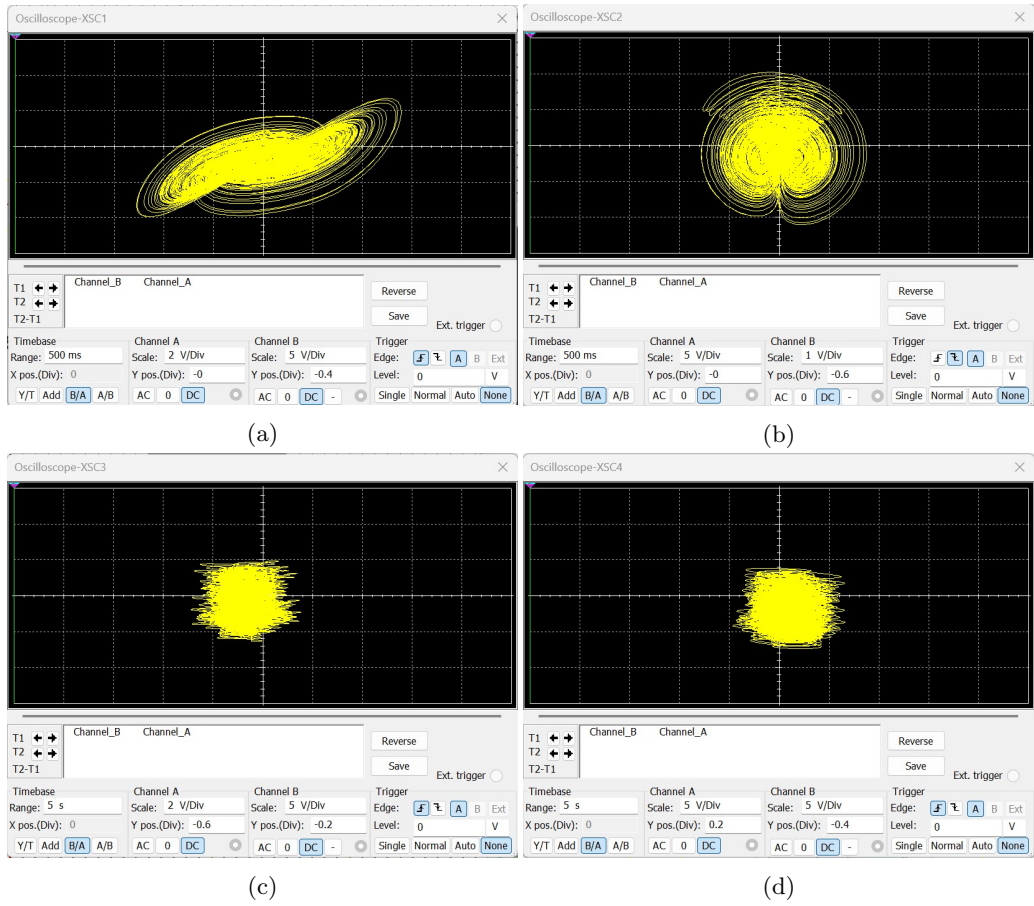


Figure 5: MultiSIM chaotic attractors of the new hyperchaotic two-scroll system (a) ξ - η plane, (b) η - ζ plane, (c) ζ - ω plane and (d) ξ - ω plane.

$$\begin{cases} \dot{\xi} = \alpha(\eta - \xi) - \eta\zeta + \omega + u_1, \\ \dot{\eta} = -\gamma\xi\zeta + \beta\eta - \varepsilon\zeta^2 + \omega + u_2, \\ \dot{\zeta} = \xi\eta - \vartheta + u_3, \\ \dot{\omega} = -\xi - \eta + u_4. \end{cases} \quad (7)$$

Consider the nonlinear control strategy [44, 45] and assume the parameters $\alpha, \beta, \gamma, \vartheta$ and ε are unknown, while u_i , $i = 1, 2, 3, 4$, is a feedback controller to be designed.

Theorem 1. *If the controller designed in (8) is implemented using a nonlinear control strategy, the system (1) will be suppressed.*

$$\begin{cases} u_1 = 0, \\ u_2 = -\alpha\xi - 2\beta\eta, \\ u_3 = \gamma\xi\eta + (\varepsilon\eta - 1)\zeta + \vartheta, \\ u_4 = -\omega. \end{cases} \quad (8)$$

Proof. Substituting the controllers (8) into the system (7), we have

$$\begin{cases} \dot{\xi} = \alpha(\eta - \xi) - \eta\zeta + \omega, \\ \dot{\eta} = -\gamma\xi\zeta + \beta\eta - \varepsilon\zeta^2 + \omega - \alpha\xi, \\ \dot{\zeta} = \xi\eta + \gamma\xi\eta + (\varepsilon\eta - 1)\zeta, \\ \dot{\omega} = -\xi - \eta - \omega. \end{cases} \quad (9)$$

Construct the Lyapunov candidate function as

$$v = \frac{1}{2}[\xi^2 + \eta^2 + \zeta^2 + \omega^2]$$

and

$$\begin{aligned} \dot{v} &= \xi\dot{\xi} + \eta\dot{\eta} + \zeta\dot{\zeta} + \omega\dot{\omega}, \\ \dot{v} &= \xi(\alpha(\eta - \xi) - \eta\zeta + \omega) \\ &\quad + \eta(-\gamma\xi\zeta - \beta\eta - \varepsilon\zeta^2 + \omega - \alpha\xi) \\ &\quad + \zeta(\xi\eta + \gamma\xi\eta + (\varepsilon\eta - 1)\zeta) \\ &\quad + \omega(-\xi - \eta - \omega), \\ \Rightarrow \dot{v} &= -\alpha\xi^2 - \beta\eta^2 - \zeta^2 - \omega^2. \end{aligned}$$

By using a theoretical method (the Lyapunov stability theorem), the system (1) was suppressed, and the accuracy of the analytical results was validated by the numerical simulations via Figure 6.

5 Conclusions

The main novelty of this work is the modelling of a new 4-D hyperchaos system with no balance point. We remark that the proposed hyperchaos system displays a hidden attractor as it has no balance point. In this research paper, invoking the use of four quadratic nonlinearities, a new nonlinear dynamical system with a hidden hyperchaotic attractor was proposed and illustrated with MATLAB signal plots. For nonlinear dynamical systems with chaotic or hyperchaotic attractors, multistability is a special property

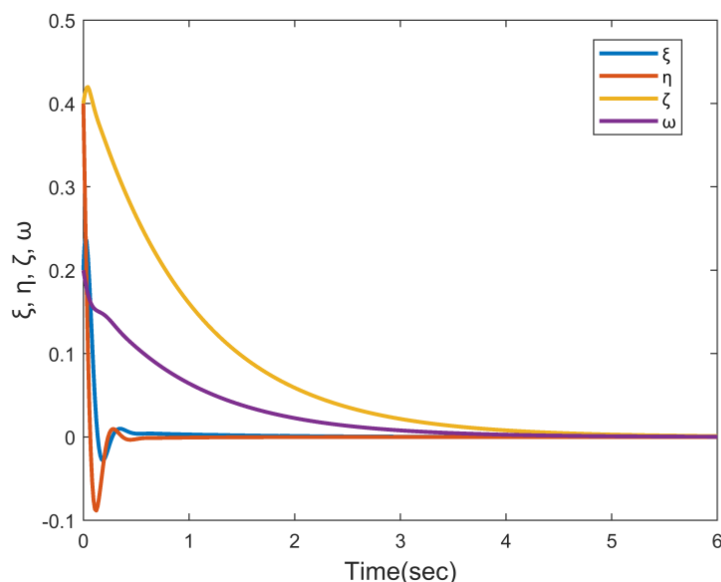


Figure 6: Convergence of attractors to zero by controller (8).

which refers to the existence of chaotic or hyperchaotic attractors for the fixed set of parameters but different choices of initial states. In this research work, we showed the existence of multistability with coexisting hyperchaotic attractors for the proposed hyperchaos system. MultiSim circuit simulation of the new hyperchaos system was designed for the validation of the new hyperchaos system with a hidden attractor, which has many practical applications in secure communication devices. Finally, the chaos control results based on feedback control are also derived for the new 4D hyperchaotic system with no equilibrium point.

References

- [1] M. Labid and N. Hamri. Chaos Synchronization between Fractional-Order Lesser Date Moth Chaotic System and Integer-Order Chaotic System via Active Control. *Nonlinear Dynamics and Systems Theory* **22**(4) (2022) 407–413.
- [2] H. Yousfi, A. Gasri and A. Ouannas. Stabilization of Chaotic h-Difference Systems with Fractional Order. *Nonlinear Dynamics and Systems Theory* **2** (4) (2023) 468–472.
- [3] S. Kumar, A. Kumar, B. Samet, J. F. Gómez-Aguilar and M. S. Osman. A chaos study of tumor and effector cells in fractional tumor-immune model for cancer treatment. *Chaos, Solitons & Fractals* **141** (2020) 110321.
- [4] S. S. Sajjadi, D. Baleanu, A. Jajarmi and H. M. Pirouz. A new adaptive synchronization and hyperchaos control of a biological snap oscillator. *Chaos, Solitons & Fractals* **138** (2020) 109919.
- [5] A. Sambas, S. Vaidyanathan, X. Zhang, I. Koyuncu, T. Bonny, M. Tuna, M. ALcin, S. Zhang, I. M. Sulaiman, A. M. Awwal and P. Kumam. A novel 3D chaotic system with line equilibrium: multistability, integral sliding mode control, electronic circuit, FPGA implementation and its image encryption. *IEEE Access* **10** (2022) 68057–68074.

- [6] A. Sambas, S. Vaidyanathan, E. Tlelo-Cuautle, B. Abd-El-Atty, A. A. Abd El-Latif, O. Guillén-Fernández and G. Gundara. A 3-D multi-stable system with a peanut-shaped equilibrium curve: Circuit design, FPGA realization, and an application to image encryption. *IEEE Access* **8** (2020) 137116–137132.
- [7] C. Xiu, R. Zhou and Y. Liu. New chaotic memristive cellular neural network and its application in secure communication system. *Chaos, Solitons and Fractals* **141** (2016) 110316.
- [8] S. Vaidyanathan. Hybrid chaos synchronization of 3-cells cellular neural network attractors via adaptive control method. *International Journal of PharmTech Research* **8** (8) (2015) 61–73.
- [9] A. J. Adéchinan, Y. J. F. Kpomahou, L. A. Hinvi and C. H. Miwadinou. Chaos, coexisting attractors and chaos control in a nonlinear dissipative chemical oscillator. *Chinese Journal of Physics* **77** (2022) 2684–2697.
- [10] H. Chaudhary, A. Khan and M. Sajid. An investigation on microscopic chaos controlling of identical chemical reactor system via adaptive controlled hybrid projective synchronization. *The European Physical Journal Special Topics* **231** (3) (2022) 453–463.
- [11] B. A. Idowu, S. Vaidyanathan, A. Sambas, O. I. Olusola, and O. S. Onma. A new chaotic finance system: Its analysis, control, synchronization and circuit design. *Nonlinear Dynamical Systems with Self-Excited and Hidden Attractors* (133) (2018) 271–295.
- [12] W. A. Wali. Application of particle swarm optimization with ANFIS model for double scroll chaotic system. *International Journal of Electrical and Computer Engineering* **11** (1) (2021) 328–335.
- [13] S. Bazzi and D. Sternad. Human control of complex objects: towards more dexterous robots. *Advanced Robotics* **34** (17) (2020) 1137–1155.
- [14] L. Moysis, E. Petavratzis, M. Marwan, C. Volos, H. Nistazakis and S. Ahmad. “Analysis, synchronization, and robotic application of a modified hyperjerk chaotic system. *Complexity* **2020** (2020) 2826850.
- [15] M. Marwan, V. Dos Santos, M. Z. Abidin and A. Xiong. Coexisting attractor in a gyrostat chaotic system via basin of attraction and synchronization of two nonidentical mechanical systems. *Mathematics* **10**, (11) (2022) 1914.
- [16] S. Bentouati, A. Tlemçani, N. Henini, and H. Nouri. Adaptive Sliding Mode Control Based on Fuzzy Systems Applied to the Permanent Magnet Synchronous Machine. *Nonlinear Dynamics and Systems Theory* **21** (5) (2021) 457–470.
- [17] M. Zhou, and C. Wang. A novel image encryption scheme based on conservative hyperchaotic system and closed-loop diffusion between blocks. *Signal Processing* **171** (2020) 107484.
- [18] C. Zhong and M. S. Pan. Encryption algorithm based on scrambling substitution of new hyperchaotic Chen system. *Chin. J. Liquid Crystals Displays*, *Chin. J. Liquid Crystals Displays* **35** (1) (2020) 91–97.
- [19] N. Nguyen, G. Kaddoum, F. Pareschi, R. Rovatti and G. Setti. A fully CMOS true random number generator based on hidden attractor hyperchaotic system. *Nonlinear Dynamics* **102** (4) (2020) 2887–2904.
- [20] L. Ding and Q. Ding, Q. A novel image encryption scheme based on 2D fractional chaotic map, dwt and 4d hyper-chaos. *Electronics* **9** (8) (2020) 1280.
- [21] A. Shakiba. A novel randomized bit-level two-dimensional hyperchaotic image encryption algorithm. *Multimedia Tools and Applications* **79** (43) (2020) 32575–32605.
- [22] F. Setoudeh and A. K. Sedigh. Nonlinear analysis and minimum L2-norm control in memcapacitor-based hyperchaotic system via online particle swarm optimization. *Chaos, Solitons & Fractals* **151** (2021) 111214.

- [23] C. Xiu, J. Fang and X. Ma. Design and circuit implementations of multimemristive hyperchaotic system. *Chaos, Solitons & Fractals* **161** (2022) 112409.
- [24] S. Vaidyanathan, V. T. Pham and C. Volos. Adaptive control, synchronization and circuit simulation of a memristor-based hyperchaotic system with hidden attractors. In *Advances in Memristors, Memristive Devices and Systems*. Springer, Cham **701** (2017) 101–130.
- [25] P. S. Swathy and K. Thamilmaran. Hyperchaos in SC-CNN based modified canonical Chua's circuit. *Nonlinear Dynamics* **78** (4) (2014) 2639–2650.
- [26] H. Lin, C. Wang and Y. Tan. Hidden extreme multistability with hyperchaos and transient chaos in a Hopfield neural network affected by electromagnetic radiation. *Nonlinear Dynamics* **99** (3) (2020) 2369–2386.
- [27] N. Wang, G. Zhang, N. V. Kuznetsov and H. Bao. Hidden attractors and multistability in a modified Chua's circuit. *Communications in Nonlinear Science and Numerical Simulation* **92** (2021) 105494.
- [28] L. Yang, Q. Yang and G. Chen. Hidden attractors, singularly degenerate heteroclinic orbits, multistability and physical realization of a new 6D hyperchaotic system. *Communications in Nonlinear Science and Numerical Simulation* **90** (2020) 105362.
- [29] X. Li, J. Mou, Y. Cao and S. Banerjee. An optical image encryption algorithm based on a fractional-order laser hyperchaotic system. *International Journal of Bifurcation and Chaos* **32** (3)(2022) 2250035.
- [30] U. Erkan, A. Toktas and Q. Lai. 2D hyperchaotic system based on Schaffer function for image encryption. *Expert Systems with Applications* **213** (2023) 119076.
- [31] J. Yu, W. Xie, Z. Zhong and H. Wang. Image encryption algorithm based on hyperchaotic system and a new DNA sequence operation. *Chaos, Solitons & Fractals* **162** (2022) 112456.
- [32] K. M. Hosny, S. T. Kamal and M. M. Darwish. Novel encryption for color images using fractional-order hyperchaotic system. *Journal of Ambient Intelligence and Humanized Computing* **13** (2) (2022) 973–988.
- [33] P. Pavithran, S. Mathew, S. Namasudra, and G. Srivastava. A novel cryptosystem based on DNA cryptography, hyperchaotic systems and a randomly generated Moore machine for cyber physical systems. *Computer Communications* **188** (2022) 1–12.
- [34] A. Alkhayyat, M. Ahmad, N. Tsafack, M. Tanveer, D. Jiang and A. A. Abd El-Latif. A novel 4D hyperchaotic system assisted josephus permutation for secure substitution-box generation. *Journal of Signal Processing Systems* **94** (3)(2022) 315–328.
- [35] J. Wu, Y. Shen and G. Xu. Integrating Fresnel diffraction, multi-phase retrieval, and hyperchaos mapping for color image encryption. *Applied Optics* **62** (4) (2023) 844–860.
- [36] D. Xu, G. Li, W. Xu and C. Wei. Design of artificial intelligence image encryption algorithm based on hyperchaos. *Ain Shams Engineering Journal* **14** (3) (2023) 101891.
- [37] Q. Lai and Y. Liu. A cross-channel color image encryption algorithm using two-dimensional hyperchaotic map. *Expert Systems with Applications* **223** (2023) 119923.
- [38] S. Vaidyanathan, I. M. Moroz, A. Sambas and W. S. M. Sanjaya. A New 4-D Hyperchaotic Two-Wing System with a Unique Saddle-Point Equilibrium at the Origin, its Bifurcation Analysis and Circuit Simulation. In: *Journal of Physics: Conference Series* **1477** (2) (2020) 022016.
- [39] W. Zhou, G. Wang, H. H. C. Iu, Y. Shen, and Y. Liang. Complex dynamics of a non-volatile memcapacitor-aided hyperchaotic oscillator. *Nonlinear Dynamics* **100** (4) (2020) 3937–3957.
- [40] A. Sambas, M. Mamat, S. Vaidyanathan, M. A. Mohamed, and W. S. M. Sanjaya. A new 4-D chaotic system with hidden attractor and its circuit implementation. *International Journal of Engineering & Technology* **7** (3) (2018) 1245–1250.

- [41] S. Vaidyanathan, E. Tlelo-Cuautle, A. Sambas, L. G. Dolvis and O. Guillén-Fernández. FPGA design and circuit implementation of a new four-dimensional multistable hyperchaotic system with coexisting attractors. *International Journal of Computer Applications in Technology* **64** (3) (2020) 223–234.
- [42] A. Sambas, S. Vaidyanathan, T. Bonny, S. Zhang, Y. Hidayat, G. Gundara and M. Mamat. Mathematical model and FPGA realization of a multi-stable chaotic dynamical system with a closed butterfly-like curve of equilibrium points. *Applied Sciences* **11** (2)(2021) 788.
- [43] A. Wolf, J. B. Swift, H. L. Swinney and J. A. Vastano. Determining Lyapunov exponents from a time series. *Physica D: nonlinear phenomena* **16** (3) (1985) 285–317.
- [44] S. F. AL-Azzawi and A. S. Al-Obeidi. Chaos synchronization in a new 6D hyperchaotic system with self-excited attractors and seventeen terms. *Asian-European Journal of Mathematics* **14** (05) (2021) 2150085.
- [45] A. S. Al-Obeidi and S. F. Al-Azzawi. A novel six-dimensional hyperchaotic system with self-excited attractors and its chaos synchronisation. *International Journal of Computing Science and Mathematics* **15** (1) (2022) 72–84.