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Numerical Solution of Neutral Double Delay Volterra Integral Equations Using Taylor Collocation Method

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Abstract: In this paper, we employ a direct collocation method using Taylor polynomials to estimate the solution of linear Volterra integral equations with two constant delays within the polynomial spline $S_{m-1}^{(-1)}(\Pi_N)$. This approach is well-suited for capturing the dynamics inherent in age and size-structured population models (Gurtin–MacCamy model), epidemiological models, chemical engineering processes (heat equation with delay in control and in state) and control theory (Roesser model). We derive an iterative formula to compute the approximate solution and prove its convergence. We confirm the validity and efficacy of this convergent algorithm by presenting numerical results.

Keywords: *neutral double delay Volterra integral equation; collocation method; Taylor polynomials; error analysis.*

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1 Introduction

Delay Volterra integral equations pose a significant mathematical challenge across various scientific and engineering domains, encompassing biology [1], epidemiology [2,3], chemical engineering [4], control theory [5], physics [6] and social sciences [7]. In [8,9], double delay Volterra integral equations (DDVIEs), which incorporate memory and delayed effects, apply to model age-structured populations, where two distinct age groups within a single population are considered. The dual delays in equation (1) correspond to the time required for maturation and reaching the maximum age, rendering analytical solutions impractical for many problems. As a result, developing efficient and precise numerical

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