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Trajectory Estimation of Amphibious Aircraft Using H-Infinity and Ensemble Kalman Filter Methods

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Abstract: An amphibious aircraft is an aircraft that has the ability to operate from a runway like a conventional aircraft or from shallow water. Such type of aircraft is widely used for tourist transportation, taking tourists to areas only reachable by water. It is not uncommon for amphibious aircraft to be used as rescue and forest fire fighting tools. The need for amphibious aircraft is in line with the development of navigation and guidance systems required by such amphibious aircraft. Navigation and motion control systems for amphibious aircraft refer to technologies and systems allowing aircraft to operate in the air and also sail on the water surface. Amphibious aircrafts are designed to have capabilities that allow them to take off and land on conventional runways and to operate to and from water area. Several navigation guidance methods in the field of robotics can also be used, one of which is for amphibious aircraft position estimation. The accuracy of aircraft position estimation is very important to ensure that the amphibious aircraft follows the specified trajectory accurately. The estimation methods used in this paper are the H-infinity and Ensemble Kalman Filter (EnKF). This study compared the numerical simulation results of the two methods, EnKF and H-infinity, aiming to estimate the position of the amphibious aircraft by generating 300 and 600 ensembles, and the simulation results with 800 ensembles had the best accuracy of about 95-98%.

Keywords: *amphibious aircraft; trajectory estimation; H-infinity and Ensemble Kalman Filter methods.*

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1 Introduction

An amphibious aircraft is a fixed-wing aircraft able to take off and land on solid ground and water. Its history dates back to World War II when, especially in the Pacific, the Allies needed search and rescue aircrafts to rescue airmen adrift at sea after their aircrafts were shot down by the enemy. An amphibious aircraft is an aircraft able to operate from a runway like a conventional aircraft or from shallow water [1]. Such type of aircraft is widely used for tourist transportation, taking tourists to areas only reachable by water [2].

In addition, that type of aircraft is needed for logistics transportation in remote areas having no airstrips. In fact, in the past, it could also be used as a naval reconnaissance aircraft due to its range. At that time the most famous amphibious aircrafts were the PBY Catalina and the Grumman HU 16 Albatross. After the end of WWII, these aircrafts were still widely used until around the 1960s, in both civilian and military environments. Some of them with excellent maintenance and modifications are still widely used in the United States, especially as a forest firefighting fleet.

An amphibious aircraft requires a technology of navigation and guidance systems, of which the navigation system is a system that directs the position of the amphibious aircraft to follow a predetermined trajectory and as much as possible has a fairly high accuracy [3]. Navigation and motion control systems for amphibious aircraft refer to technologies and systems allowing an aircraft to operate in the air and also sail on the water surface [4]. Amphibious aircrafts are designed to have amphibious capabilities that allow them to take off and land on conventional runways and operate to and from water areas. Several navigation guidance methods in the field of robotics can also be used, one of which is for amphibious aircraft position estimation. The accuracy of aircraft position estimation is very important to ensure that the amphibious aircraft follows the specified trajectory accurately.

One of the estimation methods having a small error is the Ensemble Kalman Filter method, which is very reliable for a nonlinear model. The EnKF method is often used for motion and position estimation of AUVs [5], [6], [7], mobile robots [8], and missiles [9]. Given that the EnKF method is a reliable estimation method, in this study, two methods are applied, that is, the EnKF method and the H-inifnity method. This study compared the numerical simulation results of the two methods, the EnKF and H-infinity, with the aim of estimating the position of the amphibious aircraft by generating 300 and 600 ensembles.

2 Amphibious Aircraft Model

The forces working on an unmanned amphibious aircraft can be categorized into the aircraft mass, hydrodynamic forces, aerodynamic forces and engine thrust as shown in Fig 1. The nonlinear longitudinal dynamic motion model of the unmanned amphibious aircraft is represented by the following equations.

In the earth coordinate system $X_e; Y_e; Z_e$, then the aircraft body coordinate system $X_b; Y_b; Z_b$ and the steady-translation coordinate system $X_s; Y_s; Z_s$:

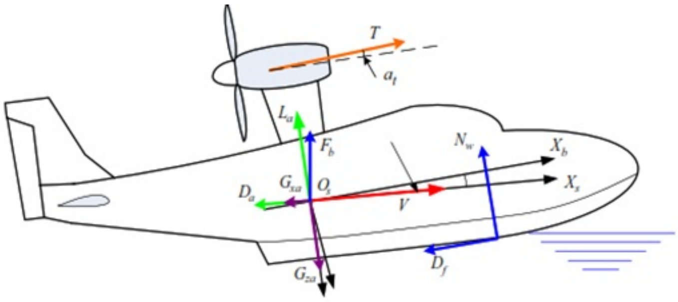

Figure 1: Forces working on amphibious aircraft [4].

$$
\begin{aligned}
m\dot{V} &= T\cos(\alpha+\alpha_t) - D_a - N_w\sin\alpha - D_f\cos\alpha + G_{xa}, \\
mV\dot{\alpha} &= mVq - T\sin(\alpha+\alpha_t) - L_a - N_w\cos\alpha + D_f\sin\alpha + G_{za}, \\
I_y\dot{q} &= M_a + M_w + M_T, \\
\dot{\theta} &= q, \\
\dot{x}_g &= u\cos\theta + w\sin\theta, \\
\dot{z}_g &= -u\sin\theta + w\cos\theta.
\end{aligned}
$$

3 Ensemble Kalman Filter Algorithm

Below is the algorithm flow of the Ensemble Kalman Filter method.

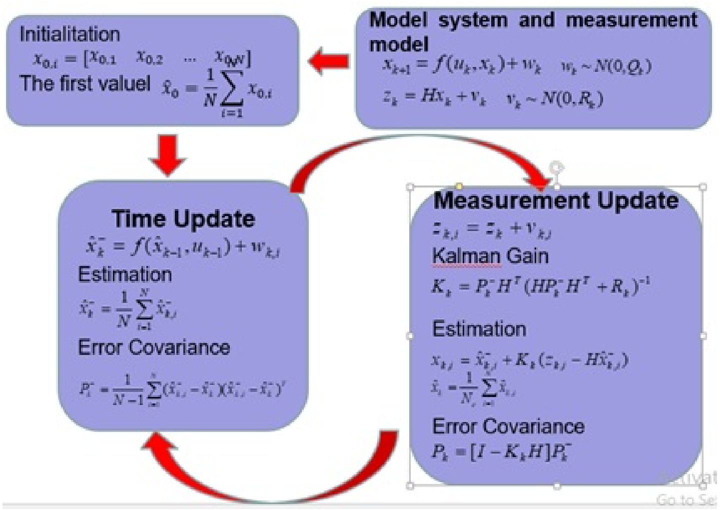

Figure 2: Ensemble Kalman Filter Algorithm [9], [10], [11].

α	Angle of attack	m	Amphibious aircraft mass
θ	Angle of pitch	D_a	Airodynamic drag force
V	Velocity	G_{xa}	gravity along X_s
α_t	Angle between engine force and X_b	G_{za}	gravity along Z_s
M_a	Total pitching moment of Air	X_g	aircraft position along X_e
M_w	Total pitching moment of Air	Z_g	aircraft position along Z_e
M_T	Total pitching moment of Engine	u	velocity component of aircraft velocity along X_b
T	Engine thrust	w	velocity component of aircraft along Z_b
N_w	Normal directional water pressure at the bottom of the aircraft	q	Pitch angular rate
D_f	Frictional force of water along the bottom of the aircraft	I_y	moment of Inertia of amphibious aircraft against Y_b
L_a	Aerodynamic lifting force	$\dot{\theta}$	Increase/decrease in angle of pitch
$\dot{V}$	Increase/decrease in velocity	$\dot{\alpha}$	Increase/decrease in angle of attack

Table 1: Description of amphibious aircraft model [4].

4 *H*-Infinity Algorithm

Below is the algorithm flow of the *H*-infinity method.

5 Simulation Results

In this paper, we compared the accuracy of simulation results, the resulted accuracy of the *H*-infinity method to that of the EnKF method by generating 300 and 600 ensembles. The simulation results are as shown in Figures 4-6, each figure represents the simulation results by the *H*-infinity and EnKF methods with 300 and 600 ensembles generated. And the errors of the simulation results are as compared in Table 2. In each figure, there are two images, that is, a and b, the image a means the simulation result by using 600 ensembles, and the image b means that by using 300 ensembles.

As seen in Figure 4, the red line is the real value of the horizontal position determined for a flight at a certain distance from the initial position of the amphibious aircraft so that the amphibious aircraft flies as high as 1400 m from the surface and advances about 2250 meters from the original place, so the position will always advance to 2250 m at the 100th iteration either by the *H*-Infinity method or by EnKF method. The horizontal position is closely related to the altitude at which when the horizontal position advances (does not turn), the altitude will follow the horizontal position to get the maximum flight

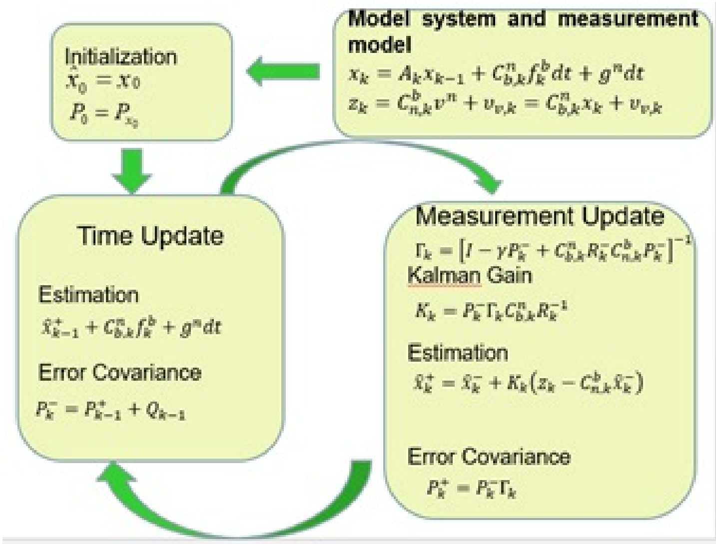

Figure 3: H-Infinity Algorithm [12], [11], [14].

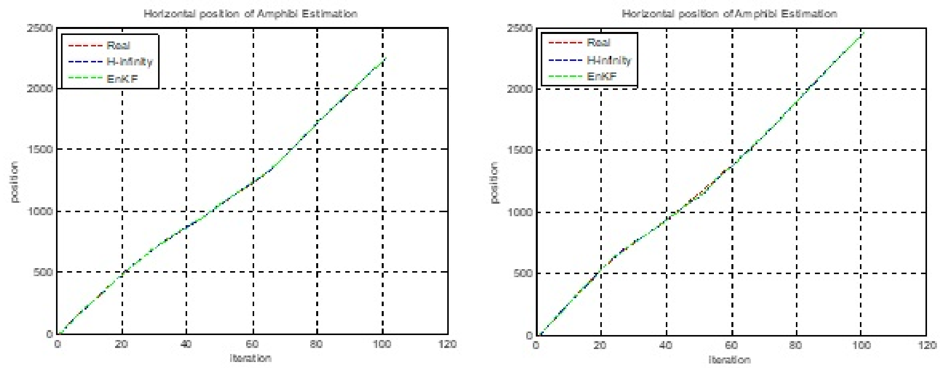

Figure 4: Horizontal position estimation of amphibious aircraft by the H-Infinity and EnKF methods, a) with 600 ensembles and b) with 300 ensembles.

altitude. In Figures 4 and 5, it can also be seen that the difference between the distance by H-infinity and that by EnKF is almost the same as the real value and the RMSE value is quite large, this can be seen in Table 2.

In Figure 5, it can be seen that the red line is the real value of the flight altitude determined for the amphibious aircraft in order to follow the predetermined trajectory. In this case, the height of the amphibious aircraft represents the maximum height the amphibious aircraft can pass, and the simulation is made for the amphibious aircraft to fly only and not to land again.

In Figure 6, it can be seen that the red line is the real horizontal position and the specified altitude that the amphibious aircraft will pass through while flying at a certain

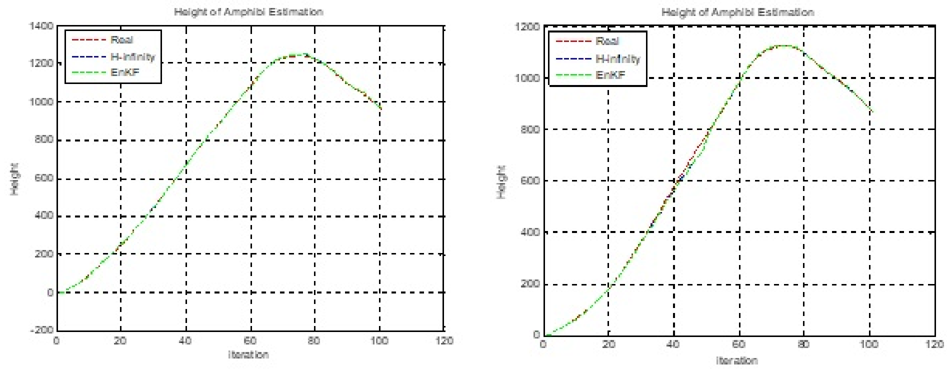

Figure 5: Height estimation of amphibious aircraft by the H-Infinity and EnKF methods, a) with 600 ensembles and b) with 300 ensembles.

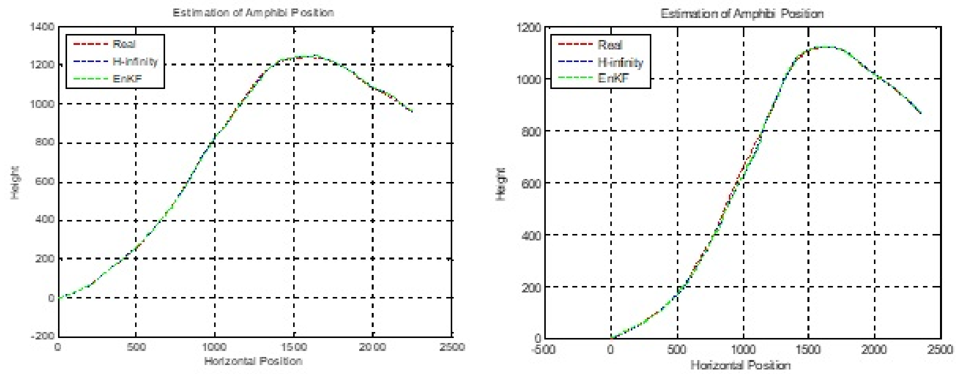

Figure 6: Position estimation of the amphibious aircraft on the predetermined trajectory by the H-Infinity and EnKF methods, a) with 600 ensembles and b) with 300 ensembles.

altitude and distance, of which Figure 6a has better accuracy than Figure 6b using 300 ensembles. Figure 6 shows that the amphibious aircraft follows the predetermined trajectory from the beginning of takeoff up to flying at a certain altitude. Table 2 also shows that the EnKF method has a smaller error than the H-infinty method.

	EnKF with 300 ensembles	H-infinity	EnKF with 600 ensembles	H-infinity
XY Motion	3.57%	4.79%	2.14%	4.56%
Time Simulation	8.45 s	7.5 s	14.73 s	12.89 s

Table 2: Comparison of the position estimation error of amphibious aircraft by the H-infinity and EnKF methods.

As shown in Table 2, these simulation results can be compared to each other. If we look at the comparison based on the accuracy of the methods, then the EnKF method has higher accuracy than the H-infinity, but it has a longer simulation time. If we look at the comparison based on the number of ensembles, then the generation of 600 ensembles indicates better accuracy than that of 300 ensembles. Overall, the EnKF method by generating 600 ensembles has the highest accuracy. In this case, both estimation methods, namely H-infinity and EnKF, have an accuracy of above 95%, so the estimation methods can be effectively used to estimate the motion of the amphibious aircraft.

6 Conclusion

Based on the results of the analysis of the two simulations above, it can be concluded that the EnKF method by generating 600 ensembles has the highest level of accuracy. If we look at the comparison based on the number of ensembles, then that with 600 ensembles has a better accuracy than that with 300 ensembles. Thus, both estimation methods, namely H-infinity and EnKF, have an accuracy of above 95%, so that the estimation methods can be effectively used to estimate the motion of amphibious aircraft.

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