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A New Chaotic Supply Chain Model, Its Bifurcation Analysis, Multi-Stability and Synchronization Using Backstepping Control

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Abstract: A supply chain is a network of interconnected organizations, people, activities, information, and resources involved in the creation and distribution of products or services from the raw material stage to the end consumer. It encompasses the entire process of transforming raw materials into finished products and delivering them to customers. In this paper, we have proposed a new mathematical model for the chaotic supply chain with one absolute nonlinearity and one quadratic function. Furthermore, we have validated stability analysis and dynamical analysis using numerical MATLAB simulation. Our finding system exhibits the index-1 spiral saddle and index-2 spiral saddle. We show that the new chaotic supply chain model exhibits multistability with coexisting chaotic attractors for different initial states. Finally, as a control application, active backstepping control has been applied to achieve complete synchronization of a pair of new chaotic supply chain models taken as the master and slave systems. The Lyapunov stability theory has been used to achieve the control result using active backstepping control.

Keywords: *chaos; dynamical system; supply chain management.*

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1 Introduction

The development of chaotic systems in supply chain management related to the study and application of chaos theory to understand and manage the complex and unpredictable behaviors that can emerge within supply chains [1]. Chaos theory is a branch of mathematics that deals with complex, non-linear systems that exhibit sensitive dependence on initial conditions [2]. In the context of supply chain management, chaotic behavior can lead to unexpected fluctuations, delays, and disruptions, which can significantly impact the efficiency and effectiveness of the supply chain [3].

Supply chains are inherently complex systems involving various interconnected entities such as suppliers, manufacturers, distributors, retailers, and customers [4–6]. The interactions among these entities can lead to non-linear and unpredictable behaviors [7]. Furthermore, chaotic behavior challenges traditional demand forecasting methods that assume linear relationships and steady-state conditions [8]. Instead, chaotic systems require more sophisticated approaches that account for sudden shifts and fluctuations in demand patterns [9, 10].

Chaos theory has also found applications in the field of economics, particularly in understanding complex and non-linear dynamics within economic systems such as financial markets [11] and bitcoin market [12], business cycles [13], decision making [14] and policy analysis [15]. Yingjin et al. [16] conducted research on the intricacies of the bullwhip effect within supply chains. The Bullwhip Effect, characterized by internal nonlinearity, was explored to comprehend its intricacies in the context of order processing under demand signals. Their aim was to establish a mathematical correlation connecting the bullwhip effect in the supply chain network with fractal and chaotic dynamics. Lei et al. [17] constructed a three-tier network for a supply chain using the dynamic and chaotic Lorenz model. They examined a nonlinear model for a three-level supply chain, which, under specific circumstances, can manifest as a set of chaotic Lorenz equations. They introduced the concept of synchronizing a chaotic supply chain network through the application of the RBF neural network technique. Additionally, the influence of external perturbations on this model was explored. Xu et al. [18] proposed an adaptive super-twisting (STW) sliding mode control (SMC) algorithm to manage the chaotic supply chain system, they demonstrated that the provided control creation combined with dynamic analysis is crucial for strategic decision-makers in contemporary supply chain management.

The main contribition of this work is assessing the stability and dynamic behavior of a new chaotic supply chain model featuring an absolute nonlinearity and a quadratic function. This analysis encompassed the use of phase portraits, Poincaré maps, Lyapunov exponents, and bifurcation diagrams.

Multistability is a complex phenomenon typically observed for chaotic nonlinear dynamical systems with the coexistence of different periodic or chaotic attractors for the same set of system parameters but different values of the initial states [19, 20]. In this research work, we also show that the new chaotic supply chain model exhibits multistability with coexisting chaotic attractors for different initial states.

Finally, as a control application, active backstepping control has been applied to achieve the complete synchronization of a pair of new chaotic supply chain models taken as the master and slave systems. The Lyapunov stability theory has been used to achieve the control result using active backstepping control. MATLAB simulations have been shown to illustrate the results presented in this research work.

2 A New Chaotic Supply Chain Model

In 2022, Hamidzadeh et al. [21] defined a chaotic supply chain model as follows:

$$\begin{cases} \dot{y}_1 = y_2, \\ \dot{y}_2 = y_3, \\ \dot{y}_3 = ay_1 - by_2 - y_3 - y_1^2, \end{cases} \tag{1}$$

where y_1 is retailers, y_2 is distributors, and y_3 is manufacturers, system (1) exhibits chaotic behavior with the parameter $a = 7.5; b = 3.8$ and the initial conditions are $y_1(0) = 1, y_2(0) = 1, y_3(0) = 1$.

In this work, we propose a chaotic supply chain with an absolute nonlinearity and a quadratic function, which is modelled as follows:

$$\begin{cases} \dot{y}_1 = y_2, \\ \dot{y}_2 = y_3, \\ \dot{y}_3 = ay_1 - by_2 - y_3 - |y_1| - y_1^2. \end{cases} \tag{2}$$

System (2) exhibits chaotic behavior with the parameter $a = 8.5; b = 3.8$ and the initial conditions are $y_1(0) = 0.1, y_2(0) = 0.1, y_3(0) = 0.1$. Using the Wolf algorithm, the Lyapunov characteristic exponents of the model (2) are $LE_1 = 0.18109, LE_2 = 0.11839$ and $LE_3 = -1.1929$. The Kaplan-Yorke dimension of the model (2) is derived as

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.251. \tag{3}$$

Subsequently, we compute the equilibrium positions of the recently introduced chaotic supply chain model (2). To achieve this, we address the resolution of the subsequent system of equations

$$\begin{cases} 0 = y_2, \\ 0 = y_3, \\ 0 = ay_1 - by_2 - y_3 - |y_1| - y_1^2. \end{cases} \tag{4}$$

It is a simple calculation to verify that the revised system of the chaotic supply chain model (2) is found to possess a pair of equilibrium points, specifically determined by

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 0 \\ 0 \end{bmatrix}. \tag{5}$$

The matrix provided below represents the Jacobian matrix associated with the chaotic supply chain model (2):

$$JE_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 8.5 - sign(x) - 2x & -3.8 & -1 \end{bmatrix}. \tag{6}$$

We find that

$$E_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 7.5 & -3.8 & -1 \end{bmatrix}, \tag{7}$$

which has the characteristics equation as follows:

$$\lambda^3 + \lambda^2 + 3.8\lambda - 7.5. \tag{8}$$

The Jacobian matrix JE_0 has the eigenvalues 1.1781, -1.0891±2.2759i. The system (2) exhibits the Index-1 spiral saddle point, which is unstable. The Jacobian matrix of the system (2) at $E_1 = (7.5,\ 0,\ 0)$ is obtained as follows:

$$E_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7.5 & -3.8 & -1 \end{bmatrix}, \tag{9}$$

which has the characteristics equation as follows

$$\lambda^3 + \lambda^2 + 3.8\lambda + 7.5 = 0. \tag{10}$$

The Jacobian matrix JE_1 has the eigenvalues -1.5858, 0.2929± 2.1548i. The system (2) exhibits the Index-2 spiral saddle point, which is unstable.

The Runge-Kutta 4th-order method is a type of the numerical integration technique that allows one to approximate the solution of an ODE by iteratively stepping through the independent variable [22]. It is particularly useful when there is no analytical solution available or when the ODE is too complex to solve directly. By using this method, in Figure 1, the MATLAB plots are displayed for the chaotic supply chain (2) with $(a, b) =$ (8.5, 3.8), with the initial conditions $Y(0) = (0.1, 0.1, 0.1)$.

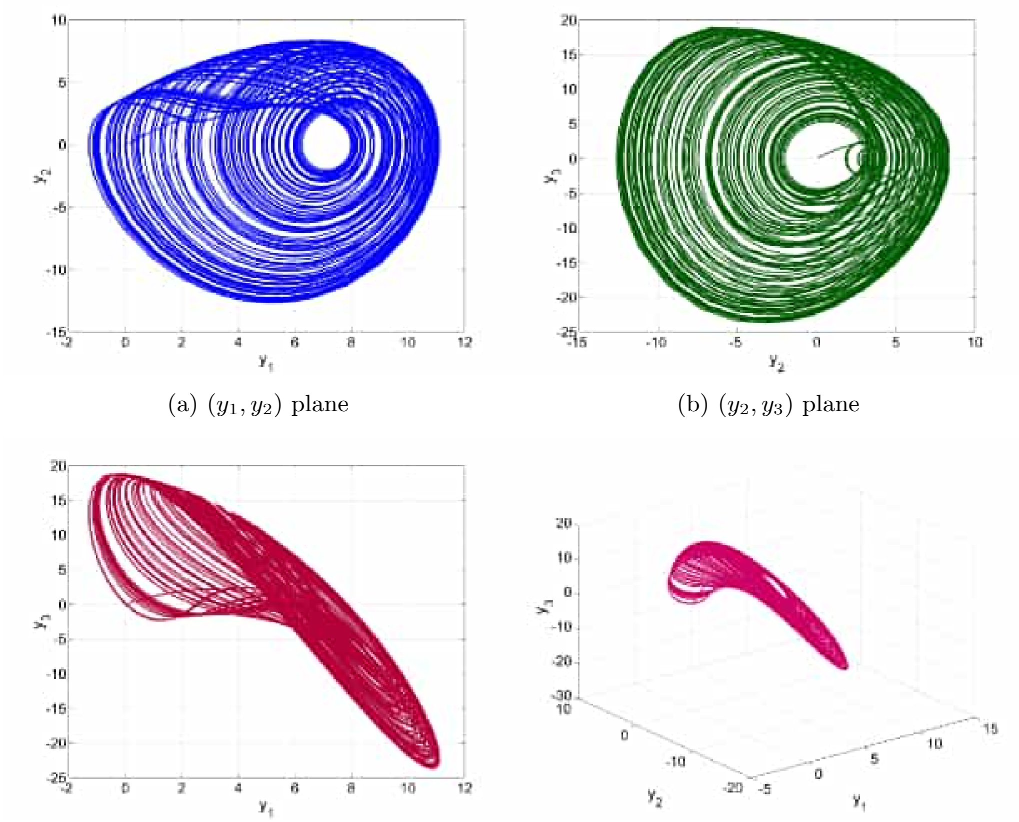
3 Dynamical Analysis of the New Chaotic Supply Chain Model

A bifurcation diagram is a graphical representation used in the field of dynamical systems and nonlinear mathematics to visualize the behavior of a system as a parameter changes [23]. It helps to illustrate how the qualitative behavior of a system changes as a parameter varies, particularly when the system undergoes bifurcations. Meanhwile, the Lyapunov exponent is a concept from the field of chaos theory and dynamical systems that quantifies the rate of exponential divergence or convergence of nearby trajectories in a nonlinear system [24]. It is used to characterize the sensitivity of a system to initial conditions, which is a fundamental aspect of chaotic behavior.

The behavior of the supply chain system (2) was explored by varying the bifurcation parameter a within the range of 6 to 9.5. The bifurcation diagram and Lyapunov exponent diagram of the chaotic supply chain system model (2) are presented in Figures 2(a) and 2(b), respectively. It is evident from these figures that the system (2) showcases both limit cycles and demonstrates chaotic patterns. Meanwhile, confirming the existence of a path to chaos through period doubling is straightforward when the parameter a is increased.

By altering the parameter b within the range of [3.8, 4.8], the behavior of the supply chain system (2) is explored. Figure 3(a) exhibits the bifurcation diagram, while Figure 3(b) presents the associated Lyapunov exponent spectrum of system (2). These figures illustrate the system's capacity to transition from chaotic to periodic behavior. The presence of a route from chaos to period doubling becomes apparent upon increasing the value of the parameter b.

A Poincaré map, also known as a Poincaré section or Poincaré surface of section, is a graphical tool used to study the behavior of chaotic systems and to analyze the dynamics of a system in a reduced-dimensional space. In addition, the Poincaré map of supply chain system (2) in Figure 4 exhibits chaotic characteristics.



(a) (y_1, y_2) plane

(b) (y_2, y_3) plane

(c) (y_1, y_3) plane

(d) The 3-D space

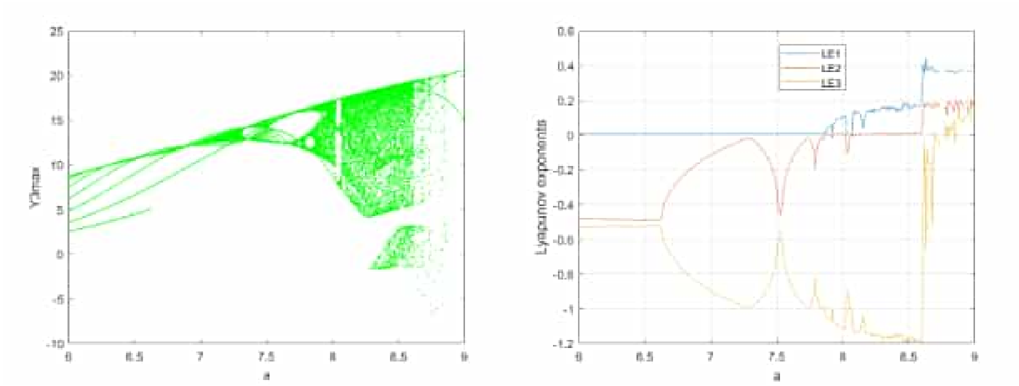
Figure 1: Chaotic attractors supply chain model (2) using MATLAB in (a) y_1 - y_2 plane (b) y_2 - y_3 plane, (c) y_1 - y_3 plane and (d) 3D plane.

4 Multistability of the New Chaotic Supply Chain Model

Multistability is a complex phenomenon typically observed for chaotic nonlinear dynamical systems with the coexistence of different periodic or chaotic attractors for the same set of system parameters but different values of the initial states [19,20]. In this research work, we also exhibit that the new chaotic supply chain model (2) exhibits multistability with coexisting chaotic attractors for different initial states.

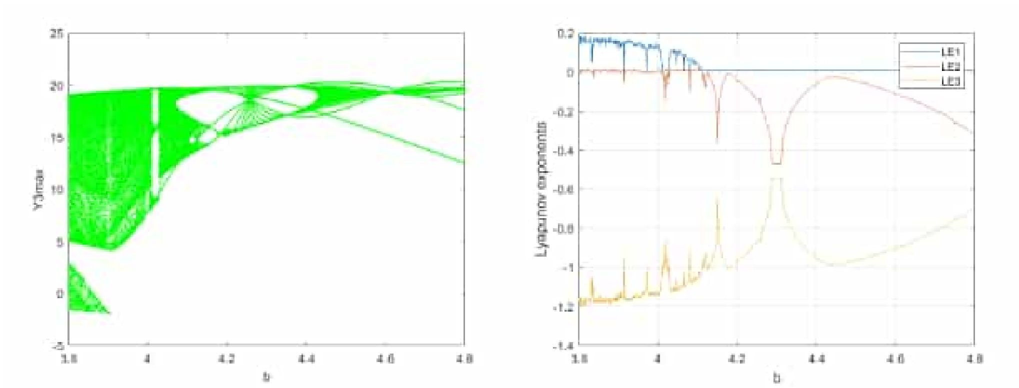
We fix the parametric values as in the chaotic case, *viz.* $a = 8.5$ and $b = 3.8$. We choose two initial states as $Y_0 = (0.1, 0.1, 0.1)$ and $Z_0 = (0.5, 0.2, 0.5)$, and we denote the corresponding state flows of the new chaotic supply chain model (2) as $Y(t)$ (in blue color) and $Z(t)$ (in red color), respectively.

Figure 5 shows the multistability with coexiting chaotic attractors of the new chaotic supply chain model (2), where the blue orbit corresponds to the chaotic attractor for $Y_0 = (0.1, 0.1, 0.1)$ and the red orbit corresponds to the chaotic attractor for $Z_0 = (0.5, 0.2, 0.5)$. In this figure, the parametric values are kept fixed at $a = 8.5$ and $b = 3.8$.



(a) Bifurcation plot (b) LE diagram

Figure 2: (a) Bifurcation plot and (b) Lyapunov exponents diagram of supply chain model (2) with variation of the parameter a.



(a) Bifurcation plot (b) LE diagram

Figure 3: (a) Bifurcation plot and (b) Lyapunov exponents diagram of supply chain model (2) with variation of the parameter b.

5 Complete Synchronization of the New Chaotic Supply Chain Models

In this section, we use the active backstepping control method [25] to solve the design problem of achieving the complete chaos synchronization of a pair of new chaotic supply chain models taken as the master and slave systems.

As the master system, we take the new chaotic supply chain model given by the jerk dynamics

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3, \\ \dot{y}_3 &= ay_1 - by_2 - y_3 - |y_1| - y_1^2. \end{aligned} \tag{11}$$

As the slave system, we consider the new chaotic supply chain model given by the

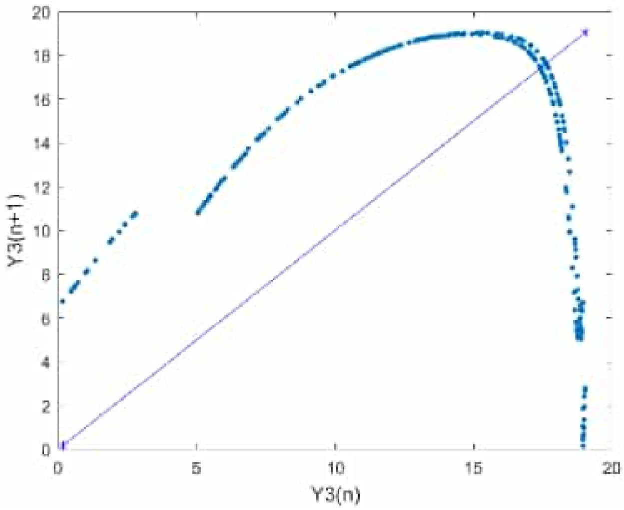
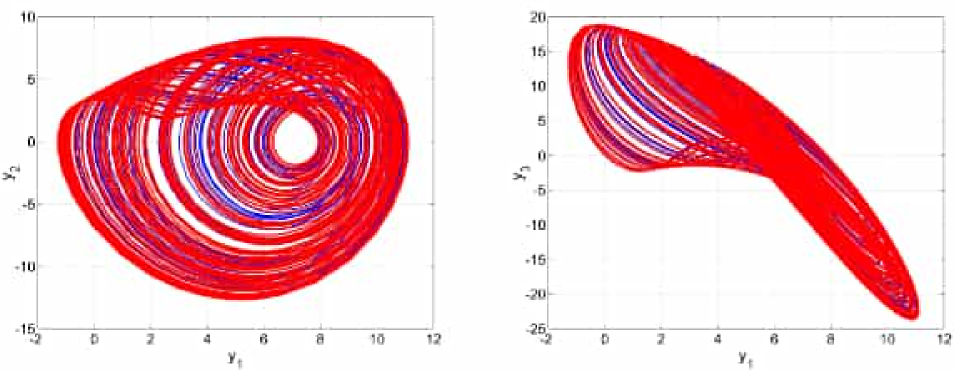


Figure 4: Poincaré section of the chaotic supply chain model (2).



(a) (y_1, y_2) plane

(b) (y_1, y_3) plane

Figure 5: Multistability of the new chaotic supply chain model (2).

jerk dynamics

$$
\begin{aligned}
\dot{z}_1 &= z_2, \\
\dot{z}_2 &= z_3, \\
\dot{z}_3 &= az_1 - bz_2 - z_3 - |z_1| - z_1^2 + v.
\end{aligned}
\tag{12}
$$

In Eq.(12), v is an active backstepping control, which is to be designed in this section.

The error between the chaotic supply chain models (11) and (12) can be defined as follows:

$$
\begin{aligned}
\mu_1 &= z_1 - y_1, \\
\mu_2 &= z_2 - y_2, \\
\mu_3 &= z_3 - y_3.
\end{aligned}
\tag{13}
$$

Using the dynamics of the systems (11) and (12), we can compute the error dynamics as follows:

$$\begin{aligned} \dot{\mu}_1 &= \mu_2, \\ \dot{\mu}_2 &= \mu_3, \\ \dot{\mu}_3 &= a\mu_1 - b\mu_2 - \mu_3 - |z_1| + |y_1| - z_1^2 + y_1^2 + v. \end{aligned} \tag{14}$$

Theorem 5.1 *The new chaotic supply chain models (11) and (12) are completely synchronized for all initial values and in* R^3 *when the active control law* v *is chosen as*

$$v = -(3+a)\mu_1 - (5-b)\mu_2 - 2\mu_3 + |z_1| - |y_1| + z_1^2 - y_1^2 - kw_3, \tag{15}$$

where $k > 0$ *is a gain constant and* $w_3 = 2\mu_1 + 2\mu_2 + \mu_3$.

Proof. We use the active backstepping control method [26] to establish the result in Theorem 5.1. First, we begin with the Lyapunov function given by

$$P_1(w_1) = \frac{1}{2}w_1^2, \tag{16}$$

where

$$w_1 = \mu_1. \tag{17}$$

Then we get

$$\dot{P}_1 = \mu_1\dot{\mu}_1 = \mu_1\mu_2 = -w_1^2 + w_1(\mu_1 + \mu_2). \tag{18}$$

In order to simplify our calculations, we define

$$w_2 = \mu_1 + \mu_2. \tag{19}$$

Using (19), we can simplify Eq.(18) as follows:

$$\dot{P}_1 = -w_1^2 + w_1w_2. \tag{20}$$

Next, we define the Lyapunov function given by

$$P_2(w_1, w_2) = P_1(w_1) + \frac{1}{2}w_2^2 = \frac{1}{2}w_1^2 + \frac{1}{2}w_2^2. \tag{21}$$

Then it is easy to show that

$$\dot{P}_2 = -w_1^2 - w_2^2 + w_2\left(2\mu_1 + 2\mu_2 + \mu_3\right). \tag{22}$$

In order to simplify our calculations, we define

$$w_3 = 2\mu_1 + 2\mu_2 + \mu_3. \tag{23}$$

Using (23), we can simplify Eq.(22) as follows:

$$\dot{P}_2 = -w_1^2 - w_2^2 + w_2w_3. \tag{24}$$

Finally, we define the Lyapunov function given by

$$P(w_1, w_2, w_3) = P_2(w_1, w_2) + \frac{1}{2}w_3^2 = \frac{1}{2}w_1^2 + \frac{1}{2}w_2^2 + \frac{1}{2}w_3^2. \tag{25}$$

Then we calculate the derivative of P along the error dynamics (14) as follows:

$$\dot{P} = -w_1^2 - w_2^2 + w_2 w_3 + w_3 \dot{w}_3 = -w_1^2 - w_2^2 - w_3^2 + w_3 Z. \tag{26}$$

A standard calculation yields the following:

$$Z = (3 + a)\mu_1 + (5 - b)\mu_2 + 2\mu_3 - |z_1| + |y_1| - z_1^2 + y_1^2 + v. \tag{27}$$

Substituting the formula given in Eq.(15) for v into Eq.(28), we get $Z = -kw_3$. Then Eq.(26) can be simplified as follows:

$$\dot{P} = -w_1^2 - w_2^2 - (1 + k)w_3^2. \tag{28}$$

Hence, we have shown that $\dot{P}$ is a quadratic and negative definite function on R^3.

Using the Lyapunov stability theory, we conclude that the error dynamics (14) is globally asymptotically stable.

Hence, the error variables $\mu_1(t)$, $\mu_2(t)$, and $\mu_3(t)$ converge to zero asymptotically as $t \to \infty$ for all initial error conditions. This completes the proof.

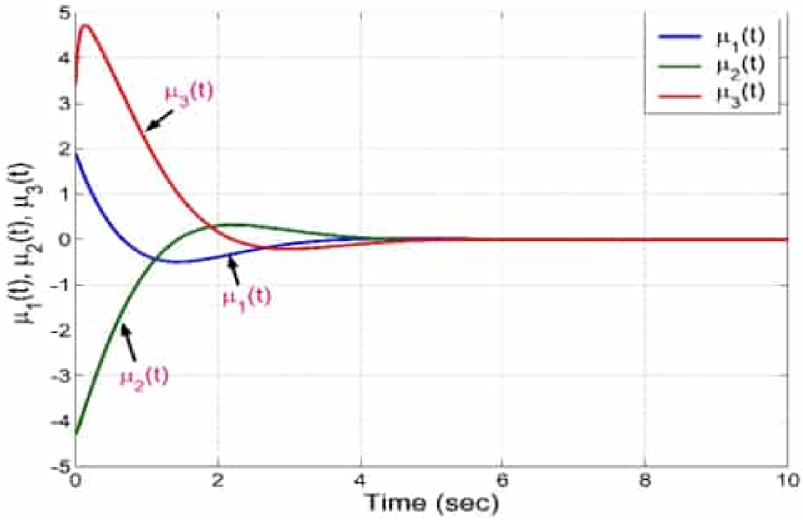


Figure 6: Convergence of the synchronization errors $\mu_1(t)$, $\mu_2(t)$, $\mu_3(t)$.

For MATLAB values, we take the values of a and b as in the chaotic case studied in Section 2, viz. $a = 7.5$ and $b = 3.8$. We pick the control gain constant k as $k = 20$.

The initial values of the master and slave chaotic supply chain systems given in the equations (11) and (12) are taken as $y(0) = (1.7,\ 5.4,\ 2.8)$ and $z(0) = (3.6,\ 1.1,\ 6.2)$, respectively.

Figure 6 shows the convergence of the synchronization errors between the master and slave chaotic supply chain systems given in the equations (11) and (12), respectively.

6 Conclusions

This study presents a new chaotic supply chain system featuring two unstable equilibrium points, which are saddle points. The main contribition of this work is assessed stability and dynamic behavior for a new chaotic supply chain model featuring an absolute non-linearity and a quadratic function. The fundamental dynamic properties of the system are explored using phase portraits, equilibrium analysis, the Kaplan–Yorke dimension, Lyapunov exponents, and bifurcation analysis. Our investigations reveal that the new chaotic supply chain system demonstrates both periodic and chaotic behaviors. Using the active backstepping control method, we derived a new control law for achieving the complete synchronization between the new chaotic supply chain models taken as the master and slave chaotic systems. In the future research, we will investigate the robustness of the proposed control method under various disturbances and uncertainties in the supply chain system. This could involve analyzing the performance of the control method in the presence of external factors that may affect the stability and synchronization.

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