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Existence of Solution for a General Class of Strongly Nonlinear Elliptic Problems

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Abstract: In this paper, we study the existence of solution for a general class of strongly nonlinear elliptic problems associated with the differential inclusion $\beta(u) + A(u) + g(x, u, Du) \ni f$, where A is a Leray-Lions operator from $W_0^{1,p}(\Omega)$ into its dual, β is a maximal monotone mapping such that $0 \in \beta(0)$, while $g(x, s, \xi)$ is a nonlinear term which has a growth condition with respect to ξ and no growth with respect to s but it satisfies a sign condition on s. The right-hand side f is assumed to belong to $L^\infty(\Omega)$.

Keywords: *inclusion problems; Leray-Lions operator; maximal monotone mapping; Sobolev spaces; truncation.*

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1 Introduction

Let Ω be a bounded domain in $\mathbb{R}^N (N \geq 1)$ with sufficiently smooth boundary $\partial\Omega$. Our aim is to show the existence of solutions for the following strongly nonlinear elliptic inclusion:

$$(E, f) \quad \begin{cases} \beta(u) + A(u) + g(x, u, Du) \ni f \text{ in } \mathcal{D}'(\Omega), \\ \\ u \in W_0^{1,p}(\Omega), \; g(x, u, Du) \in L^1(\Omega), \; g(x, u, Du)u \in L^1(\Omega), \end{cases}$$

where A is a Leray-Lions operator from $W_0^{1,p}(\Omega)$ into its dual $W^{-1,p'}(\Omega)$ $(1 < p < \infty)$ defined as $A(u) = -div(a(x, u, Du))$, $f \in L^\infty(\Omega)$, β is a maximal monotone mapping

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