



# A New Numerical Scheme for Solving Time-Fractional Variable-Order Partial Differential Equations

Soufiane Benyoussef<sup>1\*</sup>, Oumkeltoum Benhamouda<sup>2</sup>, Mohamed Dalah<sup>3</sup>  
and Khaled Zennir<sup>4,5</sup>

<sup>1\*</sup> *Department of Science and Technology, Faculty of Applied Sciences,  
University of Ibn Khaldoun, Tiaret 14000, Algeria.*

<sup>2</sup> *Faculté des Sciences et de la Technologie,  
USTHB, B. P. 32, El-Alia 16111, Bab-Ezzouar, Alger, Algeria.*

<sup>3</sup> *Department of Mathematics, Faculty of Exact Sciences,  
University Constantine 1, Frères Mentouri,*

*Applied Mathematics and Modeling Laboratory, Constantine 25 017, Algeria.*

<sup>4</sup> *Department of Mathematics, College of Science, Qassim University, SAudi Arabia.*

<sup>5</sup> *Department of Mathematics, Faculty of Science, University of 20 Aout 1955 - Skikda, Algeria.*

Received: October 22, 2023; Revised: June 9, 2024

**Abstract:** In this work, we study approximations of solutions of fractional differential equations of order  $\alpha$  by using an implicit finite difference scheme (IFDS). A discretization and development of the scheme is obtained by using different approaches to fractional derivatives. The implicit finite difference scheme (IFDS) approach is followed in order to derive a simple discretization of the space fractional derivatives. The consistency, stability and convergence of the method are proved. Several examples illustrating the accuracy of the method are given. Moreover, we study the stability and convergence of the implicit finite difference scheme (IFDS) applied to the numerical solution of the fractional differential equations of order  $\alpha$ . Two tests for our problem are solved numerically to verify the effectiveness of the proposed numerical scheme.

**Keywords:** *fractional derivatives; stability; consistence; convergence; numerical scheme.*

**Mathematics Subject Classification (2010):** 93A30, 26A33, 65N06, 34K28.

---

\* Corresponding author: <mailto:soufiane.benyoussef@univ-tiaret.dz>