Nonlinear Dynamics and Systems Theory, **24** (5) (2024) 431–441

On Localization of Spectrum of an Integro-Differential Convection-Diffusion-Reaction Operator

O. Bahi [1*], A. Khellaf [1] and H. Guebbai [2]

[1] *Ecole Nationale Polytechniques de Constantine, Algeria.*
[2] *University 8 May 1945, Guelma, Algeria. Laboratoire de Mathematiques Applqiuees et Modelisation (LMAM).*

Received: April 10, 2024; Revised: October 1, 2024

Abstract: This paper explores the spectral properties of a non-self-adjoint integral-differential operator defined on an unbounded domain. The operator is governed by the Dirichlet-type conditions. We utilize the pseudo-spectral theory to demonstrate that the operator's spectrum is localized in the real numbers.

Keywords: *non-self-adjoint operators; unbounded operators; spectral analysis; integral-differential operators.*

Mathematics Subject Classification (2010): 47Axx, 58Jxx, 35Gxx.

1 Introduction

Non-self-adjoint and unbounded operators are fundamental in numerous branches of physics and chemistry, where phenomena like convection, diffusion, and reactions are widespread, see [1–3] and references therein. In this study, we focus on the spectral analysis of a non-self-adjoint integral-differential operator of convection-diffusion-reaction type, defined on an unbounded domain and subject to the Dirichlet-type conditions. The operator under consideration, denoted as L, is defined by the expression

$$L\xi = -\Delta\xi + \begin{pmatrix} -y \\ -x \end{pmatrix} \cdot \nabla\xi + (x^2+y^2)\xi + \int_\Gamma k(x,y,z,t)\xi(z,t)dzdt.$$

Convection equations can be considered as dynamic systems [4–6], where the state of the system evolves over time. They describe the transport of a quantity under the effect of a velocity field and can be analysed using the theory of dynamical systems and

[*] Corresponding author: mailto:bahi.oussama@enpc.com