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Relationship between Persymmetric Solutions and Minimal Persymmetric Solutions of $AXA^{(*)} = B$

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Abstract: The minimal rank persymmetric solution of the quaternion matrix equation $AXA^{(*)} = B$ is defined as the matrix X that minimizes the rank of the difference $AXA^{(*)} - B$ or, equivalently, $r\big(AXA^{(*)} - B\big) = \min$, where B is persymmetric. In this paper, we focus on the quaternion matrix equation $AXA^{(*)} = B$. Our aim is to investigate the inclusion relationships between two sets, Ω_1 and Ω_2, where Ω_1, Ω_2 are, respectively, the set of persymmetric solutions and the set of minimal rank persymmetric solutions of the quaternion matrix equation $AXA^{(*)} = B$. Then, we deduce the necessary and sufficient conditions for the following relations to hold: $\Omega_1 \cap \Omega_2 \neq \varnothing$, $\Omega_1 \subseteq \Omega_2$ and $\Omega_1 \supseteq \Omega_2$.

Keywords: *linear system; persymmetric solution; Moore-Penrose inverse; rank.*

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1 Introduction

Throughout this paper, $\mathbb{R}$ and $\mathbb{C}$ stand for the real number field and the complex number field, respectively. Let $\mathbb{H}^{m\times n}$ be the set of $m\times n$ matrices over the real quaternion algebra:

$$\mathbb{H} = \left\{ a_0 + a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k} \mid \boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{ijk} = -1,\ a_0, a_1, a_2, a_3 \in \mathbb{R} \right\}.$$

The symbols A^* and $r\,(A)$ stand for the conjugate transpose and the rank of A, respectively. I_m denotes the identity matrix of order m. The Moore-Penrose generalized

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