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Complex Dynamics of Novel Chaotic System with No Equilibrium Point: Amplitude Control and Offset Boosting Control, Its Adaptive Synchronization

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Abstract: In this paper, a novel chaotic system with dissipative nature is introduced. In the proposed system, a conservative system can be modified to a dissipative system by adding a linear term. The chaotic dynamics of the new system such as Lyapunov exponents, Lyapunov dimensions, Poincare plots and attractor plots are verified through numerical simulations. The dynamical analysis is also conducted to verify the existence of chaotic attractors for the particular system parameters. It is found that the amplitude and position of the proposed chaotic attractors can be controlled. The numerical simulations revealed that the new system has many complex dynamics which can be used for various applications. Finally, the chaos synchronization for the proposed system is demonstrated by designing the nonlinear adaptive controllers. The efficiency of the synchronization methodology is verified theoretically by the Lyapunov stability theorem and numerical simulation in MATLAB environment.

Keywords: *chaotic system; no equilibrium points; amplitude control; offset boosting control; chaos synchronization*

Mathematics Subject Classification (2010): 93A30, 93B05, 93D21, 93-XX.

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1 Introduction

Chaotic systems are characterized by unpredictability, sensitivity dependent on initial conditions and system parameters, fractal dimension and Lyapunov exponents. Due to this complex behavior, for the past decades, researchers have reported many chaotic systems with unique features such as stable equilibrium [1], different equilibria [2], non-hyperbolic equilibrium points [3], fractional order [4], hidden attractors [5, 6] and found many applications in science and engineering fields [7–9]. The Lyapunov exponent values and Lyapunov dimension play important roles in the design of chaotic systems. A nonlinear system must have at least one positive Lyapunov exponent to have chaotic dynamics. For a chaotic system with dissipative flow, the sum of Lyapunov exponent values should be non-zero and negative. The Lyapunov dimension value for a dissipative chaotic system must be a real number. In a dissipative system, the system limit sets are confined to a specific limit set of zero volume, and the asymptotic motion of the chaotic system settles onto a strange attractor of the system. The chaotic system with conservative flow is reported in [10] as described in Equation (1):

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x + ayz, \\ \dot{z} &= b - y^4, \end{aligned} \tag{1}$$

where the parameter values are $a = b = 1$. The Lyapunov exponent values of the system (1) are $l_1 = 0.0836$, $l_2 = 0$ and $l_3 = -0.0836$. The Kaplan-Yorke dimension of the system (1) is $D_L = 3$. In this paper, a new chaotic system with dissipative flow is constructed by adding a liner term x and by adjusting the parameter in the third differential equation of (1).

The amplitude control [11, 12] and offset boosting control [13, 14] are the important research problems in the chaotic systems. The amplitude control in the chaotic system can be achieved by multiplying the chaotic signals with a constant parameter. The amplitude control of chaotic signals has many applications such as chaotic modulation and signal processing. The offset boosting control of the chaotic attractors can be achieved by adding a constant parameter with the particular signal. The offset boosting control can be used to convert the bipolar chaotic signal into the unipolar signal or vice versa. The proposed system has the complete amplitude control feature, that is, the amplitude of all the signals can be controlled in the new system. The attractors of the proposed system can be offset boosted along the x and z directions.

Due to the butterfly effect and sensitivity to the initial conditions, the synchronization of even the identical chaotic systems is a challenging research work. The chaos synchronization has engineering applications such as secure transmission system. In the last three decades, many methods have been reported to address the problems in chaos synchronization [15–18]. The adaptive chaos synchronization deals with the design of adaptive controllers to stabilize the chaotic signals globally. In the chaos synchronization methodology, a particular system can be considered as a master system and another or same system can be considered as a slave system. The main idea of the chaos synchronization is to design an adaptive controller so that the slave system follows the output of the master system asymptotically. In this paper, an adaptive controller is designed for the proposed system and verified the stability of the controlled system through the Lyapunov stability theory. The numerical simulation indicates the efficiency of the proposed synchronization methodology.

2 Description of New Chaotic System

The new chaotic system can be described mathematically as the following dynamics:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x + ayz, \\ \dot{z} &= \alpha - y^6 + x, \end{aligned} \tag{2}$$

where x, y, z are the state signals and α is the system parameter. The parameter values are chosen as $a = 1$ and $\alpha = 4$.

3 No Equilibrium Points

The equilibrium points of the system (2) can be calculated by equating $\dot{x} = 0$, $\dot{y} = 0$ and $\dot{z} = 0$ in Eq.(2),

$$y = 0, \tag{3a}$$

$$-x + ayz = 0, \tag{3b}$$

$$\alpha - y^6 + x = 0. \tag{3c}$$

From (3a), $y = 0$. If we replace $y = 0$ in (3b) and (3c), we will get $x = 0$ and $x = -\alpha$, respectively. It is clear that Eq.(3b) and Eq.(3c) contradict each other and no solution exists for Eq.(3). It can be concluded that the new system (2) has no equilibrium points. The system (2) can be able to generate chaotic attractors even though there are no equilibrium points.

4 Chaotic Dynamics

In this section, the various chaotic dynamics of the system (2) such as the Lyapunov exponents and Lyapunov dimension, dissipativity, sensitivity, Poincare plot, and attractors diagram are verified by numerical simulations.

4.1 Lyapunov exponents and Lyapunov dimension

The Lyapunov exponent values of the system (2) are calculated using classical Wolf's algorithm with the simulation time of 10000 sec and step size 0.1. The Lyapunov values of the system (2) are $LE_1 = 0.05223, LE_2 = 0, LE_3 = -0.07433$ for the parameters $a = 1$, $\alpha = 4$ and the initial condition $(1, -1, 1)$. The Lyapunov dimension D_L of the system (2) can be calculated as

$$D_L = 2 + \frac{l_1 + l_2}{|l_3|} = 2 + \frac{0.05223}{0.074333} = 2.7026.$$

Since the Maximum Lyapunov Exponent (MLE) of the system (2) has a positive value and the Lyapunov dimension D_L has a fractional value, the proposed system (2) satisfies the conditions required for it to be chaotic. Figure 1 represents the time history of Lyapunov exponents of the system (2) over $t \in [0, 10000]$, which indicates that the system has at least one positive Lyapunov exponent value for the entire simulation time.

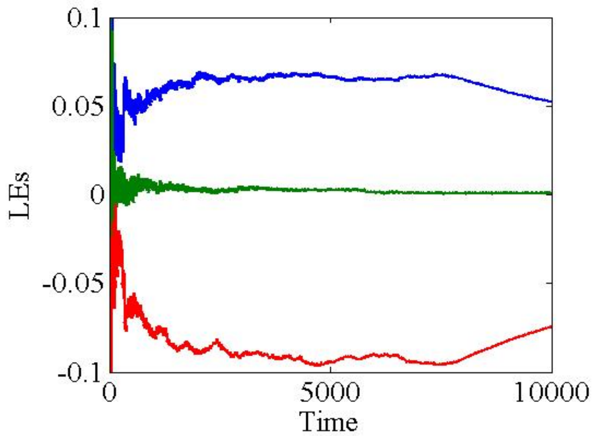


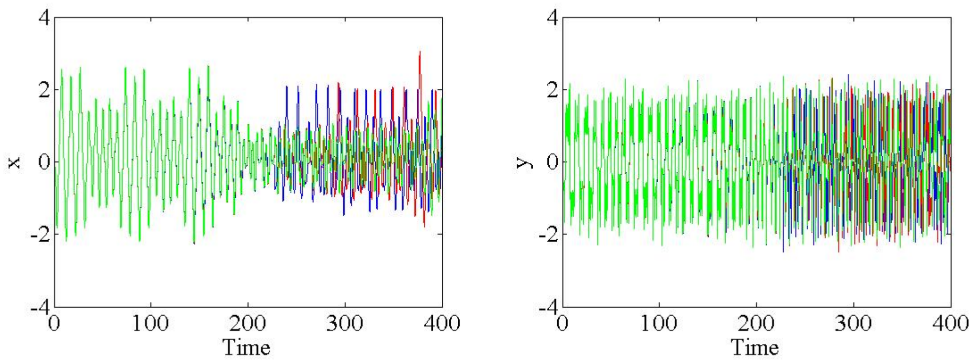
Figure 1: Time variation of the Lyapunov exponents of the system (2).

4.2 Dissipative nature

Since the sum of all the Lyapunov exponent values, i.e., $l_1 + l_2 + l_3 = -0.092494$, has a negative value, the proposed system (2) has dissipative behavior. It can be concluded that a dissipative system (2) can be constructed from the conservative system (1).

4.3 Sensitivity to initial conditions

The sensitivity to small variations in the initial conditions and unpredictability are the essential characteristics of the nonlinear dynamical system to be chaotic. Figure 2 shows the time variation of x and y signals of the system (2) with the initial conditions $(1, -1, 1)$ (Blue), $(1, -1, 1.0001)$ (Red) and $(1.0001, -1, 1)$ (Green). Figure 2 depicts that the state signals of the system (2) have different trajectories after the simulation time $t = 220sec$.

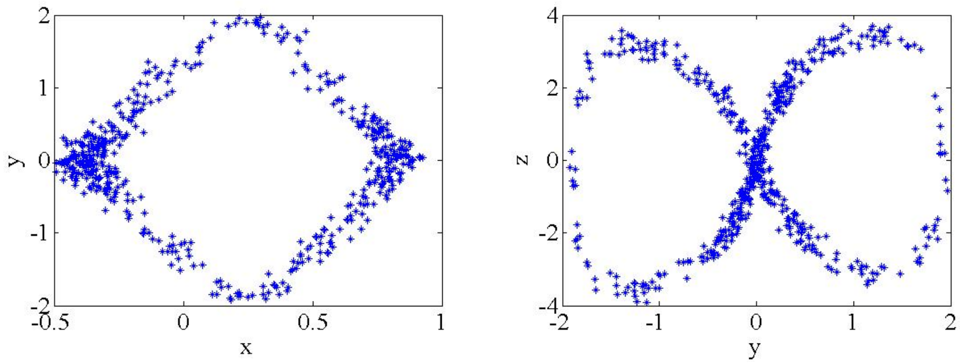


(a) x signal (b) y signal

Figure 2: Time variation of x and y signals with the initial conditions $(1, -1, 1)$ (Blue), $(1, -1, 1.0001)$ (Red) and $(1.0001, -1, 1)$ (Green).

4.4 The Poincaré plot

The Poincaré plot can be used to find out the chaotic behavior in a dynamical system by embedding a data set in a higher-dimensional state space. The Poincaré sections of the system (2) are plotted with the simulation time 50000 and step size 0.1 and presented in Figure 3. It can be observed from Figure 3 that the Poincaré sections have a set of distinct points, which indicates that the system (2) has chaotic behavior. It is interesting



(a) xy plane (b) yz plane

Figure 3: Poincaré plot of the system (2) with the initial conditions $(1, -1, 1)$.

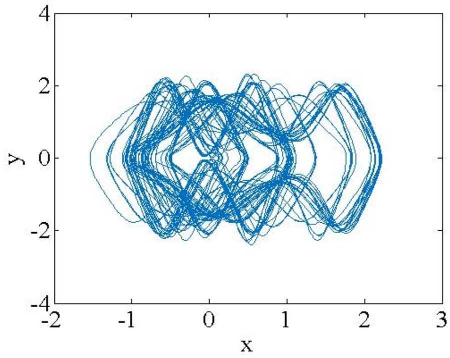
to note that the system (2) can have chaotic behavior even though there is no equilibrium point within itself. For the parameters $a = 1$ and $\alpha = 4$, the system (2) produces chaotic attractors as shown in Figure 4.

5 Dynamical Analysis

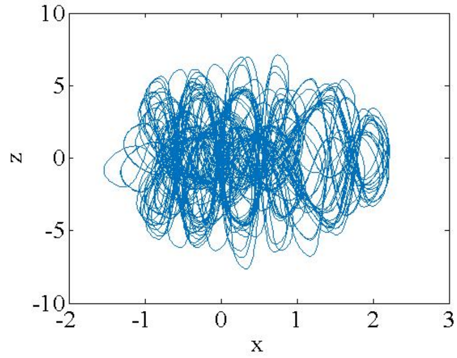
The dynamical analysis of the proposed system (2) is conducted with the help of classical non linear tools such as the bifurcation diagram and Lyapunov exponent spectrum. The bifurcation and Lyapunov exponent plots can be obtained by varying the system parameter gradually up to a particular level. The bifurcation diagram can be used to analyze the various states of a nonlinear dynamic system such as chaotic state, periodic state and limit cycle oscillation under the particular range of system parameter. The Lyapunov exponent spectrum having at least one positive Lyapunov exponent value in the region of system parameter indicates the sensitivity to the initial conditions or the existence of the chaotic dynamics in the particular system. Figure 5 depicts the bifurcation and Lyapunov exponent plots of the system (2) for the variation of the parameter $\alpha \in [0, 5]$. Figure 5a indicates that the system (2) has chaotic dynamics from $\alpha = 1.2$ to $\alpha = 5$. Figure 5b also represents that the system has chaotic dynamics in the regions $\alpha \in [1.2, 5]$ at which the system (2) has positive Lyapunov exponent values.

6 Amplitude Control

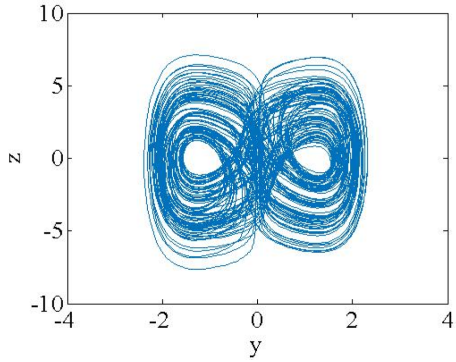
The amplitude of the chaotic signals can be controlled in the proposed system (2) by rescaling the system variables using the control parameter δ. If the signals are rescaled



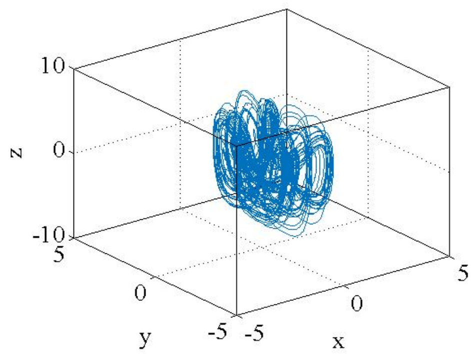
(a) xy



(b) xz

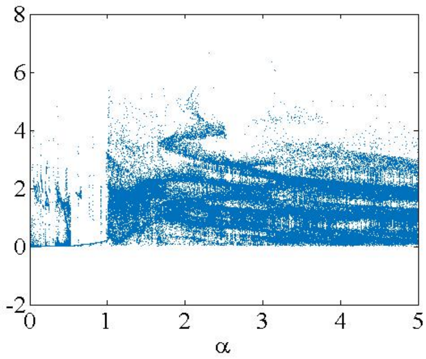


(c) yz

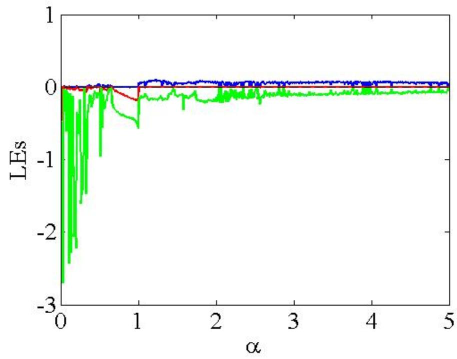


(d) xyz

Figure 4: Chaotic attractors of the new system (2) with the parameter $\alpha = 4$ and the initial conditions $(1, 1, 1)$.



(a)



(b)

Figure 5: (a) Bifurcation diagram, (b) Lyapunov spectrum of the system (2) with the initial conditions $(1, -1, 1)$.

as δx, δy and δz in the system (2), then the system (2) becomes a complete amplitude controllable system (4):

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -ax + yz\delta, \\ \dot{z} &= \alpha - \delta^5 y^6 + x. \end{aligned} \tag{4}$$

The system (4) also has no equilibrium points because of the contradiction between the second and third equations of (4). The bifurcation diagram of the system (4) with $\delta = 1$ (Blue), $\delta = 0.7$ (Red) and $\delta = 1.5$ (Green) is given in Figure 6a which indicates that the bifurcation level increases for $\delta < 1$ and reduces for $\delta > 1$. Figures 6b - 6d are the chaotic attractors of the amplitude controlled system (4) with $\delta = 1$ (Blue), $\delta = 0.7$ (Red) and $\delta = 1.5$ (Green).

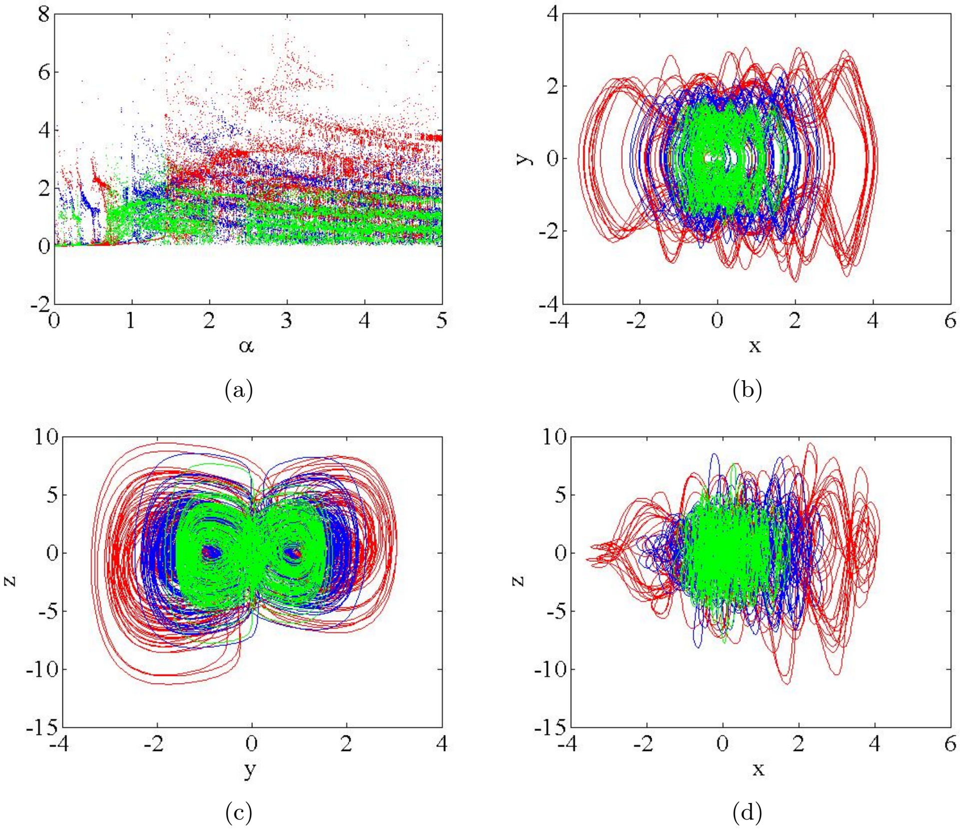


Figure 6: (a) Bifurcation diagram of the system (4), (b-d) Chaotic attractors of the system (4) with $\delta = 1$ (Blue), $\delta = 0.7$ (Red) and $\delta = 1.5$ (Green).

7 Offset Boosting Control

The offset boosting control of the chaotic signal is important for the various applications where the conversion of bipolar chaotic signals into unipolar chaotic signals and vice versa is required. The system (2) can be offset boostable along the x and z directions when we add a booster parameter with the signals. Equations (5) and (6) indicate the dc offset boosted system along the x and z directions, respectively, where β and α are the booster parameters,

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -a(x+\beta) + yz, \\ \dot{z} &= \alpha - y^6 + (x+\beta), \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -ax + y(z+\sigma), \\ \dot{z} &= \alpha - y^6 + x. \end{aligned} \tag{6}$$

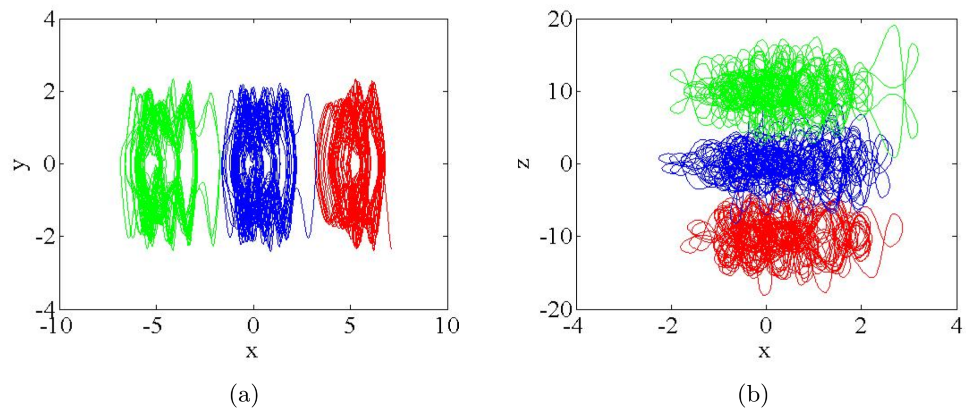


Figure 7: The offset boosted attractors of the system (2) with the initial conditions $(1, -1, 1)$. (a) x direction, (b) z direction.

Figure 7 represents the dc offset boosted attractors of the system (2) along the x and z directions with the control parameter $\beta = 0$ (Blue), $\beta = -5$ (Red) and $\beta = 5$ (Green), $\sigma = 0$ (Blue), $\sigma = 10$ (Red) and $\sigma = -10$ (Green). Note that the signals are shifted to a positive direction for the negative booster and the signals are shifted to a negative direction if the booster parameter has a positive value.

8 Adaptive Synchronization

One of the important practical applications of chaotic systems is secure communication in which the chaotic systems can act as the transmitter (master system) and receiver (slave system). In the past few decades, chaos synchronization has received great attention owing to its applications in designing secure communication systems. In this section, the adaptive synchronization of the proposed system is addressed for practical applications. The adaptive synchronization between the system (2) is achieved using nonlinear feedback control and master-slave synchronization methodology. To achieve chaos synchronization, the system (7) is considered as the master system and the system (8) is considered as the slave system. The main idea of the synchronization is to design the adaptive controllers

u by which the slave system(8) follows the master system (7) in an adaptive manner,

$$\begin{aligned} \dot{x_1} &= y_1, \\ \dot{y_1} &= -x_1 + a y_1 z_1, \\ \dot{z_1} &= \alpha - y_1^6 + x_1, \end{aligned} \tag{7}$$

$$\begin{aligned} \dot{x_2} &= y_2 + u_1, \\ \dot{y_2} &= -x_2 + a y_2 z_2 + u_2, \\ \dot{z_2} &= \alpha - y_2^6 + x_2 + u_3. \end{aligned} \tag{8}$$

The adaptive synchronization errors can be defined as given in Equation (9):

$$\begin{aligned} e_1 &= x_2 - x_1, \\ e_2 &= y_2 - y_1, \\ e_3 &= z_2 - z_1. \end{aligned} \tag{9}$$

Thus, the error dynamics can be written from the equations (7)-(9) as given in (10):

$$\begin{aligned} \dot{e}_1 &= \dot{x}_2 - \dot{x}_1 = e_2 + u_1, \\ \dot{e}_2 &= \dot{y}_2 - \dot{y}_1 = -e_1 + a(y_2 z_2 - y_1 z_1) + u_2, \\ \dot{e}_3 &= \dot{z}_2 - \dot{z}_1 = e_1 - y_2^6 + y_1^6 + u_3. \end{aligned} \tag{10}$$

According to the adaptive methodology, the adaptive controllers can be written from (10) as follows:

$$\begin{aligned} u_1 &= -e_2 - k_1 e_1, \\ u_2 &= e_1 - \hat{a}(y_2 z_2 - y_1 z_1) - k_2 e_2, \\ u_3 &= -e_1 + y_2^6 - y_1^6 - k_3 e_3, \end{aligned} \tag{11}$$

where $\hat{a}$ is the estimate value of the unknown parameter a and k_1, k_2, k_3 are the gain of the controllers. By substituting (11) in (10), the error dynamics can be modified as follows:

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1, \\ \dot{e}_2 &= e_a(y_2 z_2 - y_1 z_1) - k_2 e_2, \\ \dot{e}_3 &= -k_3 e_3. \end{aligned} \tag{12}$$

Here, the parameter error $e_a = a - \hat{a}$ and thus the parameter error dynamics can be written as

$$\dot{e}_a = -\dot{\hat{a}}. \tag{13}$$

Now, consider a positive Lyapunov definite function as follows:

$$\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_a \dot{e}_a. \tag{14}$$

By substituting (12) and (13) in the equation (14), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a[e_2(y_2 z_2 - y_1 z_1) - \dot{\hat{a}}]. \tag{15}$$

If we choose the parameter error dynamics $\dot{\hat{a}} = -e_2(y_2 z_2 - y_1 z_1)$, then the Lyapunov definite function becomes as follows:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2$$

which is a negative definite Lyapunov function. According to the Lyapunov stability theory, the equation (15) indicates that the adaptive synchronization errors e_1, e_2, e_3 and the parameter error e_a decay exponentially to zero with time. For the simulation purpose, the initial conditions are chosen for master (7) and slave system (8) as $\{1, -0.5, -2\}$ and $\{-1, 2.5, 3\}$, respectively.

Figures 8a – 8c represents the synchronized signals of the master and slave chaotic systems. Figure 8d represent the time variation of adaptive synchronization errors between the master and slave systems. Figure 8d indicates that all the signals are synchronized after 12 seconds and the error becomes zero.

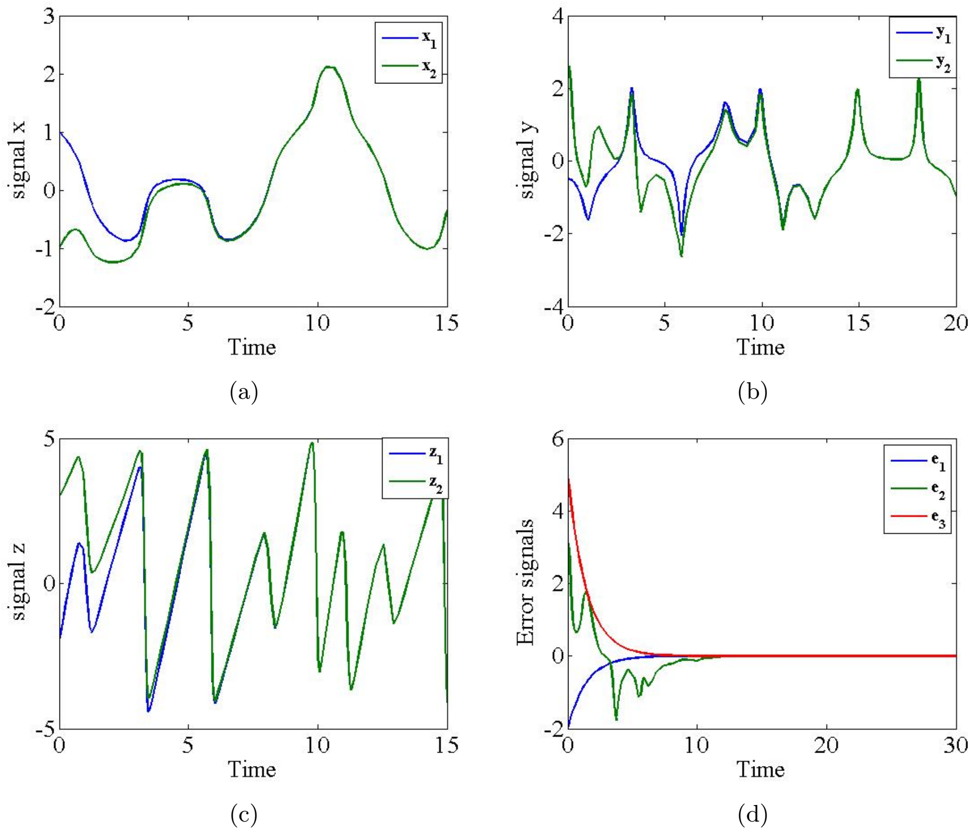


Figure 8: (a-c) The time variation of adaptively synchronized signals x, y, z, respectively, (d) The time variation of the error signals.

9 Conclusion

A novel three-dimensional chaotic system having no equilibrium points has been studied in this work. The results of the numerical simulation indicate that the proposed system satisfies all the conditions required for it to be chaotic. The dynamical analysis indicates that the new system does not lose its chaotic dynamics with a small variation of the parameter value. It is proved that the amplitude of all the signals of the proposed system can be controlled up to a particular level, which can be used to design a chaotic amplifier. The numerical simulation of the offset boosting control indicates that the x and z signals of the new system can be DC offset boosted. The DC offset boosting control behavior of the new system can be used to reduce the number of modulator devices in communication systems. The practical application of the proposed system is addressed by achieving chaos synchronization between identical systems. The adaptive synchronization methodology based on the Lyapunov stability method has been applied to synchronize the proposed system. The MATLAB results proved that the designed adaptive controllers can achieve chaos synchronization within a very short simulation time.

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