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Analysis and Optimal Control of a Mathematical Model of Malaria

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Abstract: In this work, we propose a mathematical model of malaria which takes into account the vector class represented by S_v, I_v and humans class represented by S_h, E_h, I_h and R_h. The basic reproduction number R_0 of the model is determined. We introduce two controls in our initial model. Therefore, the model with control will be presented and studied. The objective of the model with optimal control is to observe the effect of preventive measures, represented here by control u_1, and curative measures, represented by control u_2, on the evolution of malaria disease. The controls u_1 and u_2 will be characterized. Then we use the Python software for the numerical simulation of the model.

Keywords: *malaria; reproduction number; vector; simulation; optimal control.*

Mathematics Subject Classification (2010): 93B05, 93A30, 49J15, 49N90.

1 Introduction

Malaria is an acute febrile illness caused by Plasmodium parasites, which are spread to people through the bites of infected female Anopheles mosquitoes. It is preventable and curable. Malaria is a life-threatening disease primarily found in tropical countries. It was first discovered in India in the 15^{th} century. However, without prompt diagnosis and effective treatment, a case of uncomplicated malaria can progress to a severe form of the disease, which is often fatal without treatment. Malaria is not contagious and cannot spread from one person to another; the disease is transmitted through the bites of female Anopheles mosquitoes. The world's population is at risk of exposure to malaria [9]. In 2021, an estimated 247 million people contracted malaria in 85 countries. That same year, the disease claimed approximately 619000 lives [9]. The first symptoms of malaria

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usually appear within 10 to 15 days after the infectious bite of mosquitoes. Fever, headache and chills are usually signs of malaria, although these symptoms can hardly be attributed to malaria. WHO recommends rapid diagnostic testing for anyone suspected of having malaria. If Plasmodium falciparum malaria is not treated within 24 hours, the infection can progress to severe illness and death [9]. Malaria can be diagnosed using tests that determine the presence of the parasites causing the disease. There are two main types of tests, firstly, microscopic examination of blood smears and rapid diagnostic test called (TDR). Artemisinin based combination therapies (ACT) are the most effective antimalarial medicines available today. The resistance of the parasite to the different treatments leads to an endemic situation. Given all these threats, a mathematical model of malaria has been proposed to eradicate the disease. The most used means are the preventive ones. A number of recent studies of malaria show the significant direct effect of climatic factors such as temperature and rainfall on the transmission dynamics of vectors [11, 13–16, 18, 19]. B. Traore et al. [20] studied a mathematical model of malaria taking into account mosquito larvae and transmission of malaria in a periodic environment with a constant recruitment of vector and human population. Abba B. Gumel et al. [11] studied a malaria model taking into account seasonality and temperature variation in the mode of malaria transmission. Our model takes into account the preventive measures (distribution of mosquito nets, preventive medicine for children under 5 years old, spraying of areas etc.) represented by the control (u_1) and taken during a year. We use a control of cured persons (u_2) in order to allow the government to support population.

The structure of the paper is as follows. We present the mathematical model in Section 2. In Section 3, we present and study the mathematical model with control. We conclude in Section 4.

2 The Formulation of Mathematical Model

In this model, the human population is divided into four classes: the susceptible S_h, the exposed E_h, the infected I_h and cured R_h. The vector population (mosquitoes) is subdivided into two classes: susceptible vectors S_v and infected I_v. $\mu_h N_h$ is the dynamic recruitment of the human population. $\gamma_1 S_h$, $\gamma_1 E_h$, $\gamma_1 I_h$ and $\gamma_1 R_h$ are the number of susceptible, exposed, infected and cured individuals, respectively, that die naturally. $\dfrac{\beta I_v S_h}{N_h}$ is the proportions of susceptible humans that can encounter female Anopheles with a β rate. $\dfrac{\alpha_{E_h} E_h S_v}{N_h}$ and $\dfrac{\alpha_{E_h} I_h S_v}{N_h}$ are the respective proportions of Anopheles that bite exposed (E_h) and infected humans (I_h) that can infect them. γE_h is the total exposed population that manifests malaria disease at time t (exposed individuals who pass into the I_h class). θI_h is the set of sick humans who recover from malaria (humans who enter the R_h class). Λ_v is the recruitment of mosquitoes. μS_v and μI_v are the mosquitoes that die naturally, respectively, in classes S_v and I_v. The individual cured ($\alpha_1 R$) of malaria disease recontacts malaria disease in recruitment.

Remark 2.1 In our model, mosquitoes do not recover from malaria. Each person cured of malaria (R_h) is brought back into the susceptible population.

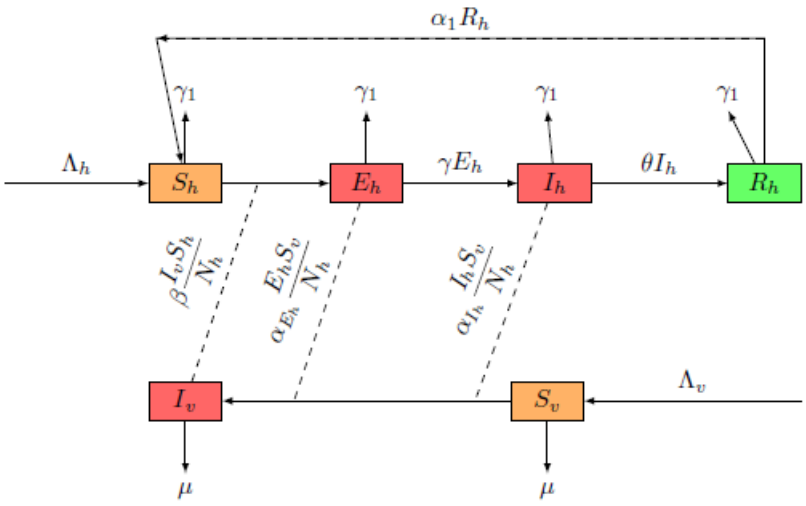


Figure 1: Transfer diagram, the black dashed arrows indicate the direction of the infection, the solid arrows represent the transition from one class to another.

The mathematical model without control is given

$$\begin{cases} \dot{S}_h = \Lambda_h + \alpha_1 R_h - \beta \dfrac{I_v S_h}{N_h} - \gamma_1 S_h, \\ \dot{E}_h = \beta \dfrac{I_v S_h}{N_h} - \gamma E_h - \gamma_1 E_h, \\ \dot{I}_h = \gamma E_h - \theta I_h - \gamma_1 I_h, \\ \dot{R}_h = \theta I_h - (\gamma_1 + \alpha_1) R_h, \\ \dot{S}_v = \Lambda_v - \alpha_{E_h} \dfrac{E_h S_v}{N_h} - \alpha_{I_h} \dfrac{I_h S_v}{N_h} - \mu S_v, \\ \dot{I}_v = \alpha_{E_h} \dfrac{E_h S_v}{N_h} + \alpha_{I_h} \dfrac{I_h S_v}{N_h} - \mu I_v \end{cases} \tag{1}$$

with the initial conditions

$$S_h(0) > 0, \quad S_v(0) > 0, \quad E_h(0) > 0, \quad I_h(0) > 0,$$

$$I_v(0) > 0, \quad R_h(0) > 0.$$

Total human population and the number of vectors are described by the following equations:

$$\dot{N}_h = \Lambda_h - \gamma_1 N_h(t) \tag{2}$$

and

$$\dot{N}_v = \Lambda_v - \mu N_v(t). \tag{3}$$

Symbols	Description	values	Sources
Λ_h	Constant recruitment rate for humans	5000	[20]
γ_1	Natural mortality rate of humans	0.00167	[20]
μ	Natural mortality rate of mosquitoes	0.01	estimate
Λ_v	Constant recruitment rate for mosquitoes	150	[20]
θ	Transfer rate of humans from I_h to R_h	0.3	estimate
α_{I_h}	Contact rate of susceptible mosquitoes with humans infected with malaria	0.05	estimate
α_{E_h}	Contact rate of susceptible mosquitoes with humans exposed to malaria	0.07	estimate
β	Contact rate of infected mosquitoes with susceptible humans	0.425	estimate
γ	Transition rate from E_h to I_h.	0.52	estimate

Table 1: The parameters of model (1).

Estimation of the total vector population at time t. Let us consider equations (3) and (2). The total vector population is estimated at time t by

$$N_v = \frac{\Lambda_v}{\mu} + \left(N_v(0) - \frac{\Lambda_v}{\mu}\right)\exp(-\mu t).$$

The total human population is estimated at time t by

$$N_h = \frac{\Lambda_h}{\gamma_1} + \left(N_h(0) - \frac{\Lambda_h}{\gamma_1}\right)\exp(-\gamma_1 t); \quad t \geq 0.$$

3 The Optimal Control Problem

In this section, we introduce two controls u_1 (prevention) and u_2 (treatment) into the model (1). Furthermore, we first prove the existence of two optimal controls u_1^*, u_2^* and then give the characterization of these two controls. So far, there is no preventive vaccine against malaria. In our study, we use the means of prevention other than vaccine.
The basic reproduction number R_0 is given by

$$R_0 = \sqrt{\frac{\beta\Lambda_v\gamma_1}{\Lambda_h\mu^2(\gamma+\gamma_1)}\left(\alpha_{E_h} + \frac{\alpha_{I_h}\gamma}{\theta+\gamma_1}\right)}.$$

3.1 Presentation of the problem

The controls u_1 and u_2 are defined as follows:

- $u_1(t) \in [0,1]$ is the control corresponding to the distribution of mosquito nets, preventive medication for children under five years and other means to prevent malaria. The rate of people sleeping under mosquito nets or protecting themselves against mosquitoes and/or preventing malaria is denoted by $u_1(t) \in [0,1]$ with $t \in [0, t_f]$. The ideal is to get the entire population to sleep under a mosquito net and to warn all children, in this case $u_1 = 1$. In reality, this is not possible, we seek to protect the maximum number of people ($u_1 = u_{1max}$).

- The second control $u_2(t) \in [0,1]$ represents the treatment of patients over the interval $[0; t_f]$. The control u_2 that we consider here can therefore represent the treatment of symptoms or the isolation of patients in hospitals to avoid possible new infection. If all patients are treated, then $u_2 = 1$.
  For all positive t, we unambiguously denote $u_i(t)$ simply by u_i for $i = 1, 2$.

By inserting the controls into the model (1), we get the following controlled equations:

$$\begin{cases} \dot{S}_h = \Lambda_h + \alpha_1 R_h - \beta(1-u_1)\dfrac{I_v S_h}{N_h} - \gamma_1 S_h, \\ \dot{E}_h = \beta(1-u_1)\dfrac{I_v S_h}{N_h} - \gamma E_h - \gamma_1 E_h, \\ \dot{I}_h = \gamma E_h - (\theta + u_2) I_h - \gamma_1 I_h, \\ \dot{R}_h = (\theta + u_2) I_h - \gamma_1 R_h, \\ \dot{S}_v = \Lambda_v - \alpha_{E_h}\dfrac{E_h S_v}{N_h} - \alpha_{I_h}\dfrac{I_h S_v}{N_h} - \mu S_v, \\ \dot{I}_v = \alpha_{E_h}\dfrac{E_h S_v}{N_h} + \alpha_{I_h}\dfrac{I_h S_v}{N_h} - \mu I_v \end{cases} \tag{4}$$

with the initial conditions

$$S_h(0) > 0, \quad S_v(0) > 0, \quad E_h(0) > 0, \quad I_h(0) > 0,$$
$$I_v(0) > 0, \quad R_h(0) > 0.$$

The basic reproduction number (R_0^c) of the model (4) is the number of cases generated by the primary infected individual under controls u_1 and u_2. This demonstrates that controls u_1 and u_2 play a role in combating malaria disease. We observe that if $u_1 = u_2 = 0$ (absence of all malaria control strategies), then we recover the same reproduction number R_0 as in the model (1) without control ($R_0^c = R_0$, if $u_1 = u_2 = 0$),

$$R_0^c = \sqrt{\frac{\beta \Lambda_v \gamma_1 (1-u_1)}{\Lambda_h \mu^2 (\gamma + \gamma_1)} \left(\alpha_{E_h} + \frac{\alpha_{I_h} \gamma}{\theta + u_2 + \gamma_1} \right)}.$$

Remark 3.1 The objective of these controls is to observe the effect of malaria treatments and the effect of preventive measures in the fight against malaria. Furthermore, we aim to propose strategies to minimize the infected population (I_h) while maximizing the recovered population (R_h) and the susceptible population (S_h).

3.2 Study of optimal control problem

In this section, we define the Hamiltonian associated with the control problem (4). Then we characterize the solutions of control problem (4) after proving their existence. Mathematically, for a fixed terminal time t_f , the problem is to minimize the functional objective J on $[0, t_f]$.

$$J(u_1, u_2) = \int_0^{t_f} \left(I_h(t) - S_h(t) - R_h(t) + \frac{A_1}{2} u_1^2(t) + \frac{A_2}{2} u_2^2(t) \right) dt. \tag{5}$$

The first terms represent the gain for the I_h that we wish to reduce. The constants A_1 and A_2 are positive, and correspond to the weights that regularize the control for prevention and treatment, respectively. As given in the literature, the costs are assumed to be quadratic functions. Indeed, costs are rarely linear, and are often presented as non -linear functions of control. Other types of functions exist in the literature [2, 4, 6, 10]. The most natural thing to do is to consider quadratic functions. These also allow us to make the analogy with the energy that is expanded here for all these measurements. Our objective is to limit the transmission of the disease by reducing the number of mosquitoes and infected humans.

We determine the optimal control (u_1^*, u_2^*) such that

$$J(u_1^*, u_2^*) = min\left\{J(u_1, u_2) : (u_1, u_2) \in \Gamma\right\}, \tag{6}$$

where

$$\Gamma = \left\{ \ (u_1, u_2), \begin{cases} u_i(t) \text{ is a continuous function by pieces on } [0,t_f] \\ a_i \leq u_i(t) \leq b_i \end{cases} \right\} \tag{7}$$

is the set of controls and a_i, b_i are constants belonging to [0;1], $i = 1, 2$. The optimal control problem is then solved when we determine $(u_1^*, u_2^*) \in \Gamma$ which minimizes the function (5).

Definition 3.1 (the Hamiltonian of the minimization problem) Pontryagin's maximum principle [12] converted (4), (5) and (6) into the problem of minimizing the Hamiltonian H defined by

$$H = -S_h - R_h + I_h + \frac{A_1}{2}u_1^2 + \frac{A_2}{2}u_2^2 + \sum_{i=1}^{6} \lambda_i f_i, \tag{8}$$

where

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{pmatrix} = \begin{pmatrix} \Lambda_h + \alpha_1 R_h - \beta(1-u_1)\dfrac{I_v S_h}{N_h} - \gamma_1 S_h \\ \beta(1-u_1)\dfrac{I_v S_h}{N_h} - \gamma E_h - \gamma_1 E_h \\ \gamma E_h - (\theta + u_2) I_h - \gamma_1 I_h \\ (\theta + u_2) I_h - (\gamma_1 + \alpha_1) R_h \\ \Lambda_v - \alpha_{E_h}\dfrac{E_h S_v}{N_h} - \alpha_{I_h}\dfrac{I_h S_v}{N_h} - \mu S_v \\ \alpha_{E_h}\dfrac{E_h S_v}{N_h} + \alpha_{I_h}\dfrac{I_h S_v}{N_h} - \mu I_v \end{pmatrix}$$

is the righ-hand side of the differential equation (1), the state variable and λ_i, $i = 1, ..., 6$, are the adjoint variables associated with their respective states.

Theorem 3.1 *Consider the optimal control problem* (4) *subject to* (5). *Then there exist an optimal pair of controls* (u_1^*, u_2^*) *and corresponding optimal states* $(S_h, E_h, I_h, R_h, S_v, I_v)$ *that minimize the objective function* $J(u_1, u_2)$ *over the set of admissible controls* Γ.

Proof. The existence of optimal control can be proved by using the results from [7] (see Theorem 2.1) and Fleming's results [3] (Theorem III.4.1), we must check the following conditions:

- the set of controls and solutions present is nonempty,
- the admissible set Γ is convex and closed,
- the vector field of the state system is bounded by a linear function of control,
- the objective function is convex,
- there exist constants $c_1, c_2 > 0$ such that the integrated part of the objective function is bounded by $c_1(|u_1|^2 + |u_2|^2)^{\frac{p}{2}} - c_2$.

(1) We verify these conditions thanks to a result of Lukes et al. [8], which assures the existence of solutions for the state system (1).

(2) The set Γ is convex and bounded by definition.

(3) The right-hand side of the state system (4) is bounded by a linear function in the state and control variables.

(4) The integrated part of the objective functional is

$$f^0(X, u_1, u_2) = I_h - S_h - R_h + \frac{A_1}{2}u_1^2 + \frac{A_2}{2}u_2^2.$$

The Hessian matrix of $f^0(X, u_1, u_2)$ is given by

$$M_{f^0} = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix},$$

$Spec(M_{f^0}) = \{A_1, A_2\} \subset \mathbb{R}_+^*$.
So, by using [1], f^0 is strictly convex over U.

(5) We have

$$\begin{aligned} f^0(X, u_1, u_2) &= I_h(t) - S_h(t) - R_h(t) + \frac{A_1}{2}u_1^2(t) + \frac{A_2}{2}u_2^2(t), \\ &= N_h - E_h - 2R_h - 2S_h + \frac{A_1}{2}u_1^2(t) + \frac{A_2}{2}u_2^2(t), \\ &\geq -E_h - 2R_h - 2S_h + \frac{A_1}{2}u_1^2(t) + \frac{A_2}{2}u_2^2(t), \\ &\geq \frac{1}{2}\min\{A_1, A_2\}\left(|u_1(t)|^2 + |u_2(t)|^2\right)^{k/2} - (E_h + 2R_h + 2S_h), \\ &\geq c_1\left(|u_1(t)|^2 + |u_2(t)|^2\right)^{\frac{k}{2}} - c_2, \end{aligned}$$

where $c_1 = \frac{1}{2}\min\{A_1, A_2\} > 0$, $c_2 = E_h + 2R_h + 2S_h$ and $k \geq 1$, so the last assertion is verified.

Since the state variables are bounded, we deduce the existence of an optimal control (u_1^*, u_2^*) which minimizes the objective function $J(u_1, u_2)$ in (5). □

We are now interested in the characterization of a control $u^* = (u_1^*, u_2^*)$, the solution of (7). Pose $Z = (S_h, E_h, I_h, R_h, S_v, I_v)$, $U = (u_1, u_2)$ and $L = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)$, being the adjoint variables. We define the Lagrangian associated with the problem (this one corresponds to the Hamiltonian increased by the penalties).

$$\begin{aligned}
\mathcal{L}(Z,U,L) = \quad & I_h - R_h - S_h + \frac{A_1}{2}u_1^2 + \frac{A_2}{2}u_2^2 \\
+ \quad & \lambda_1\left(\Lambda_h + \alpha_1 R_h - \beta(1-u_1)\frac{I_v S_h}{N_h} - \gamma_1 S_h\right) \\
+ \quad & \lambda_2\left(\beta(1-u_1)\frac{I_v S_h}{N_h} - \gamma E_h - \gamma_1 E_h\right) \\
+ \quad & \lambda_3\left(\gamma E_h - (\theta+u_2)I_h - \gamma_1 I_h\right) + \lambda_4\left((\theta+u_2)I_h - (\gamma_1+\alpha_1)R_h\right) \\
+ \quad & \lambda_5\left(\Lambda_v - \alpha_{E_h}\frac{E_h S_v}{N_h} - \alpha_{I_h}\frac{I_h S_v}{N_h} - \mu S_v\right) \\
+ \quad & \lambda_6\left(\alpha_{E_h}\frac{E_h S_v}{N_h} + \alpha_{I_h}\frac{I_h S_v}{N_h} - \mu I_v\right) - w_{11}(u_1 - a_1) - w_{12}(b_1 - u_1) \\
- \quad & w_{21}(u_2 - a_2) - w_{22}(b_1 - u_2),
\end{aligned} \tag{9}$$

where $w_{ij}(t) \geq 0, \quad i,j = 1,2$, are the penalty coefficients verifying

$$\begin{array}{c} w_{11}(u_1 - a_1) = w_{12}(b_1 - u_1) = 0 \quad \text{for optimal control} \quad u_1^* \text{ and} \\ w_{21}(u_2 - a_2) = w_{22}(b_2 - u_2) = 0 \quad \text{for optimal control} \quad u_2^*. \end{array} \tag{10}$$

Theorem 3.2 *Consider an optimal control $u^* = (u_1^*, u_2^*) \in \Gamma$ and corresponding states $X = (S_h,\ E_h,\ I_h,\ R_h,\ S_v,\ I_v)$ of system (4), there exist adjoint functions $(\lambda_i,\ i = 1, ..., 6)$ satisfying*

$$\begin{aligned}
\dot{\lambda}_1 &= -\left(-1 - \lambda_1\left((1-u_1)\frac{\beta I_v}{N_h} - \gamma_1\right) + \beta(1-u_1)\frac{I_v}{N_h}\lambda_2\right), \\
\dot{\lambda}_2 &= -\left(-(\gamma_1+\gamma)\lambda_2 + \gamma\lambda_3 - \frac{\alpha_{E_h}S_v}{N_h}\lambda_5 + \frac{\alpha_{E_h}S_v}{N_h}\lambda_6\right), \\
\dot{\lambda}_3 &= -\left(1 - \lambda_3(\theta + u_2 + \gamma_1) + \lambda_4(\theta+u_2) - \frac{\alpha_{I_h}S_v}{N_h}\lambda_5 + \frac{\alpha_{I_h}S_v}{N_h}\lambda_6\right), \\
\dot{\lambda}_4 &= -\left(\alpha_1\lambda_1 - (\gamma_1+\alpha_1)\lambda_4 - 1\right), \\
\dot{\lambda}_5 &= -\left(-\lambda_5\left(\mu + \frac{\alpha_{E_h}E_h}{N_h} + \frac{\alpha_{I_h}I_h}{N_h}\right) + \lambda_6\left(\frac{\alpha_{E_h}E_h}{N_h} + \frac{\alpha_{I_h}I_h}{N_h}\right)\right), \\
\dot{\lambda}_6 &= -\left(-\frac{\beta S_h}{N_h}\lambda_1 + \frac{\beta S_h}{N_h}\lambda_2 - \mu\lambda_6\right)
\end{aligned} \tag{11}$$

with the transversality conditions given by $\lambda_i(t_f) = 0, \quad i = 1, ..., 6$. Furthermore, the optimal controls are characterized by

$$\begin{aligned}
u_1^* &= \max\left\{a_1, \min\left\{b_1, \left(\frac{\lambda_2(t) - \lambda_1(t)}{A_1}\right)\beta\frac{S_h}{N_h}\right\}\right\}, \\
u_2^* &= \max\left\{a_2, \min\left\{b_2, \frac{(\lambda_3(t) - \lambda_4(t))}{A_2}I_h\right\}\right\}.
\end{aligned} \tag{12}$$

Proof. The differential equations for the adjoint variables are standard results from Pontryagin's maximum principle [17]. The right-hand sides of the differential equations can be easily computed. Let $w^* = (u_1^*, u_2^*)$ be the corresponding solution $X = (S_h, E_h, I_h, R_h, S_v, I_v)$ that minimizes $J(u_1, u_2)$ over Γ. By Pontryagin's maximum principle [17], there exist adjoint functions

$$p(t) = (\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t), \lambda_5(t), \lambda_6(t)), \quad t \in [0, t_f]),$$

verifying the following conditions:

$$\frac{dp(t)}{dt} = -\frac{\partial H}{\partial X}, \tag{13}$$

$$\frac{dX(t)}{dt} = \frac{\partial H}{\partial p}, \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial u_1} = \frac{\partial \mathcal{L}}{\partial u_2} = 0. \tag{15}$$

The condition (13) yields the system (11) and condition (14) yields the system (4). The optimality condition (15) gives the following system:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial u_1}\bigg|_{(u_1=u_1^*)} &= A_1 u_1^* + \lambda_1 \beta \frac{I_v S_h}{N_h} - \lambda_2 \beta \frac{I_v S_h}{N_h} - w_{11} + w_{12} = 0, \\ \frac{\partial \mathcal{L}}{\partial u_2}\bigg|_{(u_2=u_2^*)} &= A_2 u_2^* + \lambda_1 \beta \frac{I_v S_h}{N_h} - \lambda_2 \beta \frac{I_v S_h}{N_h} - w_{21} + w_{22} = 0. \end{aligned} \tag{16}$$

By solving (16) and using (10), we obtain the result (12) □

3.3 Numerical simulation

First, note that the optimality system is a problem with two boundary conditions. Indeed, the state system is solved in the direction with the initial conditions $X(0) = (100, 90, 70, 100, 60)$. The adjoint functions are solved in the opposite direction [5], with the transversality conditions $\lambda_i(t_f) = 0$, $i = 1, ..., 6$, where $t_f = 12$ months. The numerical simulations are obtained by using Python. The control curves in Figure

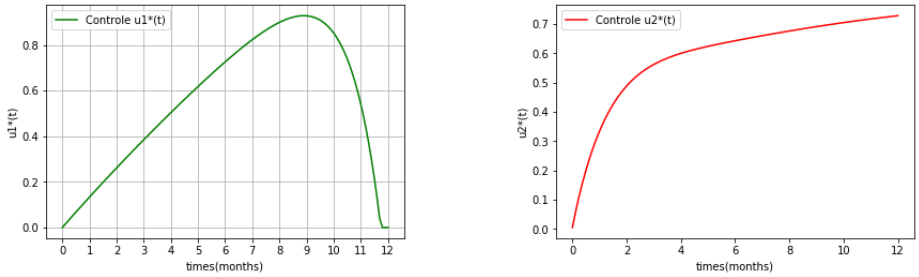


Figure 2: u_1 and u_2 control curves.

2 show the period of application for the controls u_1 and u_2. The curve of u_1 shows that the period of implementation for control u_1 are the first nine months of the year. The measurement control has no effect during the last quarter (October, November and

December) of the year. The best period for implementing preventive measures (u_1) is the month of June. Furthermore, the government's efforts should be focused on the month of June. The best time of the year for distributing mosquito nets, spraying public space and vaccinating the susceptible human population is June. For better prevention results, 50% of the population should be involved. The curve of (u_2) in Figure 2 shows that the control (u_2) of malaria patient care should be applied continuously throughout the year. For better results, over +70% of malaria patients should be supported by the government.

Figure 3: $S_h(t)$

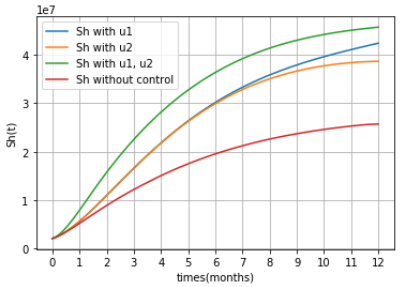


Figure 4: $E_h(t)$

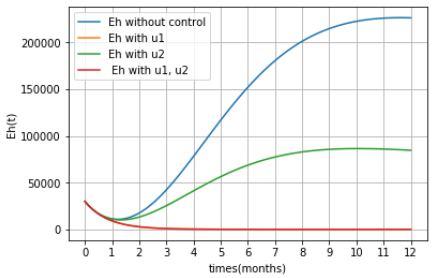


Figure 5: $I_h(t)$

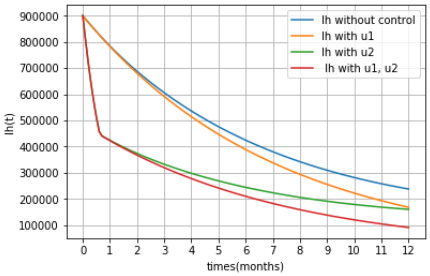


Figure 6: $R_h(t)$

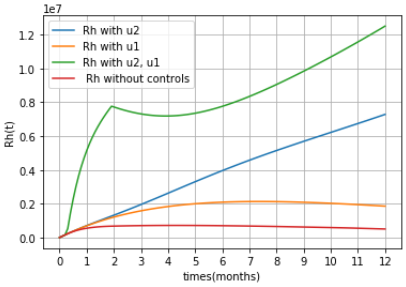


Figure 7: $S_v(t)$

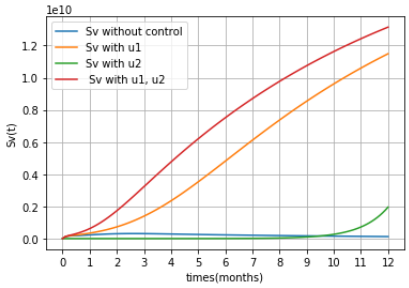
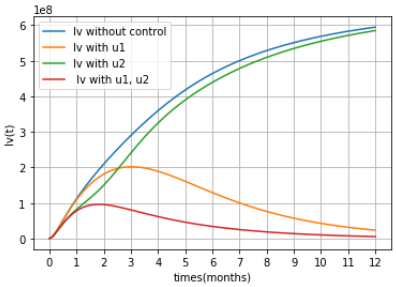


Figure 8: $I_v(t)$,



Population dynamics with different control aspects u_1 and u_2.

Comment

The curves of Figure 3 describe the dynamics of the susceptible human population (S_h) by using the treatment (u_2) and prevention (u_1) controls. This proves that treating only

malaria patients (control u_2) has a negligible effect in the fight against malaria. If we apply only treatment (control u_2), we observe a smaller effect on the susceptible population. Preventive measures are the best way to preserve the susceptible population (S_h) from malaria disease. Furthermore, if prevention and treatment are applied simultaneously, the susceptible population remains protected from malaria.

The curves of Figure 4 describe the dynamics of E_h by using u_1 and u_2 controls. This proves that treating malaria patients has a great effect in the exposed population (E_h). We have found that if all the controls are applied, the number of malaria exposed cases (E_h) falls and comes to zero after two months. We also note that if all the controls are applied, no individual is exposed to the disease after two month. Providing treatment is the most effective control strategy (control u_2) to be applied to the exposed population. Moreover, if we simultaneously apply treatment (u_2) and preventive measures (u_1) to the exposed population, then the number of the exposed individuals decreases to zero after two months.

The curves in Figure 5 show the dynamics of I_h by using u_1 and u_2 controls. This proves that treating malaria patients and using preventive measures have a great effect in the fight against malaria. We have found that if all the controls are applied, the number of malaria infected (I_h) cases falls and becomes zero after three months. We also note that if all the controls are applied, no individual is infected with the disease after three months. To observe the effect of controls on the infected human population (I_h), we need to simultaneously apply both treatment (control u_2) and preventive measures (control u_1).

The curves of Figure 6 show the dynamics of R_h by using u_1 and u_2 controls. This proves that treating malaria patients (control u_2) has a great effect in the fight against malaria. We have found that if all the controls are applied, the number of malaria cured persons increases. Treatment (control u_2) has a significant effect on the recovery of malaria patients. Preventive measures (control u_1) have a less considerable effect on recovered individuals (R_h).

The curves of Figure 7 describe the dynamics of S_v by using u_1 and u_2 controls. This proves that treating (u_2) malaria patients and using preventive measures (u_1) have a great effect in the fight against malaria. We have found that if all the controls are applied, the number of susceptible vectors increases. If preventive measures (u_1) are applied, susceptible mosquitoes cannot become infected from the infected human population (I_h, E_h).

The curves of Figure 8 describe the dynamics of (I_v) by using u_1 and u_2 controls. This proves that treating malaria patients (I_h) has a great effect in the evolution of infected vectors (I_v). We have found that if all the controls are applied, the number of infected falls and becomes zero after ten months.

4 Conclusion

In this paper, we have developed a SEIRS malaria mathematical model. We considered model (1), in which we introduced two controls, u_1 and u_2. The control u_1 in our model shows that the best way to prevent malaria disease is to prevent contact between susceptible people (S_h) and infected vectors (I_v) by using impregnated mosquito nets and spraying public spaces. If all these preventive measures are followed, malaria will disappear after a long period of application of measures. The government should support the susceptible population (by distributing mosquito nets, spraying public areas and

distributing preventive pharmaceutical products to children under the age of five years). The control u_1 in our model proves that, in order to fight against malaria, it is necessary to develop diverse strategies:

- Subsidize the access of malaria patients to hospitals or take care of all malaria patients or take care of 90% of malaria patients. Otherwise, through the infected people who are not treated for malaria, many Anopheles become infected and then spread the malaria disease.
- Reduce the population's exposure to malaria through awareness raising, free distribution of impregnated mosquito nets, access to preventive care for children under five years of age, free access to anti-malarial treatment, malaria testing, etc. We can also, in the framework of the fight against malaria, take into account the mortality rate of mosquitoes, that is to say, increase the mortality of mosquitoes by killing infected Anopheles around the population (by using insecticides or other means).

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