



# Well-Posedness of Boundary Control System of Nonlinear Chemical Reaction

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**Abstract:** We consider mixed boundary control systems inducted by non-isothermal axial dispersion chemical tubular reactors. We characterize the well-posedness, approximate controllability, and transfer function of the mixed boundary control system. There exists an admissible operator control such that the mixed boundary control system is well-posed. By constructing an extended space, the classical solution can be obtained explicitly. Sufficient conditions for approximate controllability of the mixed boundary control system are identified by the eigenvalues and eigenvectors of the Sturm-Liouville operator using an equivalence in the extended space. A proper transfer function of the associated boundary control system equipped with an output can be constructed. The proper transfer function shows that the associated boundary control system is well-posed.

**Keywords:** *chemical tubular reactor; boundary control system; well-posed; approximately controllable; Sturm-Liouville operator; transfer function.*

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## 1 Introduction

A non-isothermal reaction is a reaction in the process taking place at a temperature varying from one point to another. Dynamical analysis of non-isothermal tubular reactors has been studied massively recently, see [1–6]. The dynamics of non-isothermal axial dispersion chemical tubular reactors are described by nonlinear partial differential equations (PDEs) derived from mass and energy balance equations. The nonlinearities are usually located in the kinetic terms due to the Arrhenius law for non-isothermal reactors. In particular, let  $L$  be the length of the tubular reactor and if the reaction is

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