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The Advection-Diffusion-Reaction Equation: A Numerical Approach Using a Combination of Approximation Techniques

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Abstract: In this paper, we propose a numerical technique for solving the advection-diffusion-reaction equation. The presented approach is based on coupling two numerical methods to address the problem posed with the Robin boundary conditions perturbed with a small parameter ε, in terms of spatial and temporal variables. We start by employing a Galerkin method for the spatial descretization, using a compact basis of the Legendre polynomials to derive a system of ordinary differential equations. This system is then solved using a Crank-Nicolson scheme, with the temporal domain uniformly discretized. The obtained numerical results demonstrate the effectiveness of the proposed numerical method and the convergence of the approximate solution to the analytic solution of the classical problem with the homogeneous Dirichlet boundary conditions when ε approaches zero. This makes it particularly useful for approximating the solutions of such problems of partial differential equations appearing in reaction-diffusion systems, where the explicit solution is unknown under various types of boundary conditions.

Keywords: *advection-diffusion-reaction equation; Galerkin method; Legendre polynomials; Robin boundary conditions; Crank-Nicolson scheme.*

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