



On Stability and Convergence of a Fractional Convection Reaction-Diffusion Model

OumKeltoum Benhamouda¹, Mohamed El-Hadi Smakdji², Ammar Derbazi³, Allaoua Boudjedour^{2*}, Mohamed Dalah², Khaled Zennir⁴ and Abdelwahab Zarour²

¹ *Faculté des Sciences et de Technologie, Université USTHB, Alger, Algeria.*

² *University Constantine 1 "FM", FSE, Department of Mathematics, MAM Laboratory.*

³ *Université Mohamed El Bachir El Ibrahimi de Bordj Bou Arréridj, Algeria.*

⁴ *Department of Mathematics, College of Science, Qassim University, Saudi Arabia.*

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Abstract: In this paper, we study the one-dimensional space fractional convection-diffusion problem by using a finite difference method. First, we give the mathematical model of our first initial boundary value problem. In the second step, we develop the discretization of the mathematical model and the development of the scheme for the fractional order type linear diffusion equation. For this scheme, the stability as well as convergence are studied via the Fourier method. At the end, the solutions of some numerical examples are discussed and represented graphically using Matlab. Finally, error analysis shows that the algorithm is convergent.

Keywords: *finite difference schemes; fractional derivative; Caputo fractional derivative; stability; convergence.*

Mathematics Subject Classification (2010): N65M06, 65M12, 35R11, 65L12.

1 Introduction

In this study, we consider the one-dimensional space fractional convection-diffusion problem of Caputo type of order $0 < \alpha < 1$, which is used in the modeling of chemical convection-diffusion. Several techniques for numerical resolution of this type of equation have been studied by several authors [1] - [5]. In most of these techniques, either the solutions of the integer order differential equation versions of the given problem or the fractional differential equations with initial conditions and boundary conditions are used.

* Corresponding author: <mailto:aboudjedour@yahoo.com>