



# Inducing Chaos through Timescales in a Three-Species Food Chain Model

Khadidja Daas\* and Nasreddine Hamri

*Laboratory of Mathematics and Their Interactions,  
Departement of Mathematics, Boussouf Abdelhafid University Center,  
Mila, Algeria*

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**Abstract:** Over time, considerable attention has been devoted to understanding the complex dynamics of simplified ecosystems containing three trophic levels, revealing that complex behaviors can arise from a simple hierarchy between prey, predator, and top predator. In this study, the extension of a three-species food chain model with a Crowley-Martin-type functional response was examined by introducing discrete timescales to study its impact on system dynamics at various trophic levels. Changes in species abundance were analyzed across three different timescales: slow, fast, and intermediate. The presence of a homoclinic orbit in the subsystem (prey-predator) suggests the existence of period-doubling cascades that eventually lead to chaos in the entire system (prey-predator-top-predator). This study underscores the importance of ecological modeling and trophic interactions in understanding the diverse and intricate dynamics of ecosystems, thus highlighting the significance of research in this domain.

**Keywords:** *three-level trophic systems; time scales; chaos.*

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\* Corresponding author: <mailto:khadidja.daas@centre-univ-mila.dz>