



Boundedness in Nonlinear Oscillatory Systems over a Given Time Interval

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Abstract: This paper considers three types of perturbed motion equations with a stable linear (nonlinear) approximation. New sufficient conditions are established for the boundedness of motion on a finite interval with respect to a given Lyapunov function. The conditions are obtained on the basis of the direct Lyapunov method and the method of integral inequalities.

Keywords: *equations of perturbed motion; stable approximation; boundedness with respect to given function.*

Mathematics Subject Classification (2010): 34D40, 34D20, 70K40.

1 Introduction

Non-autonomous systems of equations, applicable in nonlinear mechanics [1], are studied by various methods (see [2–7] and the bibliography therein). The Lyapunov function method [8], combined with the method of integral inequalities (see [1, 9]), allows establishing new conditions for the boundedness of motion over a specified time interval. This paper is structured as follows.

Section 2 discusses a system of two scalar equations with nonlinear stable approximation. Definitions of motion boundedness with respect to a positive definite function are provided.

In Section 3, an estimation of the Lyapunov function is established.

Section 4 presents conditions for the boundedness of motion with respect to a positive definite function.

Section 5 addresses the problem of boundedness of solutions to equation systems with autonomous stable approximation.

In Section 6, conditions for boundedness are established in the case of stability of non-autonomous linear approximation.

Section 7 provides conditions for the boundedness of solutions over a specified time interval for perturbed motion equations in the normal Cauchy form.

The concluding section offers comments on the obtained results.

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