



## $\Phi$ -Hilfer Proportional Fractional Differential System: Uniqueness and Stability Result

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**Abstract:** This work derives uniqueness and Ulam-Hyers (UH) stability results for a coupled system with the  $\Phi$ -Hilfer proportional fractional derivative. We first construct a new Bielecki-type vector-valued norm in weighted space and then use the fixed-point argument to achieve a new uniqueness criterion. Secondly, the UH and the generalized Ulam-Hyers (GUH) stability is established. To verify the obtained result, an example is provided.

**Keywords:**  $\Phi$ -Hilfer proportional fractional derivative; uniqueness; Ulam-Hyers stability; fixed point theorem.

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