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Simulation of Tsunami Wave Propagation Using the Finite Difference Method for Disaster Early Warning System

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Abstract: Indonesia is a country that is the meeting point of three active tectonic plates in the world and it is located in the Pacific Ring of Fire, which makes it prone to disasters. One of the disasters that may occur is a tsunami that appears due to an earthquake or an active volcano that erupts in the sea. Tsunami waves are one of the fluid problems, tsunami waves modelling can be solved using the Finite Difference Method. This research aims to model the propagation of tsunami waves. This modelling is important for early detection of a tsunami formed and can calculate its impact. This research uses the finite difference as a method for constructing discretization equations to find a numerical solution for tsunami propagation modelling. From the modelling results, the highest tsunami wave was 0.005 km at the 0th km, then the wave height decreased at the 30^{th} and 60^{th} seconds by 0.0034552 km and 0.0031604 km at the 3rd and 9th km, and at the 90th second, the waves rose to 0.0036794 km at the 14^{th} km.

Keywords: *finite difference method; modeling; tsunami wave propagation; tsunami.*

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1 Introduction

As a maritime country, Indonesia has an ocean area larger than the land area, it is located in Southeast Asia and is flanked by two Asian countries and the Australian continent. Indonesia holds the title of a country that has the second longest coastline in the world, consisting mostly of islands, with a length of 99,093 km [1]. Indonesia is a country that is the meeting place of three active tectonic plates of the world, these three active tectonic plates make Indonesia a country that has many active mountains and is the epicenter of earthquakes. The factors above make Indonesia an earthquake-prone area [2–4]. The Indo-Australian Ocean Plate in the south, the Pacific Ocean Plate in the east, the Eurasian Plate in the north, the Philippine Sea Plate added, are the world's active tectonic plates with dynamic movements and mutual urges between the plates that make the territory of Indonesia prone to tsunami disasters [5].

A very long water wave commonly called a tsunami is a large displacement of sea water from the sea to the mainland. The cause of a tsunami is due to an external force such as an underwater earthquake, volcanic eruption, or a meteor so that the waves radiate outward in all directions [6]. More than 10% of tsunamis are caused by underwater and aerial landslides with the resulting water waves reaching a maximum run-up of possibly 10-15 km from the tsunami source along the coastline and 5% are caused by volcanic eruptions [7]. One of the biggest tsunamis in Indonesia was in Aceh in 2004, which was caused by an earthquake measuring 9.1 on the Richter Scale. As a result of the tsunami, approximately 130,013 people lost their lives and 37,066 were reported missing [8]. In addition, the aftermath of the Aceh tsunami had an impact on people's livelihoods. A lot of damage was experienced, in particular in the construction sector, the fishing sector, the agricultural sector because the people living around the coast depend on livelihood strategies such as fishing, gardening, which utilize the surrounding nature [9].

A tsunami is a collection of tidal water waves that move quickly towards the mainland, it usually occurs on the coast, especially in Indonesia. Tsunamis are caused by active volcanic eruptions at sea or by earthquakes [10]. Indonesia often experiences tsunamis, the largest tsunamis that have ever occurred in Indonesia were in Aceh, 2004 [11, 12], in Pangandaran, 2006, and in Mentawai, 2010. A tsunami in Indonesia in the last five years occurres in Palu and the Sunda Strait in 2018. Many sectors of life are affected if a tsunami occurres, including the Construction Sector, Fisheries Sector, Agriculture Sector, it has impacts on the coastline and non-structural impacts.

Research on computational fluid dynamics has been carried our intensively, including the Airway Pressure Valve [13], Solitary Wave [14]. To reduce the impact of losses due to the tsunami disaster, a prediction can be made by simulating the propagation of tsunami waves using the finite difference method equation. The finite difference method is a method that is often used to solve partial differential equations [15, 16]. This method has the advantage that it is easy to understand and simple [17]. The finite difference method is also easy to use to solve physical problems that have regular geometric shapes, for example square domains in two dimensions, cubic in three dimensions and intervals in one dimension [18].

The finite difference method is widely used for research, including the analysis of the convergence of the finite difference method in approaching the diffusion equation, it is found that the numerical solution in the finite difference method converges to the analytical solution of the diffusion equation. Research on simulations of tsunami waves includes the application of the implicit Crank-Nicolson finite difference method to determine the case of 2D advection-diffusion in the distribution of pollutants in the water by simulating the concentration distribution pattern in the Kapuas River, showing a more distant distribution and a very small RMSE value from several time variations so that the results obtained are close to the expected ones [19]. Then, in the simulation study of the impact of the barrier on the tsunami wave using the finite difference method, it was found that the shallow water equation used can identify tsunami waves with the construction of the barrier [20].

Extensive research has been conducted on simulating tsunami wave propagation. A study by Nisa [2] highlighted the significance of modeling tsunami wave propagation, particularly in light of the substantial losses incurred following a tsunami. Another study by Chasanah [10] aimed to model the propagation of tsunami waves caused by the eruption of Mount Anak Krakatau and explored multiple scenarios to mitigate tsunami wave height in Jakarta Bay.

Based on the explanations and previous studies that have been described, research will be conducted on the simulation of tsunami wave propagation using the finite difference method. From the simulation, one can determine the wave height and the distance of the wave propagation from the initial point of the wave formation and the effect of seabed material on the wave propagation. The purpose of this study is to create a model for tsunami wave propagation so that it can be used to predict the impacts that occur due to the tsunami disaster. In addition, the height and propagation distance of the tsunami waves obtained in this study can be used as information for disaster mitigation, so that people, especially those living in coastal areas, can evacuate earlier to higher ground.

2 Literature Review

2.1 Tsunami

A tsunami is a movement of sea water caused by a seaquake, an underwater volcanic eruption, or a meteor strike. The speed of tsunami waves can reach 500-1000 km/hour [1]. Tsunamis often occur after an earthquake, but not always. A tsunami occurs when an earthquake has a strength of more than 6.5 on the Richter Scale and occurs out at sea at a depth of 30 km. The height of the tsunami waves will increase as they get closer to the shore due to the reduction in sea depth [21].

Tsunami waves can have a dangerous impact on humans, from material losses to loss of life. This disaster can occur at any time so it is almost unavoidable [22]. The tsunami disaster in the Sunda Strait in 2018, provides a clear picture because it occurred suddenly, caused by an underwater landslide due to the eruption of Mount Anak Krakatau. The resulting impact was very large, 437 fatalities were recorded [23]. Therefore, it is very necessary to understand the implementation of disaster mitigation so that it does not cause a very large impact.

2.2 Tsunami wave propagation modeling

A tsunami results from the movement of tectonic plates with the strength of an earthquake, causing vibrations. Even though they occur in very deep waters, tsunamis have a wavelength between 2 peaks of more than 100 km on the high seas and the time difference between peaks is estimated to be 10 minutes to 1 hour [24]. It is necessary to model the propagation of tsunami waves to be able to predict disasters that occur. Modeling can be done using shallow water theory or the so-called shallow water equation (SWE) which is a simple differential equation for water waves and is close to the actual behavior of ocean waves [2].

The shallow water equations used are based on the derivation of the law of conservation of mass and conservation of momentum in the fluid. The following are the equations for shallow water waves expressed through the three-dimensional equations [2]:

1. Mass Conservation Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{1}$$

2. Momentum Equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial \rho}{\partial y} + \frac{1}{\partial \rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) = 0, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{1}{\rho}\frac{\partial \rho}{\partial y} + \frac{1}{\partial \rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) = 0, \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial \rho}{\partial y} + \frac{1}{\partial \rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right) = 0. \tag{4}$$

x and y are the horizontal axes, while z is the vertical axes. It shows the height of the water surface from a balanced state at a position (x, y), τ_{ij} is a shear or tangential stress. In the direction of the axis, the momentum equation has a surface $\rho = 0$ under dynamic conditions, which results in hydrostatic pressure. In tsunami wave propagation, the mass and momentum conservation equations are used with dynamic and kinematic boundary conditions. The kinematic boundary conditions are divided into two kinds, namely the kinematics on the free surface and at the bottom of the channel. The kinematic boundary conditions of the free surface are stated as follows:

$$\frac{\partial \eta}{\partial t} = -u\frac{\partial \eta}{\partial x} - v\frac{\partial \eta}{\partial y} + w. \tag{5}$$

The kinematic boundary conditions at the bottom of the channel are stated as follows.

$$u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + w = 0. \tag{6}$$

In uniform flow, in general, the friction on the seabed can be expressed as follows [2]:

$$\frac{\tau_x}{\rho} = \frac{1}{2g}\frac{f}{D^2}M\sqrt{M^2 + N^2}, \tag{7}$$

$$\frac{\tau_y}{\rho} = \frac{1}{2g}\frac{f}{D^2}N\sqrt{M^2 + N^2}. \tag{8}$$

The equations (7) and (8) represent the coefficient of friction. The relationship between the Manning roughness value and the coefficient can be expressed in the following equations:

$$n = \sqrt{\frac{fD^{\frac{1}{3}}}{2g}}, \tag{9}$$

$$f = \frac{2gn^2}{D^{\frac{1}{3}}}. \tag{10}$$

Material type	n value
Pure cement, fine metal	0.010
Rubble	0.017
Fine soil	0.018
Natural channel in good condition	0.025
Natural channel with seabed rocks and plants	0.035
Natural channel with bad condition	0.060

Table 1: Sea Level Condition.

By substituting equation (9) and equation (10) into equation (7) and equation (8), we get equation (11) and (12) as follows:

$$\frac{\tau_x}{\rho} = \frac{n^2}{D^{\frac{7}{3}}} M\sqrt{M^2+N^2}, \tag{11}$$

$$\frac{\tau_y}{\rho} = \frac{n^2}{D^{\frac{7}{3}}} N\sqrt{M^2+N^2}. \tag{12}$$

3 Research Methods

This study aims to determine the height and distance of the tsunami wave propagation from the center of the vibration by using the finite difference method with an explicit scheme.

The finite difference method is a numerical method for solving problems of physical phenomena or technical problems. This method can be used in converting partial differential equations into linear equations [23]. The equation for the finite difference method is obtained using the Taylor series from equations (13) and (14) of one variable function around x as follows [24]:

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{f''(x)}{2!}\Delta x^2 + \cdots, \tag{13}$$

$$f(x-\Delta x) = f(x) - f'(x)\Delta x + \frac{f''(x)}{2!}\Delta x^2 + \cdots. \tag{14}$$

The finite difference method is divided into two types, namely explicit and implicit. The explicit finite difference method uses an approximation of the forward difference $u(x, t+\Delta t)$ around the point t and the center difference $u(x+\Delta x, t)$ and around the point x. Meanwhile, the implicit finite difference method uses backward difference an approximation $u(x,t)$ of the approx $t+\Delta t$ and the center $u(x+\Delta x, t+\Delta t)$ and $u(x-\Delta x, t+\Delta t)$ approx x [25].

In modeling the propagation of tsunami waves using an explicit finite difference method with a forward difference approach of more than one independent variable, the equation [26]

$$\frac{\partial f}{\partial x} \approx \frac{f(x_{i+1}, t_j) - f(x_i, t_j)}{\Delta x} \tag{15}$$

can be simplified to

$$\frac{\partial f}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}.$$

4 Result and Discussion

By deriving the shallow water formula from the mass and momentum conservation equations, and by spatial discretization using the finite difference method using equations (1) to (6), we obtain

$$\eta_{i,j}^{n+1} = rM_{i,j}^{n} - rM_{i+1,j}^{n} + rN_{i,j}^{n} - rN_{i,j+1}^{n} + \eta_{i,j}^{n}, \tag{16}$$

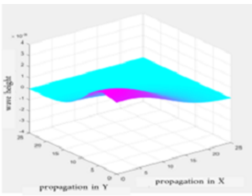
$$M_{i,j}^{n+1} = r\rho\eta_{i,j}^{n} - r\rho\eta_{i+1,j}^{n} + M_{i,j}^{n}, \tag{17}$$

$$N_{i,j}^{n+1} = r\rho\eta_{i,j}^{n} - r\rho\eta_{i+1,j}^{n} + N_{i,j}^{n}. \tag{18}$$

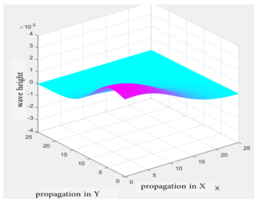
Equations (16)-(18) are the result of discretization using the finite difference method and the equations (16)-(18) are a numerical solution that will be used in the tsunami wave propagation simulation. The simulation was carried out by several trials using the approximate values of $\Delta x, \Delta y$ and Δt which varied.

4.1 Trial 1

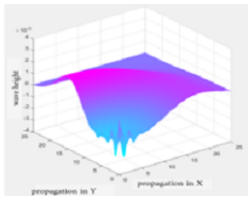
The value of Δx and Δy is 1 and Δt is equal to 0.009. The amplitude value used is 0.004 km, the wavelength is 4 km, and the distance from the center of the wave formation is 25 km with the seabed material used being fine cement or fine metal. The following is the result of the simulation in Trial 1 of the propagation of tsunami waves with the Manning roughness value.



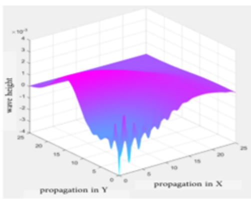
(a) Tsunami Propagation in 0 second



(b) Tsunami Propagation in 30 seconds



(c) Tsunami Propagation in 60 seconds



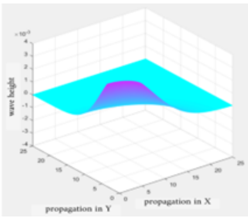
(d) Tsunami Propagation in 90 seconds

Figure 1: Simulation in the first trial of 3D Tsunami Propagation.

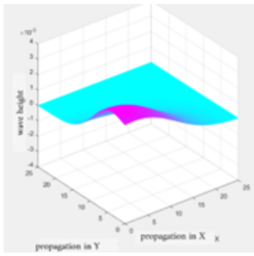
The tsunami wave is formed as high as 4 m at 0 second and is located at 0 km and moves in the positive x-axis direction on the coast which is as high as 1.3175×10^{-8} m. At the 30^{th} second, the initial wave height decreased to 2.6214 m with the highest wave crest located at 4 km from the initial point of the wave formation and the wave condition on the shore as high as 2.696×10^{-7} m. Then, at the 60^{th} second, the highest wave crest is located at 9 km from the initial point of wave formation with a wave height of 30008 m and a wave condition on the shore as high as 1.0302×10^{-5} m. At the 90^{th} second, the highest wave crest is located at 13 km from the initial point of wave formation with a wave height of 4.2654 m and a wave condition on the shore as high as $3,6287 \times 10^{-4} m$.

4.2 Trial 2

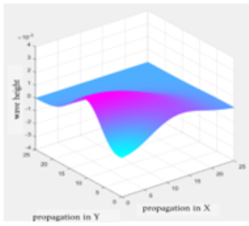
The value of Δ_x and Δ_y is 1 and Δ_t is 0.01. The amplitude value used is 0.005 km, the wavelength is 5 km, and the distance from the center of the wave formation is 25 km with the seabed material used being fine cement or fine metal. The following is the result of the simulation in Trial 2 of the propagation of tsunami waves with the Manning roughness value.



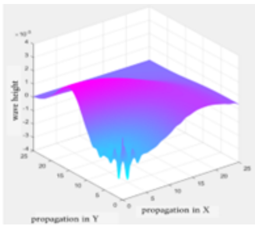
(a) Tsunami Propagation in 0 second



(b) Tsunami Propagation in 30 seconds



(c) Tsunami Propagation in 60 seconds



(d) Tsunami Propagation in 90 seconds

Figure 2: Simulation in the second trial of 3D Tsunami Propagation.

The wave is formed as high as 5 m at the 0^{th} second and is located at 0 km from the starting point of the wave formation and the state of the wave on the beach is as high as $1.8633 \times 10^{-5} m$. Then the wave decreased to 3.4552 m at the 30^{th} second with the highest wave crest located at 3 km from the initial point of the wave formation and the wave condition on the shore only as high as 1.7682×10^{-4} m. At the 60^{th} second, the wave height decreased to 3.1604 m with the highest wave crest located at 9 km from the starting point of the wave formation and the condition of the waves on the shore as high as $2.9034 \times 10^{-3} m$. At the 90^{th} second, the highest wave crest is located at 14 km from the starting point of the wave formation with a wave height of 3.6794 m and the condition of the waves on the shore only as high as 3.9867×10^{-2} km.

According to the simulation results obtained through several trials, the height of the tsunami waves moving to the coast has increased from the center of the wave formation. The highest wave that moves to the shore, according to the tests carried out in the third trial, in 90 seconds is 0.28291 meter, with the initial wave height of 6 meters. However, from the simulations carried out, the material that makes up the seabed does not affect the formation of tsunami waves because the waves produced have the same shape and height.

From the simulation results, three values are obtained, namely the highest wave height, the distance the wave travels, and the wave height when it reaches the beach. The results of this study are different from those of the research that also uses the theory of shallow water equations in shallow seas but with a case study in Manado Bay [25]. The results obtained in this study were the height of the tsunami waves that occurred after the earthquake on the coast of Manado Bay , it was about 1.5-3 m with a period of 20-24 minutes. The wave height can reach 3 m due to the conditions of the Manado beach which is in the form of a bay so that tsunami wave amplification can occur [26].

Then the results of the tsunami simulation in another study using the continuity and momentum equations with a case study in the city of Palu obtained the initial height of the tsunami wave reaching 1.5 m and spreading to the mouth of the Palu bay with a height of 1.6 m. The maximum wave height obtained is close to the actual height of 3.65 m. The time taken by the tsunami waves to reach the mouth of the bay, which is 62 km away, is 10 minutes. While the tsunami waves from the mouth of the bay to the city of Palu, which is 32 km away, run for 12 minutes [27].

In contrast, the research on tsunamis used for disaster mitigation at Manakarra Beach, stated that the waves started hitting the beach 39 minutes after the earthquake. The maximum wave height occurred at the 50^{th} minute was a height of 8,306 m. At the highest tide, the height of the Manakarra Beach pier is 2 m. So, the waves that came at the 50^{th} minute were 6,306 m high. Therefore, there are about 30 minutes to evacuate to higher ground [28]. Therefore, for further research, it is expected to use data in areas that have been affected by the tsunami so that the accuracy of the simulation in this study can be known. The modeling of tsunami wave propagation uses the shallow water equation where the equation is based on the derivation of the law of conservation of mass and conservation of momentum [29]. The shallow water equation can be used in the presence of dynamic and kinematic boundary conditions. The equation used is derived from the formula with known initial conditions. The method used in the modelling propagation of tsunami waves is an explicit finite difference method, where the equations used will be discretized without any friction on the seabed. The discretization results using the finite difference method provide a numerical solution or simulating the propagation of tsunami waves.

5 Conclusion

The shallow water equation can be used in the presence of dynamic and kinematic boundary conditions. The method used for modelling the propagation of tsunami waves is an explicit finite difference method, where the equations used are discretized without any friction on the seabed. The discretization results using the finite difference method provide a numerical solution for simulating the propagation of tsunami waves. According to the simulation results obtained through several trials, the height of the tsunami waves moving to the coast increases from the center of the wave formation. The highest wave that moves to the shore, according to the tests carried out in the third trial, in 90 seconds, is 0.28291 meters with the initial wave being 6 meters. However, from the simulations carried out, the material that makes up the seabed does not affect the formation of tsunami waves because the waves produced have the same shape and height. Three values are obtained, namely the highest wave height, the distance the wave travels, and the wave height when it reaches the beach.

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