



## On Neumann Systems with Singularity Applied in Quenching Phenomena in Museilack Spaces

A. Elouardani<sup>1</sup>, A. Aberqi<sup>2</sup> and M. Elmassoudi<sup>1\*</sup>

<sup>1</sup> Faculty of Sciences Dhar El Mahraz, Sidi Mohamed Ben Abdellah University, Fez, Morocco.

<sup>2</sup> National School of Applied Sciences, Sidi Mohamed Ben Abdellah University, Fez, Morocco.

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**Abstract:** In this study, by the uniform control of the singularity and energy estimates, we establish the existence of solutions for the reaction-diffusion model with singularity due to Quenching phenomena in the boundary.

$$\begin{cases} \frac{\partial u_1}{\partial t} - \operatorname{div}\left(a(x, t, \nabla u_1)\right) = f_1(t, x)h_\gamma(u_1, u_2) & \text{in } Q_T, \\ \frac{\partial u_2}{\partial t} - \operatorname{div}\left(a(x, t, \nabla u_2)\right) = f_2(t, x)h_\gamma(u_1, u_2) & \text{in } Q_T, \end{cases} \quad (1)$$

where the operator  $A(u) = \operatorname{div}\left(a(x, t, \nabla u_i)\right)$  is a generalized Leray-Lions operator defined on the inhomogeneous Musielak-Orlicz spaces (the vector field  $a(x, t, \nabla u_i)$  has a growth prescribed by a generalized N-function).

**Keywords:** *quenching phenomena, Museilack-Orlicz spaces, local singularity, non-linear dynamical systems.*

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\* Corresponding author: <mailto:elmassoudi09@gmail.com>