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Estimation of Amphibious Aircraft Trajectory Using Particle Filter and Square Root Ensemble Kalman Filter

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Abstract: An amphibious aircraft is a type of aircraft able to take off and land on water. Amphibious aircraft play an important role in air transportation, especially in areas having a lot of water. Amphibious aircraft perform many functions including passenger and freight transportation, medical rescue, tourism and maritime surveillance. Amphibious aircraft require navigation and guidance systems so as to steadily follow a predetermined trajectory. Several navigation guidance system algorithms in the aerospace field can be used, one of which is for amphibious aircraft trajectory estimation. The accuracy of aircraft position estimation is very important to ensure that the amphibious aircraft follows a predetermined trajectory. For this, the navigation and guidance system algorithm requires modeling the motion system of the amphibious aircraft. In this study, the Particle Filter and Square Root Ensemble Kalman Filter methods were used to estimate the trajectory to be followed by the amphibious aircraft. The main purpose of the study was to compare the performance of the two methods to find out which one proved to have a higher accuracy. Based on the simulation results, both methods had a high accuracy, that is, the Particle Filter method had an accuracy of about 98.2%, and the EnKF-SR method had an accuracy of 99.3%.

Keywords: *amphibious aircraft; trajectory estimation; particle filter method; EnKF-SR method.*

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1 Introduction

An amphibious aircraft is a type of aircraft designed to operate both in the air and on the water. Amphibious aircraft have multiple functions including passenger and freight transportation, medical rescue, tourism, and maritime surveillance. The amphibious aircraft ability is to take off and to land both on airstrips and on water surfaces such as wide lakes or rivers [1]. Amphibious aircraft have an important role in the field of air transportation and defence of the Homeland, as well as strategic purposes such as military operations or humanitarian assistance in emergency situations. They enable fast and targeted delivery in hard-to-reach places [2].

In addition to defence and military purpose, amphibious aircraft are needed in remote and outermost areas of the Republic of Indonesia because these remote areas are often difficult to reach by land transport, and the airstrips there are inadequate. With amphibious aircraft, logistics and supplies can be transported directly to the site via the large and accessible watersways. So, in view of the many benefits provided by amphibious aircraft, especially for logistics distribution, the technology related to amphibious aircraft needs to be developed and researched so that it can be utilized by a wider community [1].

The development of amphibious aircraft, which can take off and land on either land or water, requires advanced and reliable navigation systems to ensure operational safety and efficiency. When landing on water or land, the navigation system helps the aircraft to land safely in a pre-selected area, avoiding obstacles such as rocks or floating objects. A good navigation and control system helps the aircraft avoid obstacles such as islets or uneven terrain in the water, and can increase efficiency and reduce risk. Navigation and control systems on seaplanes play a crucial role in the operational safety and efficiency as well as their mission effectiveness [3]. Navigation system is a system that directs the position of an amphibious aircraft to follow a predetermined trajectory with a high degree of accuracy [4]. This amphibious aircraft can operate in the air and land on the water. Several navigation guidance methods in the field of robotics can also be used, one of which is to estimate the position of an amphibious aircraft. The precision and accuracy of the aircraft trajectory estimation is very important to ensure the amphibious aircraft follows the specified trajectory accurately [5].

One of the algorithms that gives a small error in estimation is the Particle Filter and Square Root Ensemble Kalman Filter, in which the EnK-SR method is a development of the EnKF method frequently used for motion and position estimation in the field of robotics, for example, for ASV [6], AUV trajectory [7], and arm robot motion [8]. Given that the EnKF method is a very reliable estimation method, in this study, two methods are used, that is, the EnKF-SR and Particle Filter methods for estimating the trajectory of unmanned amphibious aircraft. And, in this study, the focus is on comparing the numerical simulation results of the two methods, the EnKF-SR and Particle Filter methods by generating 300 and 500 ensembles.

2 Amphibious Aircraft Model

In this study, we used an amphibious aircraft model with the ability to operate in the shallow water areas of Indonesia. The forces acting on an unmanned seaplane can be categorized into the aircraft mass, hydrodynamic forces, aerodynamic forces, and engine thrust as shown in Figure 1.

We consider the Earth coordinate system $X_e; Y_e; Z_e$, then the aircraft body coordinate

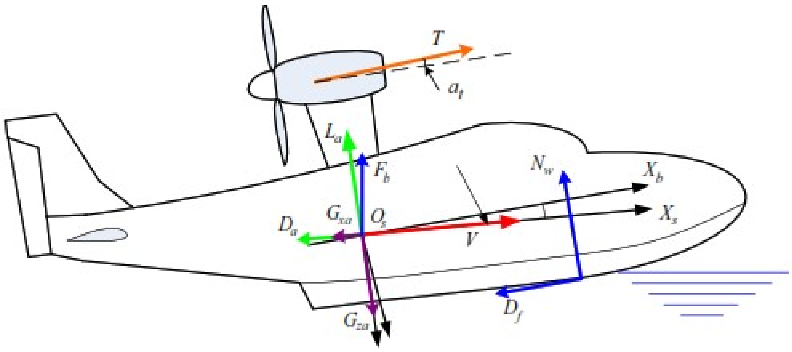

Figure 1: Forces working on amphibious aircraft [9].

system $X_b; Y_b; Z_b$ and the steady-translation coordinate system $X_s; Y_s; Z_s$.

The nonlinear longitudinal dynamic motion model of unmanned seaplanes is represented by the following equations [9]:

$$m\dot{V} = T\cos(\alpha + \alpha_t) - D_a - N_w \sin\alpha - D_f \cos\alpha + G_{xa}, \quad (1)$$
$$mV\dot{\alpha} = mVq - T\sin(\alpha + \alpha_t) - L_a - N_w \cos\alpha + D_f \sin\alpha + G_{za}, \quad (2)$$
$$I_y\dot{q} = M_a + M_w + M_T, \quad (3)$$
$$\dot{\theta} = q \quad (4)$$
$$\dot{x}_g = u\cos\theta + w\sin\theta, \quad (5)$$
$$\dot{z}_g = -u\sin\theta + w\cos\theta. \quad (6)$$

3 Particle Filter Algorithm

The Particle Filter algorithm is shown in Figure 2 [10].

4 Ensemble Kalman Filter Square Root Algorithm

The EnKF-SR algorithm, see Figure 3, is applied to the modeling of the amphibious aircraft motion system [11].

5 Result Simulation

In this study, the researchers simulated two methods at once, that is, the Particle Filter and EnKF-SR methods, and compared the accuracy achieved by the two methods. The study compared the accuracy of the two methods. The EnKF-SR method used a combination of 300 and 500 ensembles. The algorithms in Figures 2 and 3 were implemented on the motion system equations of the unmanned amphibious aircraft. The simulation results, after applying the Particle Filter and EnKF-SR algorithms to the unmanned amphibian motion system modeling, were obtained as shown in Figures 4-7.

α	Angle of attack	m	Amphibious aircraft mass
θ	Angle of pitch	D_a	Airodynamic drag force
V	Velocity	G_{xa}	gravity along X_s
α_t	Angle between engine force and X_b	G_{za}	gravity along Z_s
M_a	Total pitching moment of Air	X_g	aircraft position along X_e
M_w	Total pitching moment of Air	Z_g	aircraft position along Z_e
M_T	Total pitching moment of Engine	u	velocity component of aircraft velocity along X_b
T	Engine thrust	w	velocity component of aircraft along Z_b
N_w	Normal directional water pressure at the bottom of the aircraft	q	Pitch angular rate
D_f	Frictional force of water along the bottom of the aircraft	I_y	moment of Inertia of amphibious aircraft against Y_b
L_a	Aerodynamic lifting force	$\dot{\theta}$	Increase/decrease in angle of pitch
$\dot{V}$	Increase/decrease in velocity	$\dot{\alpha}$	Increase/decrease in angle of attack

Table 1: Description of amphibious aircraft model [4].

In Figure 4 a and b, it can be seen that the red line is the real value of the horizontal position determined to fly at a certain distance from the place of the amphibious aircraft so that the amphibious aircraft flies as high as 1180 m from the surface and advances about 2500 meters from the original place, so the position will always advance to 2500 m at the 100th iteration either by the EnKF-SR method (blue line) or the Particle Filter one (green line). Figure 4a shows the results of estimating the horizontal position of the unmanned amphibious aircraft. Figure 4b represents the position estimation results for the height reached by the amphibious aircraft. In Figure 4a and 4b, it can also be seen that the difference in distance or error between the EnKF-SR and Particle Filter methods with real values is quite small with a fairly high level of accuracy, this can be seen in Table 2. The resulting error for horizontal estimation is around 0.046% and the error in altitude estimation is around 0.2%.

Figure 5 represents an unmanned amphibious aircraft taking off and flying at an altitude of 1200 meters. It can be seen that the red line in Figure 4 is the real data of the horizontal position and altitude predetermined to be passed by the amphibious aircraft flying at a certain height and distance. The resulting error for the motion in the XY plane is about 0.253% for the EnKF-SR method and 0.254% for the Particle Filter method. In this case, the height of the unmanned amphibious aircraft represents the maximum height that can be passed by the amphibious aircraft. In this case, this simulation is only made for flying amphibious aircraft and not for landing on the ground

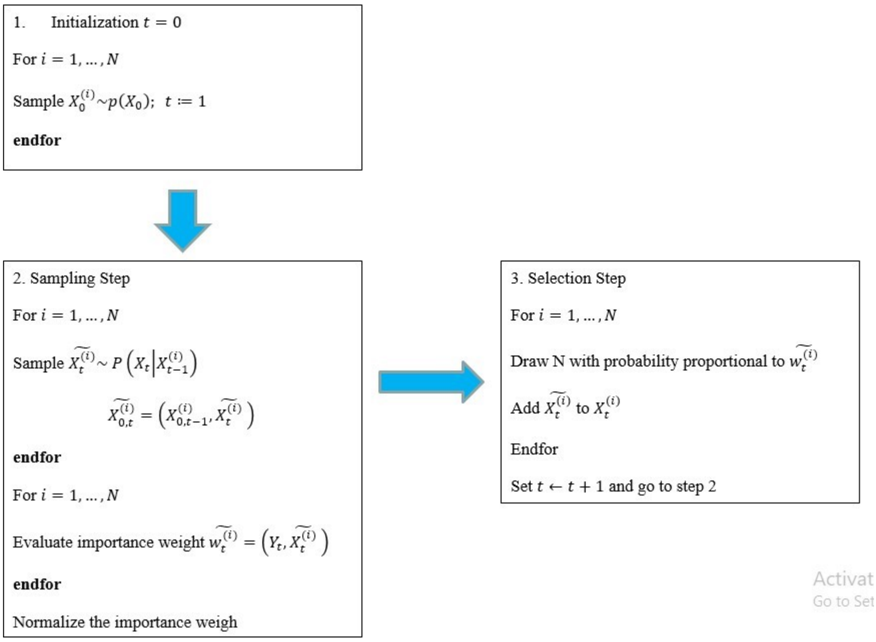

Figure 2: Particle Filter Algorithm

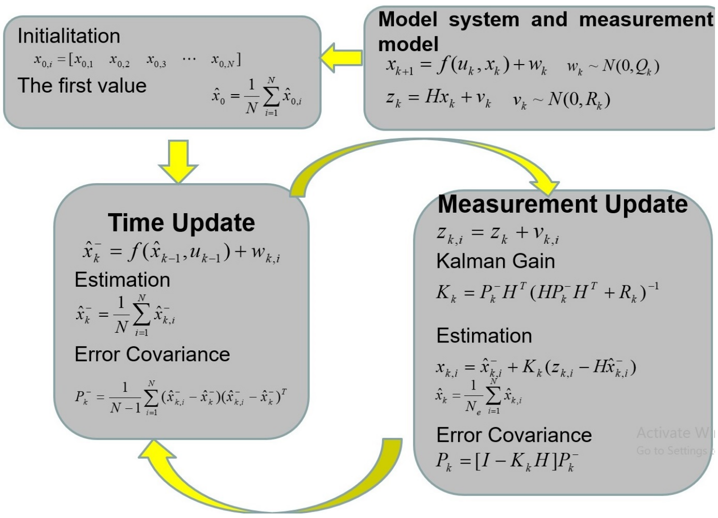

Figure 3: Square Root Ensemble Kalman Filter Algorithm.

again. Figure 4a shows a better accuracy than Figure 4b with 300 ensembles. In Figure 5, the amphibious aircraft follows the specified trajectory from the start of takeoff and flies at a certain height. Table 2 also shows that the EnKF-SR method has a smaller error of 0.001% than the Particle Filter method.

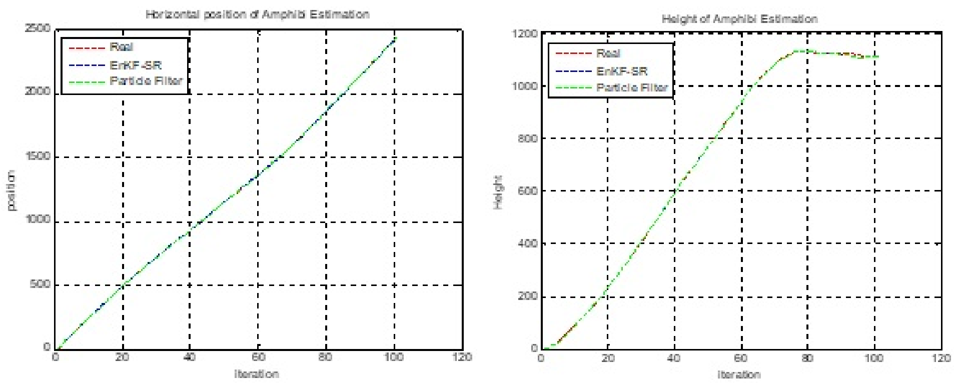

Figure 4: Numerical Simulation Results by the Particle Filter and EnKF-SR methods (500 ensembles), a) estimation of horizontal position of amphibious aircraft and b) estimation of altitude of amphibious aircraft.

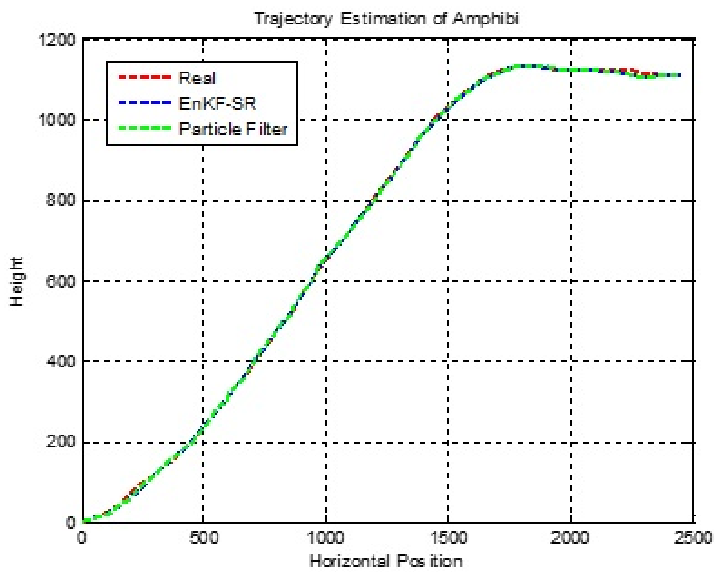

Figure 5: Amphibious aircraft position estimation by the EnKF-SR and Particle Filter methods with 500 ensembles.

In Figure 6 a and b, it can be seen that the red line is the real value of the horizontal position determined to fly at a certain distance from the place of the amphibious aircraft so that the amphibious aircraft flies as high as 1400 m from the surface and advances about 2000 meters from the original place, so the position will always advance to 2000 m at the 100th iteration either by the EnKF-SR method (blue colored line) or by the Particle Filter one (green colored line). In Figure 6a and 6b, it can also be seen that the difference in distance or error between the EnKF-SR and Particle Filter real values is quite small with a fairly high level of accuracy, as seen in Table 2. The resulting error

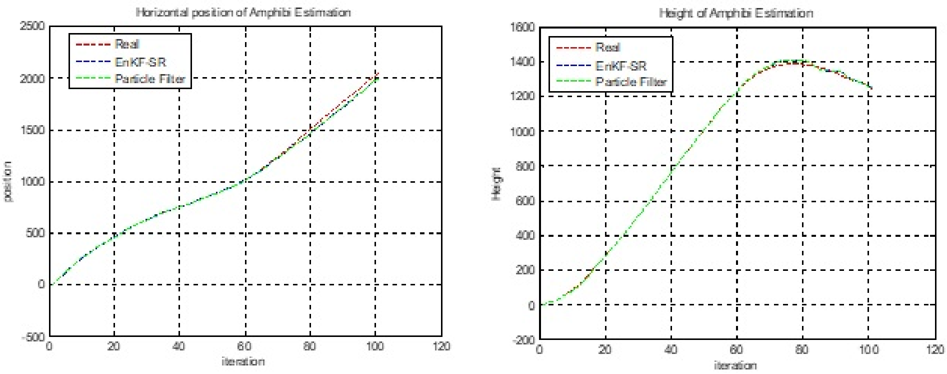

Figure 6: Numerical Simulation Results by the Particle Filter and EnKF-SR methods (300 ensembles), a) estimation of horizontal position of amphibious aircraft and b) estimation of altitude of amphibious aircraft.

for the horizontal estimation is around 0.095%, and the error in altitude estimation is around 0.5%.

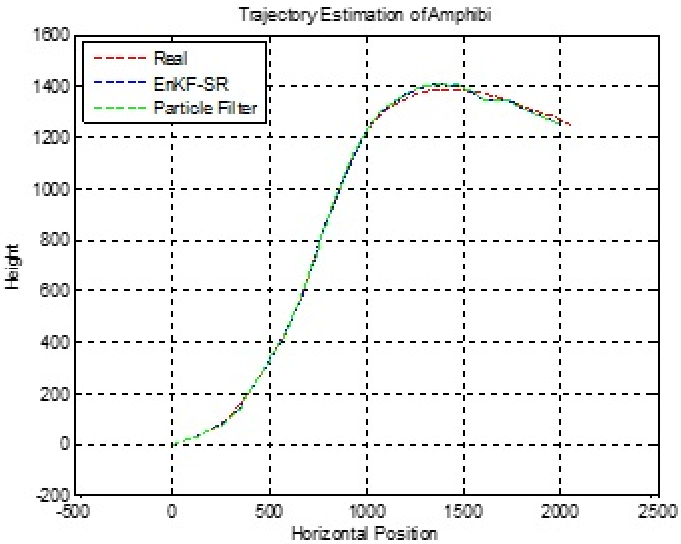

Figure 7: Amphibious aircraft position estimation by the EnKF-SR and Particle Filter methods with 300 ensembles.

Figure 7 represents an unmanned amphibious aircraft taking off and flying at an altitude of 1400 meters and traveling about 2 kilometers. It can be seen that the red line in Figure 4 is the real data of the horizontal position and altitude predetermined to be passed by the amphibious aircraft flying at a certain altitude and distance. The resulting error for the motion in the XY plane is about 0.522% for the EnKF-SR method and 0.523% for the Particle Filter method. In this case, the height of the unmanned amphibious aircraft represents the maximum height that can be passed by the amphibious aircraft, also, in this case, this simulation is only made for the flying amphibious aircraft and not for landing on the ground again. Figure 6a has better accuracy than Figure 6b when using 300 ensembles. In Figure 7, the amphibious aircraft follows the specified trajectory from the start of takeoff and flying at a certain height. Table 2 also shows that the EnKF-SR method has a smaller error of 0.001% than the Particle Filter method.

	EnKF with 300 ensembles	Particle Filter	EnKF with 500 ensembles	Particle Filter
RMSE XY Motion	8.3604	8.3742	3.0422	3.0575
Error XY Motion	0,522%	0,523%	0,253%	0,254%
Time Simulation	3,125 s	2,546 s	7,046 s	5,187 s

Table 2: Comparison of amphibious aircreft trajectory estimation error using the Particle Filter and EnKF-SR methods.

Overall, based on the simulation results listed in Table 2, some of these simulation results can be compared to each other. As regards the comparison of the accuracy of the methods, the EnKF-SR method has a higher accuracy than the Particle Filter one, but it has a longer simulation time. If we consider the comparison in terms of the number of ensembles, then generating 500 ensembles has a higher accuracy than 300 ensembles. Overall, the EnKF-SR method by generating 500 ensembles has the highest accuracy. In this case, both estimation methods, the Particle filter and EnKF-SR ones, have an accuracy of above 98%, so the estimation methods can be effectively used to estimate the motion of unmanned amphibious aircraft.

6 Conclusion

Based on the results of analyzing the two simulations presented in Figures 4-7, it can be concluded that the EnKF-SR method by generating 500 ensembles has the highest accuracy. If we consider the comparison in terms of the number of ensembles, then 500 ensembles produced a better accuracy than 300 ensembles. In this case, both estimation methods, the EnKF-SR and the Particle Filter ones, have an accuracy of above 98%, so the estimation methods can be effectively used to estimate the motion of unmanned amphibious aircraft.

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