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General Stability Result for a Nonlinear Viscoelastic Wave Equation With Boundary Dissipation

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Abstract: In this paper, we consider a model of a dynamic viscoelastic wave equation with a nonlinear source and boundary dissipation. Our fundamental goal is to establish the general decay rates of the energy solutions under a class of generality of the relaxation function $g : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the inequality $g'(t) \leq -H(g(t))$ for all $t \geq 0$, where H is a function satisfying some specific properties. This work extends the previous works with a viscoelastic wave equation and improves earlier results in the literature.

Keywords: *viscoelastic wave equation; relaxation function; general decay; convexity.*

Mathematics Subject Classification (2020): 35B37, 93D15, 93D20, 74D05, 35L55.

1 Introduction

In this paper, we are concerned with the following nonlinear viscoelastic wave equations:

$$\begin{cases} u_{tt} - k_0 \Delta u + \int_0^t g(t-s)\, div(a(x) \nabla u(s)) ds + b(x) u_t = |u|^{p-2} u & \text{in } \Omega \times \mathbb{R}^+, \\ k_0 \frac{\partial u}{\partial \nu} - \int_0^t g(t-s) (a(x) \nabla u(s)) \nu ds + h(u_t) = 0 & \text{on } \Gamma_1 \times \mathbb{R}^+, \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), & x \in \Omega, \\ u = 0 & \text{on } \Gamma_0 \times \mathbb{R}^+, \end{cases} \tag{1}$$

where $k_0 > 0$, and Ω is a bounded domain in $\mathbb{R}^n$ $(n \geq 1)$ with a smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$. Hence Γ_0 and Γ_1 are closed and disjoint with $mes(\Gamma_0) > 0$ and ν is the unit outward normal to Γ. $b : \Omega \to \mathbb{R}^+$ is a function, and

$$2 < p \leq \frac{2n}{(n-2)}, \quad n \geq 3, \tag{2}$$

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